

Forward Kinematic Modeling and Motion Behavior Analysis of a Four-Wheel Swerve Drive Mobile Robot

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Abstract - This study develops and validates a kinematic model for a four-wheel swerve (FWS) drive mobile robot during combined translational and rotational motion, selecting point P at the center of the front end of the robot as the trajectory tracking point. The robot consists of four independently driven and steered modules, enabling simultaneous adjustment of the rolling velocity and orientation angle of each wheel. Based on planar rigid-body motion principles and pure rolling conditions, the kinematic relationships among the required velocity at P, the body twist of the robot at its geometric center O, and the states of the four modules are established. The inverse kinematic model determines the linear velocity, steering orientation angle, and angular velocity of each wheel from the motion requirements at P; meanwhile, the forward kinematic model utilizes the Jacobian matrix and a least-squares solution to reconstruct the body motion of the robot and the trajectory of P. Numerical simulation results in MATLAB demonstrate that the four modules generate distinct yet stable velocities and orientation angles, consistent with the geometric configuration and rigid-body motion constraints. The deviation between the required twist and the reconstructed twist is negligible (10^{-17}), confirming numerical consistency between the inverse and forward kinematics. Notably, point P closely tracks the reference trajectory throughout the simulation, yielding a final position error of approximately 2.88×10^{-4} m after 5 s. The results confirm the validity of the proposed model and establish a foundation for developing trajectory tracking control for swerve robots during coupled motions.

Keywords- Swerve drive robot; forward kinematics; inverse kinematics; coupled translational-rotational motion; trajectory tracking.

I. INTRODUCTION

Wheeled mobile robots are undergoing rapid development for applications in logistics, surveillance, agriculture, inspection and maintenance, and autonomous transportation due to their high energy efficiency, relatively compact structure, and strong adaptability to diverse operating environments. Among omnidirectional mobility architectures, the swerve drive configuration is particularly noteworthy because each wheel module integrates two independently controllable motions: a rolling motion that generates tangential velocity and a steering motion around the vertical axis to orient the wheel. When four modules are independently arranged, the robot can produce three planar motion components-longitudinal translation, lateral translation, and rotation about the vertical axis, while utilizing standard rubber tires instead of the passive rollers found in Omni or Mecanum wheels. This characteristic enables

the robot to achieve omnidirectional maneuverability, rapid orientation changes, in-place rotation, crab walking, and superior traction on outdoor surfaces [1]. Tagliavini et al. demonstrated that swerve configurations can be described as a robotic system composed of multiple independently steerable locomotion units, thereby broadening their operational scope across logistics, precision agriculture, healthcare, inspection, and maintenance [2]. For four-wheel independently steered robots, the hardware architecture can be designed with four steering mechanisms and four drive mechanisms, thereby enhancing overall flexibility and pose adjustment capabilities [3]; studies on agricultural robots also show that four-wheel steering mechanisms can support multiple motion modes and adapt better to diverse operational conditions [4]. Recently, Lee and Joe developed an outdoor 4WIS robot employing four independent drive-steer assemblies, highlighting the potential of this architecture for navigating uneven terrain and executing high-maneuverability tasks [5].

In recent years, research on swerve drives and four-wheel independently steered robots has progressively shifted from mechanical mechanism design toward kinematic modeling, wheel velocity-steering angle coordination, and motion control. Purbowaskito and Hsu constructed a kinematic model for a 4WIS robot and utilized it as the foundation for Lyapunov-stability-based motion control [6], while Li et al. established kinematic and steering geometry models for a four-wheel independently steered robot to develop an improved Pure Pursuit algorithm [7]. Chen et al. investigated an independent four-wheel control strategy aimed at distributing appropriate steering states among wheels [8], whereas Jeong and Yim developed an integrated model predictive control (MPC) algorithm to allocate control actions to independent steering and drive actuators [9]. Liu et al. further exploited the maneuverability of 4WIS robots for high-speed trajectory tracking, demonstrating a strong coupling among the motion model, steering angle, and robot velocity state [10]. Ding et al. treated the redundantly actuated four-wheel independently steered robot as a non-linear over-actuated system, categorizing motion modes into independent-wheel steering, differential steering, and Ackermann steering, and subsequently employed MPC to resolve velocity distribution [11]. Shi et al. formulated geometric relationships for 4WID-4WIS systems for trajectory planning and position estimation [12], while Jeong and Yim simultaneously investigated trajectory tracking capability and

lateral stability for 4WIS systems under low-friction conditions [13]. Zhang et al. focused on oblique driving for 4WIS vehicles and developed a trajectory tracking scheme tailored to this motion mode [14], whereas Bae and Lee investigated a four-wheel steering mechanism alongside an adaptive steering mode switching algorithm to reduce the turning radius [15]. Regarding true swerve configurations, a 2023 study directly developed and controlled a four-module swerve robot based on a kinematic model where each module was independently controlled in drive velocity and wheel direction [16]. Tagliavini et al. subsequently proposed a more general kinematic framework for swerve robots with two or more modules, noting that many previous studies primarily established models for specific robot configurations [2]. At an advanced control level, Caran et al. researched mobile robots with four independently steered and actuated wheels and formulated optimal trajectory tracking algorithms [17], Auh et al. developed model predictive contouring control for 4WIS-4WID systems [18], while Chu et al. utilized the instantaneous center of rotation (ICR) as a geometric parameter to maintain consistency among wheels during large-curvature turning [19]. Recent studies have continued to expand integrated kinematic models for multiple motion modes of 4WIS robots [20], simultaneously coordinate body path and orientation [21], and handle steering angle coordination mismatches through geometric constraints [22]. The latest trends also incorporate the kinematic-geometric features of 4WIS into reinforcement learning for outdoor environments [5] and apply ICR constraints to large-curvature trajectories [23].

This study proposes a vector- and Jacobian matrix-based forward kinematic model for a four-wheel swerve drive robot, wherein the velocity states of the four modules are mapped into the robot body motion vector using the pseudoinverse solution of the over-actuated kinematic system. On this basis, the proposed model is investigated via numerical simulations for coupled translational-rotational motion modes; additionally, a sensitivity analysis is conducted to quantify the impact of module velocity and orientation deviations on robot velocity, heading direction, and trajectory. Through this, the paper aims to elucidate the relationship between the local states of the four swerve modules and the overall kinematic behavior of the robot body, establishing a theoretical foundation for subsequent research on motion control and trajectory tracking.

II. SWERVE ROBOT MODELING

2.1 Kinematics model of Swerve Robot

The studied swerve robot consists of four modules S_1 , S_2 , S_3 , and S_4 symmetrically arranged at the four corners of a rectangular robot frame with wheelbase length L and track width W , where $L > W$, as shown in Figure 1.

To describe the motion of the robot, two coordinate frames are established: an inertial frame $\{G\}$ fixed to the environment and a body frame $\{B\}$ attached to the robot, with its origin O located at the geometric center of the four modules. The x_B axis is selected along the heading direction of the robot, the y_B axis points to the left, and the z_B axis is perpendicular to the motion plane according to the right-hand rule. The pose of the robot in

frame $\{G\}$ is defined by vector $\mathbf{q} = [X \ Y \ \psi]^T$, where X and Y are the global coordinates of center O , and ψ is the heading angle of the robot measured counterclockwise from x_G to x_B .

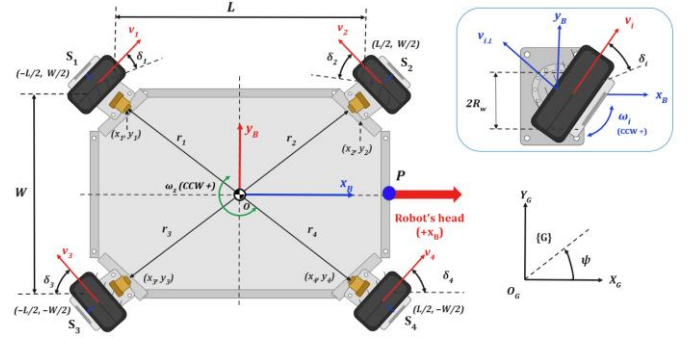


Figure 1: Kinematic model of the 4-wheel swerve drive mobile robot

Correspondingly, the instantaneous body velocity twist in frame $\{B\}$ is represented by $\xi_B = [v_x \ v_y \ \omega_z]^T$, where v_x and v_y are the longitudinal and lateral translational velocities, and ω_z is the angular velocity about the z_B axis. The coordinates of the four modules in the body frame $\{B\}$ are (x_i, y_i) , $i = 1, 2, 3, 4$, as given in Eq. (1); therefore, the position vector from the robot center to the i th module is denoted as $\mathbf{r}_i = [x_i \ y_i]^T$. Each module has the capability to independently adjust the steering angle δ_i and the wheel angular velocity ω_i ; with the wheel radius R_w , the linear velocity $v_i = R_w \omega_i$ is determined along the wheel's rolling direction and perpendicular to the wheel's rotation axis. Based on these conventions, the geometric configuration of the robot is uniquely defined by:

$$\begin{aligned} S_1: & \left(-\frac{L}{2}, \frac{W}{2}\right), \quad S_2: \left(\frac{L}{2}, \frac{W}{2}\right) \\ S_3: & \left(-\frac{L}{2}, -\frac{W}{2}\right), \quad S_4: \left(\frac{L}{2}, -\frac{W}{2}\right) \end{aligned} \quad (1)$$

In addition to the geometric center O , this study introduces a feature point P located at the exact midpoint of the robot body's front edge. Since the x_B -axis is conventionally defined as the robot's heading direction, P lies on the x_B -axis at a distance $a = L/2$ from O , and its position vector in the body frame is denoted as $\mathbf{r}_P = [a \ 0]^T$. The feature point P does not replace the center O in constructing the fundamental kinematic model; instead, it serves as a trajectory tracking point for combined motions. This selection is particularly significant when $\omega_z \neq 0$, as the trajectories of O and P do not coincide in this case. The global coordinates of P are determined from the robot's pose by:

$$\begin{aligned} X_P &= X + a \cos \psi \\ Y_P &= Y + a \sin \psi \end{aligned} \quad (2)$$

As a result, the position vectors $\mathbf{r}_i$, the feature point P , the steering angles δ_i , and the wheel velocities v_i form the geometric foundation to establish the relationship between the motion state of each individual module and the overall motion v_x , v_y , ω_z of the robot body, while also allowing the model to be extended to the trajectory tracking problem at a functional point on the robot body.

2.2. Parameters and Velocity Model of Swerve Module

It should be noted that the kinematic model is constructed under the assumptions that the robot body acts as a planar rigid body, wheels undergo pure rolling without slipping, wheel and chassis deformations are neglected, and all four swerve modules can independently adjust steering angle and wheel rolling speed. These assumptions make it possible to focus on the geometric–kinematic relationship between the body motion and the state of each module, without yet considering the effects of forces, moments, wheel elasticity, or actuator dynamics.

Consider module i with coordinates (x_i, y_i) and position vector $\mathbf{r}_i$ from center O . According to the velocity relationship of a rigid body, the velocity at each module is the sum of the translational velocity of the robot center and the velocity component generated by the rotational motion around the center O :

$$\mathbf{v}_i = \mathbf{v}_O + \omega_z \mathbf{k} \times \mathbf{r}_i \quad (3)$$

with $\mathbf{v}_O = [v_x \ v_y]^T$, $\mathbf{r}_i = [x_i \ y_i]^T$, and $\mathbf{k}$ the unit vector along z_B . Therefore, the two velocity components of the module in the body frame are $v_{ix} = v_x - \omega_z y_i$ and $v_{iy} = v_y + \omega_z x_i$, respectively. On the other hand, under the assumption of pure rolling without slipping, the velocity vector at the module must lie along the wheel's rolling direction, which is perpendicular to the wheel's rotation axis. If δ_i is the steering angle of the module, measured counterclockwise from the x_B -axis to the wheel's rolling direction, then the unit vector along the rolling direction is represented by $\mathbf{e}_i = [\cos\delta_i \ \sin\delta_i]^T$. Therefore, $\mathbf{v}_i = v_i \mathbf{e}_i$, where v_i is the wheel's linear velocity. From these two velocity representations, the fundamental kinematic relationship between the state of the i -th module and the motion of the robot body is determined by:

$$\begin{aligned} v_i \cos\delta_i &= v_x - \omega_z y_i, \\ v_i \sin\delta_i &= v_y + \omega_z x_i, \quad i = 1, 2, 3, 4 \end{aligned} \quad (4)$$

Equation (4) shows that the velocity of each module depends on the translational velocities v_x , v_y , the angular velocity ω_z of the robot body, and the geometric position (x_i, y_i) of the module relative to the center O . This is the foundational kinematic relationship used to establish both the forward kinematics (FK) and inverse kinematics (IK) problems. In the case where $\omega_z = 0$, the four modules share the same velocity vector, and the robot executes a pure translational motion; conversely, when $v_x = v_y = 0$, the velocity at each module is solely generated by the rotational motion of the robot body and is tangent to the circle centered at O passing through the corresponding module. In the general case where $\omega_z \neq 0$ and $(v_x, v_y) \neq (0, 0)$, the motion of each module is a combined result of both translational and rotational components, thereby allowing the robot to simultaneously perform longitudinal, lateral, and heading orientation changes.

For point P , since $\mathbf{r}_P = [a \ 0]^T$, applying the same rigid body velocity relationship yields

$$\mathbf{v}_P = \mathbf{v}_O + \omega_z \times \mathbf{r}_P = \begin{bmatrix} v_x \\ v_y + a\omega_z \end{bmatrix}. \quad (5)$$

Therefore,

$$\begin{aligned} v_{Px} &= v_x \\ v_{Py} &= v_y + a\omega_z \end{aligned} \quad (6)$$

This relationship is particularly important in cases where the robot simultaneously translates and rotates. It shows that when $\omega_z \neq 0$, the lateral velocity of point P differs from the lateral velocity of the center O by an amount of $a\omega_z$. Therefore, if the desired trajectory is specified for the robot's front head, the controlled variable must be $\xi_P = [v_{Px} \ v_{Py} \ \omega_z]^T$, which is then transformed into ξ_B at O before solving the inverse kinematics problem.

TABLE 1: Geometric and Kinematic Parameters of the Swerve Robot

Parameter Description	Symbol	Unit
Wheelbase length along x_B	L	m
Track width along y_B	W	m
Wheel radius	R_w	m
Coordinates of module S_i in frame $\{B\}$	(x_i, y_i)	m
Position vector from O to S_i	$\mathbf{r}_i$	m
Distance from O to tracking point P	$a = L/2$	m
Position vector from O to P	$\mathbf{r}_P$	m
Center O coordinates in frame $\{G\}$	x, y	m
Tracking point P coordinates in frame $\{G\}$	X_P, Y_P	m
Robot heading angle	ψ	rad
Longitudinal body velocity	v_x	m/s
Lateral body velocity	v_y	m/s
Body angular velocity	ω_z	rad/s
Velocity of point P along x_B	v_{Px}	m/s
Velocity of point P along y_B	v_{Py}	m/s
Module steering angle	δ_i	rad
Wheel linear velocity along rolling direction	v_i	m/s
Module velocity component along x_B	v_{ix}	m/s
Module velocity component along y_B	v_{iy}	m/s
Wheel angular rotational speed	ω_i	rad/s

2.3. Forward Kinematic Analysis of the Swerve Robot

The forward kinematic model is formulated to determine the overall motion state of the robot body from the actual motion states of the four Swerve modules, thereby evaluating the effects of individual wheel linear speeds and steering orientation angles on the translational, rotational, and trajectory tracking capabilities of the robot. This also serves as the basis for investigating kinematic behavior across various motion

modes and analyzing system sensitivity in the presence of velocity or orientation discrepancies among the modules. Accordingly, the input to the forward kinematic problem comprises the kinematic parameters of the four modules, namely the linear velocities v_i and steering orientation angles δ_i , $i = 1, \dots, 4$, whereas the direct output is the body velocity vector ξ_B . When combined with the initial pose $\mathbf{q}_0$ and the elapsed motion time, the model further yields the overall position and heading angle $\mathbf{q} = [X \ Y \ \psi]^T$ as output, enabling the reconstruction of the robot's motion trajectory within the inertial coordinate frame. From the kinematic relationships of each individual module, the wheel velocity components are aggregated into the vector:

$$\mathbf{U} = \begin{bmatrix} v_1 \cos \delta_1 \\ v_1 \sin \delta_1 \\ \vdots \\ v_4 \cos \delta_4 \\ v_4 \sin \delta_4 \end{bmatrix} \quad (7)$$

The relationship between the four modules and the body velocity vector of the robot is expressed in matrix form as:

$$\mathbf{U} = \mathbf{J} \xi_B \quad (8)$$

where the Jacobian matrix $\mathbf{J} \in \mathbb{R}^{8 \times 3}$ is established corresponding to the coordinates x_i and y_i , $i = 1, \dots, 4$:

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & -y_1 \\ 0 & 1 & x_1 \\ \vdots & \vdots & \vdots \\ 1 & 0 & -y_4 \\ 0 & 1 & x_4 \end{bmatrix}$$

Because the four modules generate eight velocity components while the robot body possesses only three planar degrees of freedom, Eq. (8) constitutes an over-determined system of equations. Under ideal conditions where module states fully satisfy rigid-body motion constraints, the system yields a unique exact solution (v_x, v_y, w_z) . However, in real physical systems or during kinematic mismatch investigations, measured values may not be perfectly compatible. Consequently, the body velocity vector of the robot is solved in the least-squares sense using the Moore-Penrose pseudoinverse of the Jacobian matrix:

$$\xi_B = [v_x \ v_y \ w_z]^T = \mathbf{J}^+ \mathbf{U} = (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T \mathbf{U} \quad (9)$$

Therefore, the solution in Eq. (9) can be expanded into explicit forward kinematic equations. The two translational velocity components of the robot body correspond to the arithmetic mean of the respective velocity components across the four modules, whereas the angular velocity depends simultaneously on the positions and velocity vectors of each module:

$$\begin{aligned} v_x &= \frac{1}{4} \sum_{i=1}^4 v_i \cos \delta_i \\ v_y &= \frac{1}{4} \sum_{i=1}^4 v_i \sin \delta_i \\ w_z &= \frac{\sum_{i=1}^4 (x_i v_i \sin \delta_i - y_i v_i \cos \delta_i)}{L^2 + W^2} \end{aligned} \quad (10)$$

Equation (10) allows the direct determination of three characteristic parameters describing the instantaneous motion of the robot body from the states of the four modules. In particular, the first two equations indicate that v_x and v_y result from the synthesis of wheel velocity components along the two axes of the $\{B\}$ frame, whereas the third equation reflects the contribution of each module to the rotational motion through the lever arm length $r_i = \sqrt{x_i^2 + y_i^2}$. Consequently, even when the modules operate at identical velocity magnitudes, varying δ_i can induce completely distinct motion states, ranging from pure translation to in-place rotation or combined translation-rotation.

To evaluate the robot's trajectory, the velocity ξ_B determined from Eq. (10) which is initially expressed in the body-fixed frame $\{B\}$, must be transformed into the inertial frame $\{G\}$. Given the instantaneous heading angle ψ , the relationship between body velocity and the time rate of change of robot pose is defined by:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ w_z \end{bmatrix} \quad (11)$$

or equivalently, $\dot{X} = v_x \cos \psi - v_y \sin \psi$, $\dot{Y} = v_x \sin \psi + v_y \cos \psi$, and $\dot{\psi} = w_z$. By integrating Eq. (11) over time with initial conditions $\mathbf{q}_0 = [X_0 \ Y_0 \ \psi_0]^T$, the full position and orientation of the robot in the inertial coordinate frame can be obtained. For the trajectory tracking point P , after determining (X, Y, ψ) of center O , the position of point P is reconstructed by:

$$\begin{aligned} X_P &= X + a \cos \psi \\ Y_P &= Y + a \sin \psi \end{aligned} \quad (12)$$

Simultaneously, from the reconstructed body twist of the robot, the velocity of point P is given by:

$$v_{Px} = v_x, \quad v_{Py} = v_y + a w_z. \quad (13)$$

Thus, the forward kinematic model not only establishes a sequential mapping chain from the local states of the four modules $\{v_i, \delta_i\}_{i=1}^4$ through the body velocity state v_x, v_y, w_z to the global motion state X, Y, ψ , but also enables direct determination of the motion state of functional point P . This provides the critical foundation for evaluating whether the front end of the robot accurately tracks the desired trajectory, rather than merely verifying the trajectory of the geometric center O .

2.4. Inverse Kinematic Model of the Swerve Robot

The inverse kinematic model determines the required states of the four Swerve modules from the desired motion state of the robot body. This model directly serves the control process: given the overall velocity command of the robot, the system must simultaneously resolve the corresponding rolling speed and steering orientation angle for each module. Therefore, the input to the primary inverse kinematic problem is the desired velocity $\xi_{B,ref} = [v_x, v_y, w_z]^T$, while the output consists of eight quantities $\{v_i, \delta_i\}_{i=1}^4$, or equivalently four wheel angular velocities $\omega_i = \frac{v_i}{R_w}$ and four steering angles δ_i .

For module $S_i(x_i, y_i)$, the two required velocity components are directly determined by:

$$\begin{aligned} v_{ix} &= v_x - \omega_z y_i \\ v_{iy} &= v_y + \omega_z x_i \end{aligned} \quad (14)$$

Accordingly, the magnitude of the rolling velocity and the corresponding steering orientation angle of the module are solved via:

$$\begin{aligned} v_i &= \sqrt{v_{ix}^2 + v_{iy}^2} \\ \delta_i &= \text{atan2}(v_{iy}, v_{ix}) \end{aligned} \quad (15)$$

where the atan2 function is utilized instead of the standard arctangent function to guarantee correct angle determination across all four quadrants. Substituting the specific coordinates of the four modules, the velocity components are expanded as:

$$\begin{aligned} S_1: \quad v_{1x} &= v_x - \frac{W}{2} \omega_z, \quad v_{1y} = v_y - \frac{L}{2} \omega_z, \\ S_2: \quad v_{2x} &= v_x - \frac{W}{2} \omega_z, \quad v_{2y} = v_y + \frac{L}{2} \omega_z, \\ S_3: \quad v_{3x} &= v_x + \frac{W}{2} \omega_z, \quad v_{3y} = v_y - \frac{L}{2} \omega_z, \\ S_4: \quad v_{4x} &= v_x + \frac{W}{2} \omega_z, \quad v_{4y} = v_y + \frac{L}{2} \omega_z. \end{aligned} \quad (16)$$

Equation (16) clarifies the individual contributions of translational and rotational motion components to each module.

The v_x component is modified by $\pm \frac{W\omega_z}{2}$ depending on the module's lateral position along y_B , whereas v_y is modified by $\pm \frac{L\omega_z}{2}$ based on its longitudinal position along x_B . This spatial distribution inherently produces distinct rolling velocities and steering angles across the four modules during coupled translational-rotational maneuvers.

In matrix form, the inverse kinematics process can be concisely expressed using the same Jacobian matrix employed in forward kinematics:

$$\mathbf{U} = \mathbf{J} \mathbf{\xi}_B \quad (17)$$

After determining v_{ix} and v_{iy} , the rolling velocity v_i and steering angle δ_i are calculated according to Eq. (15), while the wheel angular velocity is given by:

$$\omega_i = \frac{v_i}{R_w}, \quad i = 1, 2, 3, 4. \quad (18)$$

In conventional formulation, (v_x, v_y, ω_z) is directly assigned at the geometric center O. However, when the robot undergoes combined translation and rotation and the physical requirement demands that the front center point P track the trajectory, this direct command assignment is no longer suitable. Since P is offset from O by a distance $a = L/2$, the velocity relationship between the two points is:

$$\begin{bmatrix} v_{Px} \\ v_{Py} \end{bmatrix} = \begin{bmatrix} v_x \\ v_y + a \omega_z \end{bmatrix} \quad (19)$$

Therefore, when the control input is specified at point P, consisting of $(v_{Px}, v_{Py}, \omega_z)$, the equivalent twist that must be applied at center O is pre-calculated via:

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -a \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{Px} \\ v_{Py} \\ \omega_z \end{bmatrix}. \quad (20)$$

Following the transformation in Eq. (20), the resulting twist at O is fed into Equations (14)–(18) to determine the required states of the four modules.

Finally, the IK and FK models are integrated into a closed-loop validation structure. The required motion at P is mapped to O, after which IK determines $(v_i, \delta_i, \omega_i)$; these module states are subsequently fed into FK to reconstruct $\mathbf{\xi}_B$, re-evaluate velocity at P, integrate pose $\mathbf{q}$, and calculate trajectory (X_P, Y_P) .

III. NUMERICAL SIMULATION AND RESULT EVALUATION

3.1. Simulation Objectives and Problem Setup

Numerical simulation is conducted to validate the correctness and consistency of the four-module Swerve drive robot kinematic model formulated above. Specifically, the simulation procedure sequentially executes three primary transformations: (i) mapping the required motion state at trajectory tracking point P to the equivalent motion state at the robot geometric center O; (ii) employing inverse kinematics (IK) to calculate linear velocity v_i , steering orientation angle δ_i , and wheel angular velocity ω_i for the four modules; and (iii) utilizing these calculated module states directly as inputs to forward kinematics (FK) to reconstruct body motion, integrate robot pose in the inertial coordinate frame, and determine the actual trajectory of point P. This closed-loop verification structure allows simultaneous evaluation of kinematic equation accuracy and motion response capability.

In this study, a combined translational-rotational motion mode is selected for investigation because it represents a general motion scenario in which the robot simultaneously exhibits non-zero longitudinal velocity, lateral velocity, and angular velocity. In this motion scheme, point P located at the front center of the robot is designated as the trajectory tracking point. Accordingly, motion requirements are pre-specified at P, then transformed into equivalent commands at O prior to solving the IK problem. This setup ensures that the functional point at the front end of the robot—rather than the geometric center—directly executes the desired trajectory.

The simulation is implemented in MATLAB with a time step $\Delta t = 0.01$ s and a total duration $T = 5$ s. The parameters used in the simulation are summarized in Table 2.

TABLE 2. Simulation parameters

Parameter	Symbol	Value	Unit
Robot length	L	0.60	m
Robot width	W	0.50	m
Wheel radius	R_w	0.05	m
Distance O P	$a = L/2$	0.30	m
Time step	Δt	0.01	s
Simulation time	T	5.00	s
Velocity along x_B	v_{Px}	0.40	m/s
Velocity along y_B	v_{Py}	0.30	m/s
Angular velocity	ω_z	0.20	rad/s
Initial coordinates of P	(X_{P0}, Y_{P0})	(0, 0)	m
Initial heading angle	ψ_0	0	rad

An important note regarding initial conditions is that the initial position is set for tracking point P at (X_{P0}, Y_{P0}) , rather than directly placing center O at the origin. With $\psi_0 = 0$, the initial coordinates of the robot center O must satisfy:

$$\begin{bmatrix} X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} X_{P0} - a \cos \psi_0 \\ Y_{P0} - a \sin \psi_0 \end{bmatrix} \quad (21)$$

Condition (21) guarantees that at $t=0$, point P lies precisely at the starting point of the reference trajectory, eliminating initial geometric offsets between the robot center and the tracking point.

3.2. Trajectory Determination for Combined Translation and Rotation

Point P is required to simultaneously translate along both axes of the body coordinate frame while the robot rotates about axis z_B . The motion state vector specified at P is defined by:

$$\xi_P = \begin{bmatrix} v_{Px} \\ v_{Py} \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.3 \\ 0.2 \end{bmatrix} \quad (22)$$

Because $\omega_z \neq 0$, ξ_P cannot be directly applied as the velocity vector ξ_B of center O. In this scenario:

$$\begin{aligned} v_x &= v_{Px} \\ v_y &= v_{Py} - a \omega_z \end{aligned} \quad (23)$$

Thus, the actual reference motion that must be supplied to the inverse kinematic model at center O is $\xi_{B,ref} = [0.4 \ 0.24 \ 0.2]^T$. The distinction between ξ_P and ξ_B holds significant kinematic importance. As the robot body rotates with ω_z , point P receives an additional lateral velocity component $a\omega_z = 0.06$ m/s due to rotation. Consequently, center O needs to translate laterally with $v_y = v_{Py} - a\omega_z = 0.24$ m/s so that the total lateral velocity at P matches $v_{Py} = 0.30$ m/s precisely. If this correction were neglected and v_{Py} were directly applied at O, the actual lateral velocity of P would become $v_{Py} + a\omega_z = 0.36$ m/s, causing the front end of the robot to deviate from the intended trajectory.

From the motion vector at center O, inverse kinematic equations are applied to determine module states, including linear rolling velocities, steering orientation angles, and wheel angular velocities.

3.3. Verification of the Forward Kinematic Problem

After determining module states (v_i, δ_i) via IK, these values are directly passed as inputs to the FK model. The module velocity vector at each computation step is constructed as:

$$U_k = \begin{bmatrix} v_1 \cos \delta_1 \\ v_1 \sin \delta_1 \\ \vdots \\ v_4 \cos \delta_4 \\ v_4 \sin \delta_4 \end{bmatrix} \quad (24)$$

The reconstructed body motion of the robot is then evaluated via:

$$\hat{\xi}_B = J^+ U = [\hat{v}_x \ \hat{v}_y \ \hat{\omega}_z]^T \quad (25)$$

To evaluate numerical consistency between the two kinematic problems, reconstruction errors are defined individually as:

$$\begin{aligned} e_{vx} &= \hat{v}_x - v_x \\ e_{vy} &= \hat{v}_y - v_y \\ e_{\omega} &= \hat{\omega}_z - \omega_z \end{aligned} \quad (26)$$

Under ideal simulation conditions without measurement noise, wheel slip, mechanical tolerances, or actuator errors, the FK solution using module states generated by IK should theoretically produce near-zero reconstruction errors, bounded only by floating-point rounding precision.

The velocity of point P is reconstructed from the motion of center O:

$$\begin{aligned} \hat{v}_{Px} &= \hat{v}_x \\ \hat{v}_{Py} &= \hat{v}_y + a \hat{\omega}_z \end{aligned} \quad (27)$$

3.4. Construction of Reference and Actual Trajectories of Point P

Once the state of center O in the inertial coordinate frame is determined by integrating Eq. (11) at time step $t_k = k\Delta t$ using Euler's method with $\Delta t = 0.01$ s, updates are executed according to:

$$\begin{aligned} X_{k+1} &= X_k + \Delta t (\hat{v}_x \cos \psi_k - \hat{v}_y \sin \psi_k) \\ Y_{k+1} &= Y_k + \Delta t (\hat{v}_x \sin \psi_k + \hat{v}_y \cos \psi_k) \\ \psi_{k+1} &= \psi_k + \hat{\omega}_z \Delta t \end{aligned} \quad (28)$$

The actual coordinates of tracking point P at each time step are determined by:

$$\begin{aligned} X_P(t) &= X(t) + a \cos \psi(t) \\ Y_P(t) &= Y(t) + a \sin \psi(t) \end{aligned} \quad (29)$$

Meanwhile, the reference trajectory of point P is constructed independently directly from motion commands (v_{Px}, v_{Py}) :

$$\begin{aligned} \dot{X}_{P,ref} &= v_{Px} \cos \psi - v_{Py} \sin \psi \\ \dot{Y}_{P,ref} &= v_{Px} \sin \psi + v_{Py} \cos \psi \\ \dot{\psi} &= \omega_z \end{aligned} \quad (30)$$

Thus, the actual trajectory P is obtained by transforming the motion from IK to FK, integrating the motion of the center O, and transforming from O to P, whereas $(X_{P,ref}, Y_{P,ref})$ is built directly from motion commands at the tracking point. Establishing agreement between these two trajectories provides a rigorous, comprehensive validation beyond merely comparing velocity components.

Because v_{Px} , v_{Py} , and ω_z are constant, Equation (30) possesses an analytical solution:

$$\begin{aligned} \psi(t) &= \psi_0 + \omega_z t \\ X_{P,ref} &= X_{P0} + \frac{1}{\omega_z} [v_{Px} (\sin \psi - \sin \psi_0) + v_{Py} (\cos \psi - \cos \psi_0)] \\ Y_{P,ref} &= Y_{P0} + \frac{1}{\omega_z} [-v_{Px} (\cos \psi - \cos \psi_0) + v_{Py} (\sin \psi - \sin \psi_0)] \end{aligned} \quad (31)$$

Analytical solution (31) provides an independent benchmark to check discretization errors associated with numerical Euler integration. The trajectory of point P is a curve rather than a straight line, despite v_{Px} and v_{Py} being constant in

frame {B}, because frame {B} continuously rotates relative to inertial frame {G}.

3.5. Determination of Trajectory Tracking Errors at Point P

To quantify trajectory tracking capability, position errors along the two global axes are defined as:

$$\begin{aligned} e_{x_p}(t) &= X_p(t) - X_{p,ref}(t) \\ e_{y_p}(t) &= Y_p(t) - Y_{p,ref}(t) \end{aligned} \quad (32)$$

From these, the Euclidean distance error between the actual and reference points is determined:

$$e_p(t) = \sqrt{e_{x_p}^2(t) + e_{y_p}^2(t)} \quad (33)$$

The quantity e_p serves as the primary metric for evaluating tracking performance at the front center point of the robot. In the present ideal kinematic model, both actual and reference trajectories are generated from identical kinematic relations and time steps, so e_p is expected to remain exceptionally small, near numerical precision limits. In physical systems, e_p would further reflect steering angle errors, wheel velocity errors, wheel slip, actuator response delays, positioning errors, and sensor noise.

In addition to evaluating errors between (X_p, Y_p) and $(X_{p,ref}, Y_{p,ref})$, the analytical solution in Eq. (31) is utilized to assess discretization errors arising from numerical Euler integration.

3.6. Numerical Simulation Results

Based on the simulation parameters established above, computation results for the forward and inverse kinematic problems are presented in detail from Figure 2 to Figure 10.

Figure 2 illustrates that the linear velocities of the four wheels remain stable throughout the simulation, taking values of approximately $v_1 = 0.394$, $v_2 = 0.461$, $v_3 = 0.485$ and $v_4 = 0.541$ m/s. Velocity variations across modules are a direct consequence of combined translational-rotational motion: the

ω_z component induces position-dependent velocity vectors at (x_i, y_i) , requiring each module to operate at a distinct speed to maintain rigid-body motion consistency across the robot frame. Module S_4 exhibits the highest linear velocity, while S_1 exhibits the lowest, matching velocity distributions predicted by inverse kinematics. The absence of velocity fluctuations over time aligns with constant motion commands v_{px}, v_{py} and w_z , confirming initial kinematic model consistency.

Figure 3 demonstrates that the steering orientation angles of the four modules remain constant, with $\delta_1 = 27.2^\circ$, $\delta_2 = 40.6^\circ$, $\delta_3 = 21.8^\circ$ and $\delta_4 = 33.7^\circ$. These distinct steering angles reflect the core kinematic feature of combined translational-rotational motion: each module must orient its velocity vector along an individual direction due to its unique offset (x_i, y_i) relative to center O. Specifically, δ_2 is the largest and δ_3 is the smallest, perfectly matching velocity component distributions derived from inverse kinematics.

Figure 4 shows that the wheel angular velocities remain constant at approximately $\omega_1 = 7.87$, $\omega_2 = 9.22$, $\omega_3 = 9.69$ and $\omega_4 = 10.82$ rad/s. This distribution corresponds directly to linear

velocities shown in Figure 2 via $\omega_i = \frac{v_i}{R_\omega}$, as all wheels share an identical radius $R_\omega = 0.05$ m.

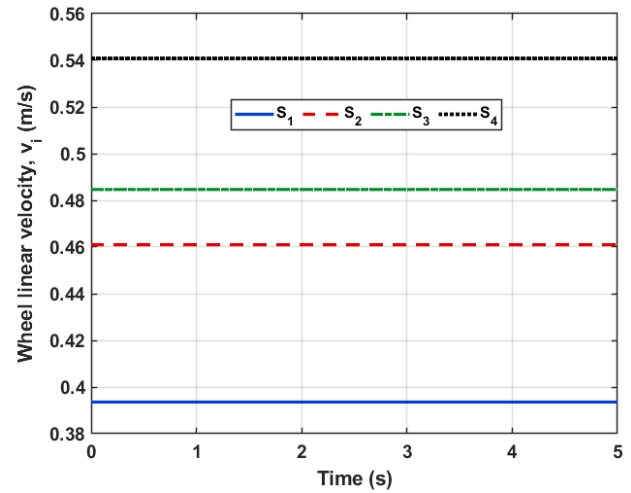


Figure 2: Linear velocities of individual wheels

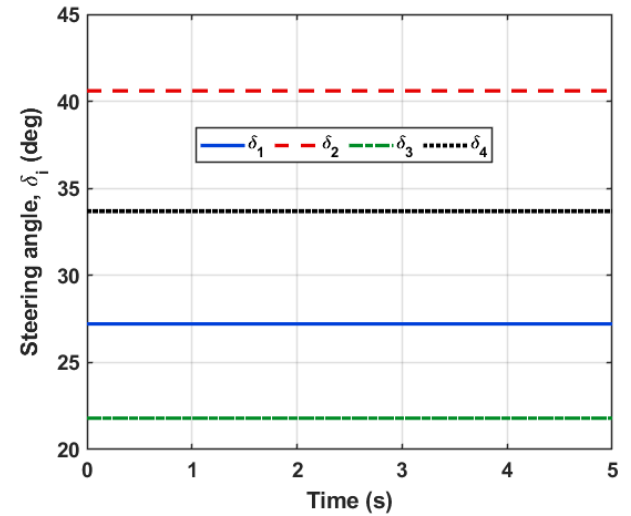


Figure 3: Steering orientation angles of individual wheels

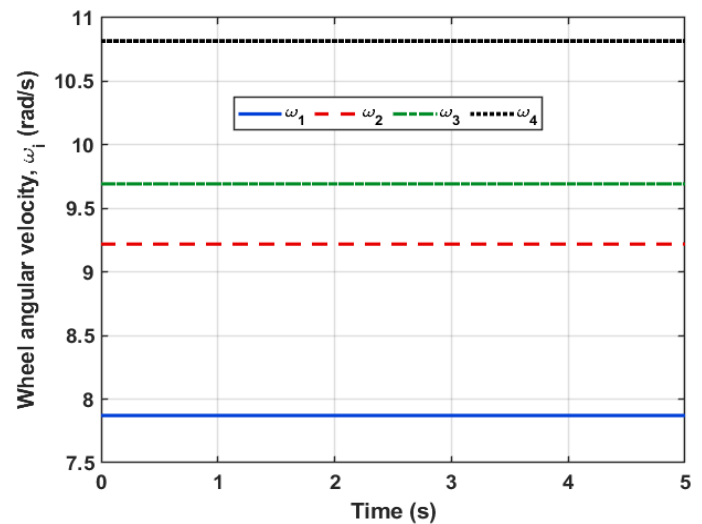


Figure 4: Angular velocities of individual wheels

Wheel S_4 exhibits the highest angular velocity, while S_1 exhibits the lowest, reflecting localized kinematic demands during coupled translation and rotation. Steady-state behavior confirms that under constant body twist commands, required actuator speeds remain constant, validating consistency between inverse kinematics and drive system transmission relations.

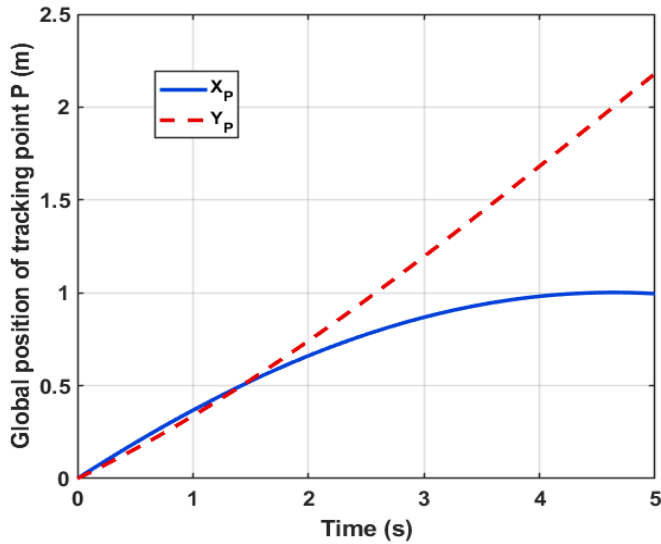


Figure 5: Position of point P in inertial frame

Figure 5 displays the variation of global coordinates of tracking point P. While Y_P increases steadily to reach approximately 2.2m at $t=5s$, X_P rises rapidly in the initial phase before reaching a peak of around 1.0m. This non-linear trajectory trend stems directly from velocity transformations from body-fixed frame to inertial frame as the robot rotates with non-zero ω_z . Although v_{Px} and v_{Py} remain constant in $\{B\}$, their projections onto $\{G\}$ continually evolve with $\psi(t)$, making dot X_P and dot Y_P time-varying. Thus, results accurately reflect coupled translational-rotational Swerve kinematics and predict a curved trajectory for point P in the global plane.

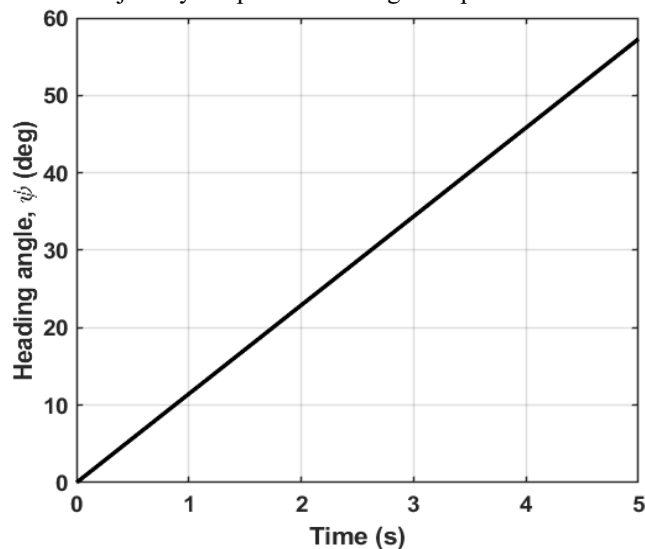


Figure 6: Heading angle of the robot

Figure 6 shows that robot heading angle ψ increases linearly from 0 to approximately 57.3° after 5s, agreeing with constant angular velocity $\omega_z = 0.2 \text{ rad/s}$ via $\psi(t) = \psi_0 + \omega_z t$. This result confirms that the rotational component of motion S_s is maintained stably and consistently with input commands. Kinematically, continuous variation in ψ causes body frame $\{B\}$ to rotate relative to inertial frame $\{G\}$; hence, while local velocities at point P remain constant, global velocity components vary continuously. This is precisely what produces the non-linear variation of (X_P, Y_P) in Figure 5 and the curved trajectory of point P in the plane of motion.

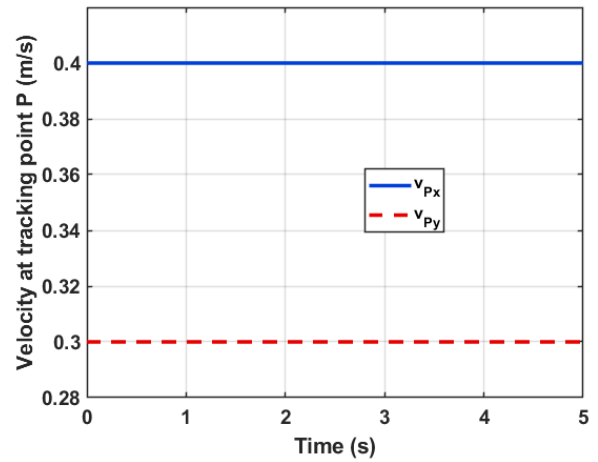


Figure 7: Velocity components of point P in body frame

Figure 7 demonstrates that the two velocity components of tracking point P remain constant at $v_{Px}=0.40\text{m/s}$ and $v_{Py}=0.30\text{m/s}$ throughout the simulation. This confirms that kinematic velocity transformation from point P to center O, with $v_y = v_{Py} - a \times \omega_z$, accurately compensates for velocity components induced by body rotation. These local velocity values in frame $\{B\}$ project into time-varying velocity components in frame $\{G\}$ as ψ changes, explaining why point P maintains constant local velocity while describing a curved path in the global coordinate frame.

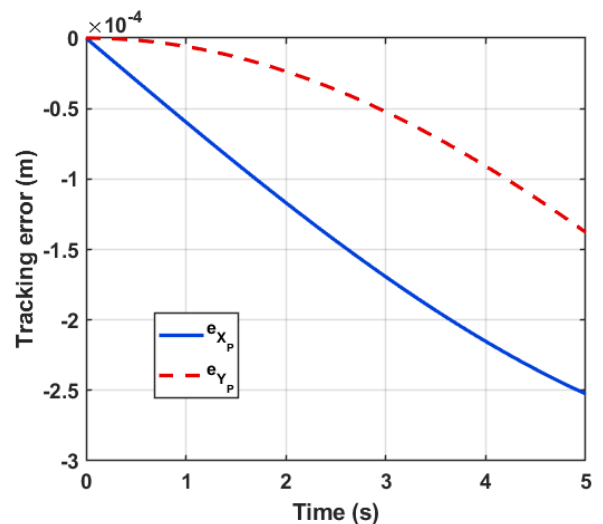


Figure 8: Position tracking errors of point P relative to reference trajectory

Figure 8 reveals that position tracking errors of point P relative to the reference trajectory are exceptionally small, with $|e_{x_p}|$ and $|e_{y_p}|$ reaching only approximately 2.5×10^{-4} m and 1.4×10^{-4} m at $t=5$ s (0.25 mm and 0.14 mm, respectively). The slight error growth over time stems primarily from cumulative numerical discretization error in discrete Euler integration rather than kinematic model discrepancy. Over meter-scale traversal distances, position errors below 0.3 mm demonstrate high numerical consistency across the $P \geq O$ transformation, inverse kinematics, and forward kinematics, confirming that the front center of the robot tracks the reference path with high precision under ideal conditions.

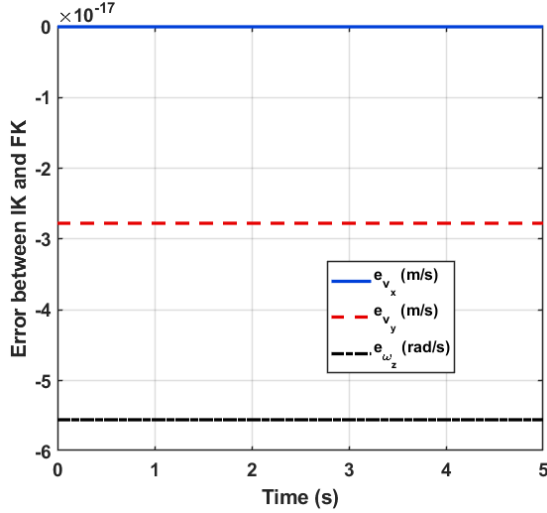


Figure 9: Robot body velocity reconstruction errors

Figure 9 indicates that reconstruction errors between IK commands and FK outputs for v_x , v_y and ω_z are on the order of 10^{-17} , which is at the floating-point precision limit of numerical computation and technically zero. This demonstrates that module states (v_i, δ_i) solved by IK reconstruct the requested body twist almost perfectly when fed into FK. Crucially, non-accumulating error over time confirms that the Jacobian matrix and pseudoinverse formulation are strictly consistent with coordinate definitions across the four modules. Consequently, minor position errors observed in Figure 8 are attributable to numerical integration rather than IK-FK transformation mismatch.

Figure 10 visualizes the complete motion S_5 , demonstrating that tracking point P at the front center of the robot aligns almost perfectly with the reference trajectory throughout the motion, while geometric center O follows a distinct inner trajectory. The divergence between the two paths is a natural outcome of rigid-body mechanics when $\omega_z \neq 0$, underscoring the necessity of velocity transformation from P to O. Robot pose snapshots across multiple time steps illustrate simultaneous translation and rotation while all four modules maintain rolling directions consistent with required body twists. At $t=5$ s, P reaches approximately (1.00; 2.18) m with heading angle $\psi=57.3^\circ$ and a final position error of only 2.88×10^{-4} m, thereby confirming the effectiveness of the trajectory-tracking model at point P.

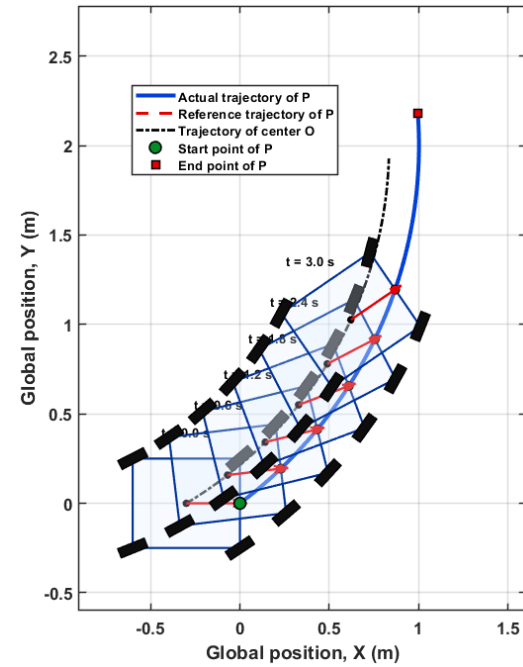
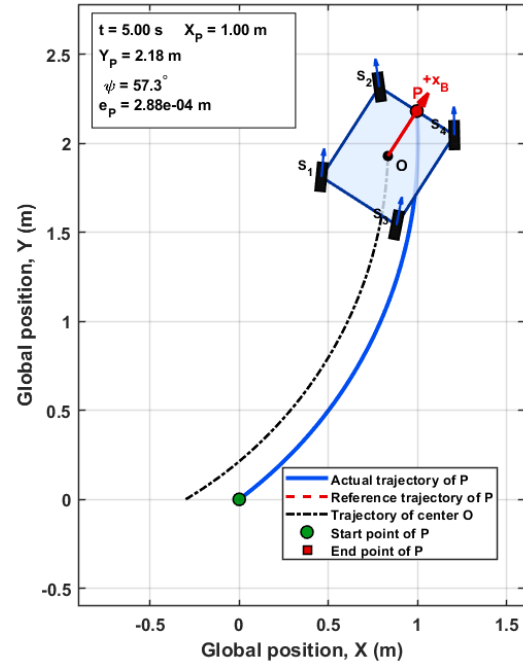


Figure 10: Robot trajectory tracking visualization

IV. CONCLUSION

The study has established a kinematic model for a four-wheel Swerve Drive robot under combined translational and rotational motion, in which point P, located at the center of the robot's front end, is defined as the trajectory-tracking point from the initial formulation of the problem. The rigid-body velocity relationship between P and the geometric center O is combined with the kinematics of the four modules to establish a unified transformation chain from the required velocity at P to the equivalent motion at O, after which the linear velocity, steering angle, and angular velocity of each wheel are determined using inverse kinematics (IK). In the reverse

direction, forward kinematics (FK) based on the Jacobian matrix is used to reconstruct the motion of the robot body, determine its global pose, and obtain the actual trajectory of P. This formulation ensures that the trajectory-tracking requirement is applied directly at the front point of the robot rather than at the geometric center, making it particularly suitable for simultaneous translational and heading-change motion.

The simulation results for the combined translational-rotational trajectory show that the four modules maintain distinct yet stable velocity and steering-angle states, consistent with the geometric position of each wheel. The deviation between the required motion and the reconstructed motion is approximately zero, demonstrating the numerical consistency of the forward-inverse kinematic model. While center O follows its own trajectory due to the rotation of the body, point P almost coincides with the reference trajectory and reaches a final position error of approximately 2.88×10^{-4} m after 5 s. This result confirms the kinematic feasibility of using P as the trajectory-tracking point for combined translational-rotational motion. Future work should focus on closed-loop control and experimental validation, while also considering the effects of wheel slip, velocity and steering-angle limits, actuator dynamics, sensor noise, and localization errors on the trajectory-tracking accuracy of the physical robot.

REFERENCES

- [1] Mısırlı, O., Beyazlı, M., Alp, S., Taşkın, G. B., & Isık, Z. (2024). Design and mechanical analyses of mobile robot with Swerve driving system. *Turkish Journal of Nature and Science*, 13(4), 66-84. DOI: 10.46810/tjns.1506516.
- [2] Tagliavini, L., Botta, A., Colucci, G., Baglieri, L., Duretto, S., & Quaglia, G. (2024). Kinematic modelling of Swerve-drive-based mobile robots. *Proceedings of the 7th Jc-IFTOMM International Symposium*, 225-231. DOI: 10.57272/jciftomm.7.0_225.
- [3] Bae, B., & Lee, D.-H. (2023). Design of a four-wheel steering mobile robot platform and adaptive steering control for manual operation. *Electronics*, 12(16), 3511. DOI: 10.3390/electronics12163511.
- [4] Qu, J., Li, H., Zhang, Z., Xi, X., Zhang, R., & Guo, K. (2022). Performance analysis and optimization for steering motion mode switching of an agricultural four-wheel-steering mobile robot. *Agronomy*, 12(11), 2655. DOI: 10.3390/agronomy12112655.
- [5] Lee, H., & Joe, H.-M. (2026). Reinforcement learning-based control of a 4-wheel independent steering mobile robot for robust path tracking in outdoor environments. *Sensors*, 26(6), 1761. DOI: 10.3390/s26061761.
- [6] Purbowaskito, W., & Hsu, C.-H. (2020). Kinematics model based motion control of a 4WIS mobile robot with Lyapunov stability. In 2020 12th International Conference on Information Technology and Electrical Engineering (ICITEE). IEEE. DOI: 10.1109/ICITEE49829.2020.9271742.
- [7] Li, J., Wu, Q., Wang, J., Qin, H., & Li, J. (2020). Autonomous tracking control for four-wheel independent steering robot based on improved Pure Pursuit. *Journal of Beijing Institute of Technology*, 29(4), 466-473. DOI: 10.15918/j.jbit1004-0579.19110.
- [8] Chen, H., Chen, S., Zhou, R., Huang, X., & Zhu, S. (2021). Research on four-wheel independent steering intelligent control strategy based on minimum load. *Concurrency and Computation: Practice and Experience*, 33(9), e6145. DOI: 10.1002/cpe.6145.
- [9] Jeong, Y., & Yim, S. (2021). Model predictive control-based integrated path tracking and velocity control for autonomous vehicle with four-wheel independent steering and driving. *Electronics*, 10(22), 2812. DOI: 10.3390/electronics10222812.
- [10] Liu, X., Wang, W., Li, X., Liu, F., He, Z., Yao, Y., Ruan, H., & Zhang, T. (2022). MPC-based high-speed trajectory tracking for 4WIS robot. *ISA Transactions*, 123, 413-424. DOI: 10.1016/j.isatra.2021.05.018.
- [11] Ding, T., Zhang, Y., Ma, G., Cao, Z., Zhao, X., & Tao, B. (2022). Trajectory tracking of redundantly actuated mobile robot by MPC velocity control under steering strategy constraint. *Mechatronics*, 84, 102779. DOI: 10.1016/j.mechatronics.2022.102779.
- [12] Shi, P., Chen, X., Yang, A., & Zhang, L. (2022). Ackerman steering trajectory planning and position estimation of 4WID-4WIS intelligent vehicle. *Journal of Engineering Design*, 29(2), 123-132. DOI: 10.3785/j.issn.1006-754X.2022.00.032.
- [13] Jeong, Y., & Yim, S. (2022). Integrated path tracking and lateral stability control with four-wheel independent steering for autonomous electric vehicles on low friction roads. *Machines*, 10(8), 650. DOI: 10.3390/machines10080650.
- [14] Zhang, J., Zhang, X., Zhang, X., & Zhang, K. (2023). Trajectory tracking control strategy for oblique driving of 4WIS unmanned vehicle. *Transactions of Beijing Institute of Technology*, 43(2), 161-169. DOI: 10.15918/j.jbit1001-0645.2022.052.
- [15] Bae, B., & Lee, D.-H. (2023). Design of a four-wheel steering mobile robot platform and adaptive steering control for manual operation. *Electronics*, 12(16), 3511. DOI: 10.3390/electronics12163511.
- [16] Kunal, B., Madhuka, G., & Satyajeet, N. (2023). Design and control of Swerve Drive robot using kinematic model. In 2023 14th International Conference on Computing Communication and Networking Technologies (ICCCNT). IEEE. DOI: 10.1109/ICCCNT56998.2023.10307118.
- [17] Čaran, B., Milić, V., Švaco, M., & Jerbić, B. (2024). Optimal control-based algorithm design and application for trajectory tracking of a mobile robot with four independently steered and four independently actuated wheels. *Actuators*, 13(8), 279. DOI: 10.3390/act13080279.
- [18] Auh, E., Jung, H., Pico, N., Choi, H., Koo, J., & Moon, H. (2024). Model predictive contouring control for four-wheel independent steering and driving mobile robots. In 2024 IEEE International Conference on Real-Time Computing and Robotics (RCAR), 259-264. DOI: 10.1109/RCAR61438.2024.10671326.
- [19] Chu, Y., Wang, C., Zhou, X., Zhang, Z., & Zhao, W. (2024). Instantaneous center of rotation tracking control of four-wheel independent steering vehicles under large-curvature turning conditions. *IEEE Transactions on Intelligent Transportation Systems*, 25, 17965-17978. DOI: 10.1109/TITS.2024.3432885.
- [20] Kim, S., & Song, Y.-E. (2025). Integrated-kinematic modeling and autonomous navigation performance analysis of four-wheel independent steering mobile robot. *IFAC*.
- [21] Takanezawa, K., Ozaki, R., Takesue, N., Hiruma, J., & Mikado, J. (2025). Four-wheel independent steering and four-wheel independent driving robot and its simultaneous path and orientation tracking control. *International Journal of Automation Technology*, 19(4), 521-534. DOI: 10.20965/ijat.2025.p0521.
- [22] Yang, Z., Dai, Y., Hu, M., Bian, Y., Cui, Q., & Li, Y. (2025). Improving path tracking performance of 4WIS vehicles via constraint-oriented consistent coordinated steering. *Chinese Journal of Mechanical Engineering*, 38, 158. DOI: 10.1186/s10033-025-01298-4.
- [23] Lv, S., Leng, H., & Zhang, F. (2026). Adaptive sliding mode trajectory tracking control for four-wheel independent steering vehicles based on instantaneous center of rotation constraints. *World Electric Vehicle Journal*, 17(7), 330. DOI: 10.3390/wevj17070330.