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A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics

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Abstract**—** *This paper presents a systematic and innovative framework for classifying physical quantities in classical mechanics, categorizing them as vectors, scalars, and generalized quantities. A critical component of this study is the introduction of the 'parent-offspring' technique, which allows for a hierarchical and rigorous classification of derived parameters based on their relationship to fundamental quantities like length, mass, and time. By examining how derived quantities emerge from fundamental parameters, we offer a deeper understanding of their true nature. This technique is essential for resolving common ambiguities in the classification of physical quantities such as area and pressure, which can be mistakenly treated as vectors or scalars without careful consideration of their derivations. The paper discusses the mathematical operations (addition, dot product, and cross product) that govern the classification of quantities and highlights how these operations impact the transformation behavior of true vectors, true scalars, pseudovectors, and pseudoscalars. The proposed method provides a more comprehensive approach to identifying and categorizing quantities in various fields. Through the use of this classification framework, the paper aims to improve conceptual clarity, reduce misconceptions, and enhance the accuracy of applying physical laws in various scientific and engineering disciplines, offering a valuable tool for educational, research, and practical purposes.*

Keywords**—** *Vector; scalar; mechanics; fundamental parameters; derived parameters; pressure force; momentum.*

I. INTRODUCTION

Classical mechanics forms the foundational pillars of physics and engineering, providing a systematic framework to describe, predict, and understand the motion of bodies under various forces [1–3]. Central to this framework is the accurate representation and classification of physical quantities. These quantities, which express measurable properties of physical systems, fall broadly into two categories: scalars, which possess magnitude only, and vectors, which possess both magnitude and direction [4]. At first glance, this distinction appears elementary and intuitive; however, the rigorous identification of vectors and scalars is deeply rooted in transformation theory, geometric structure, and the physical meaning of measurements (Fig. 1). Correctly distinguishing these quantities is essential for applying Newtonian mechanics consistently and accurately, both in introductory contexts and in advanced analytical modeling. Classical texts such as those by Goldstein *et al*. [5] and Feynman *et al*. [6] emphasize the interplay between mathematics and physical interpretation in mechanics. Scalars—such as mass, time, energy, temperature, and density—remain invariant under coordinate transformations and are described fully by their magnitudes. Mathematics primarily deals with numbers and vectors, whereas mathematical physics focuses on scalar and vector quantities, respectively [7].

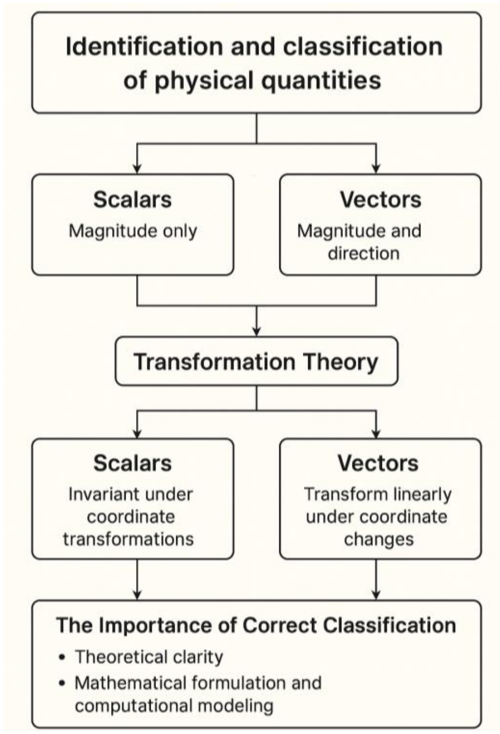

Fig. 1. Classification of physical quantities in classical mechanics. Physical systems can be classified into two main categories: scalars, which have only magnitude and invariant under coordinate transformation; and vectors, which have both magnitude and direction, being transform under coordinate changes.

The distinction between vectors and scalars affects the mathematical modeling and solutions to physical problems [8]. As such, understanding the nature of physical quantities is fundamental not only for theoretical clarity but also for correct mathematical formulation and computational modeling [9, 10]. Misidentification of vectors and scalars can lead to conceptual misunderstandings, inappropriate use of mathematical tools, and inaccuracies in problem-solving. For instance, treating displacement or velocity as scalars restricts the ability to capture directional changes in motion, while attempting to treat energy or temperature as vectors results in nonsensical operations. Errors of this kind are common among students and researchers beginning their study of mechanics and are also encountered in interdisciplinary fields where practitioners may not have a formal background in vector analysis. These challenges motivate a systematic investigation into how vectors and scalars are identified, classified, and utilized within classical mechanics.

Identifying whether a physical quantity is a scalar, a vector, or a more generalized quantity can be achieved through two widely used complementary techniques: consideration of magnitude and direction and analysis based on coordinate transformations, as summarized in Table 1 and Figs. 2 and 3. The first method relies on the sensation or intuitive observation that scalars possess only magnitude, while vectors encode both magnitude and directional information. The second method provides a rigorous and generalizable framework, examining how quantities transform under rotations, translations (shifting the origin), and reflections (mirror transformation) of the coordinate system, thus ensuring consistency across reference frames [11]. As stated above, a scalar is fundamentally defined as a quantity that remains invariant under coordinate transformations. Conversely, a vector transforms in a specific linear manner under such operations, preserving its geometric meaning across coordinate frames. This transformation-based definition provides a rigorous and generalizable framework that extends beyond simple magnitude-and-direction descriptions. It also naturally leads to the recognition of generalized quantities. These include pseudovectors (axial vectors), such as torque and angular momentum, which change sign under improper transformations (e.g., reflections). In addition, pseudoscalars (e.g., volume) exhibit analogous behavior for scalar quantities. Magnot [12] explored the category of diffeological vector pseudo-bundles and investigated a potential extension of classical differential geometric tools used for finite-dimensional vector bundles. These tools include the automorphism group, the frame bundle, the space of connection 1-forms, and the space of covariant derivatives.

In addition to these established approaches, we presently propose a novel technique: 'parent-offspring identification' (introduced later in this article), which is incorporated into Table 1 and Fig. 4 for completeness. In this approach, a quantity is classified as a vector, scalar, or generalized entity based on the relationship between derived quantities (offspring) and their fundamental or parent parameters. Specifically, the formation and properties of an offspring quantity are traced back to its parental quantity, providing a hierarchical and physically motivated method for identification. This technique is particularly useful in complex systems where derived quantities arise from combinations, derivatives, or integrals of more fundamental parameters, allowing a systematic classification even when conventional magnitude-and-direction or coordinate-based analyses are insufficient or complex.

Classical mechanics relies heavily on both scalar and vector quantities, and many of its laws emerge from the interplay between them. Newton's second law, $\mathbf{F} = m\mathbf{a}$, is inherently vectorial, relating vector force $\mathbf{F}$ to vector acceleration $\mathbf{a}$ through the scalar quantity mass m [13]. Momentum, impulse, velocity, and displacement are vector quantities that carry directional information essential for understanding trajectories, collisions, and inertia. In contrast, time, mass, energy, and temperature are scalar quantities, providing invariant measures that describe how systems store, transform, or dissipate physical properties. The dot and cross

Table 1. Comparison between different approaches of vector and scalar identification

Approaches → Properties ↓	**Magnitude & direction**	**Coordinate transformation**	**Parent-offspring (new)**
Description	Observing the physical properties of a quantity	Examining behavior under rotations, translations, and reflections	Classifying quantities based on relationship between derived ('offspring') quantities and fundamental ('parent') quantities
Scalars/Vectors Identification	Scalars: magnitude only Vectors: magnitude plus direction	Scalars: invariant under all transformations Vectors: components transform linearly; pseudovectors change sign under reflections	Scalars/Vectors are identified by how offspring inherit properties from parents
Advantages	Intuitive and simple for basic quantities	Rigorous, frame-independent, applicable to complex and generalized quantities	Provides hierarchical, physically motivated classification; useful for derived quantities
Limitations	Fails for derived or complex quantities; may not capture transformation behavior	Requires understanding of transformation rules; can be abstract	Requires knowledge of the derivation or dependency relationships; may be more complex to implement

products used throughout mechanics encode the physical meanings of work, power, torque, and angular momentum; these operations fundamentally depend on correct classification [6, 14, 15]. For example, work is defined as the dot product of force and displacement, yielding a scalar energy transfer, whereas torque is defined as the cross product of position vector and force, producing a pseudovector that governs rotational motion. These definitions are meaningful only when the underlying quantities are correctly identified and employed.

computational errors. These examples highlight the need for a thorough and systematic classification framework.

In advanced mechanics and engineering, the classification of quantities extends beyond simple vectors and scalars to include tensors, but the foundational distinction remains essential. Even in tensor calculus, vectors (first-order tensors) and scalars (zero-order tensors) play a primary role. For researchers progressing toward continuum mechanics, fluid dynamics, elasticity, thermodynamics, and electromagnetism, a rigorous grounding in the identification of vectors and scalars ensures a smoother transition to higher-level concepts.

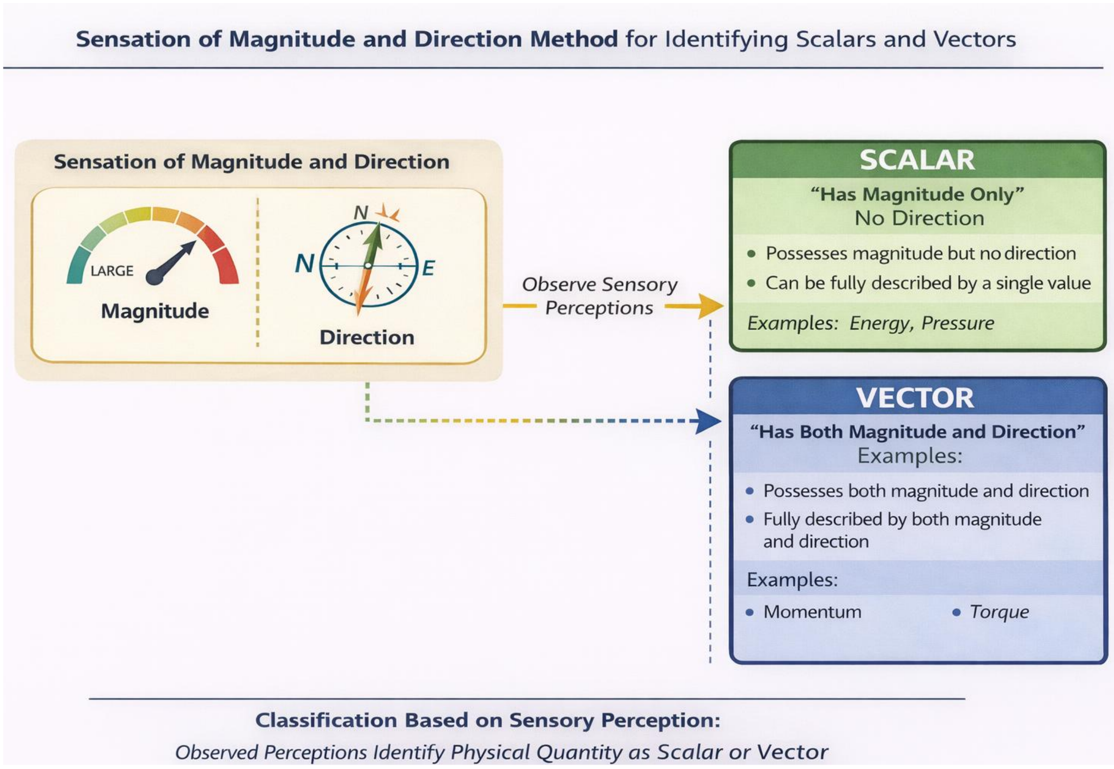

Fig. 2. Identification of scalars and vectors based on sensation of magnitude and direction. Human perception is inherently subjective and varies from one individual to another. Scalars can be described by a single value that represents their magnitude, whereas vectors require both magnitude and direction to be fully defined. This subjective nature of perception can lead to inconsistent or non-congruent results, as different people may interpret the same stimuli in different ways.

Despite its central role, the identification of vectors and scalars is often oversimplified in educational settings, where students are encouraged to rely on intuition rather than formal definitions. This approach leads to several recurring misconceptions. A common error is to assume that any quantity associated with direction is a vector, even when the direction arises from context rather than from its inherent definition. For example, heat flux and electric current are vector quantities, but heat and charge themselves are scalars. Similarly, some quantities that appear to lack direction, such as moment of momentum or vorticity, are nevertheless classified as vectors due to their rotational transformation properties. Another misconception arises in the treatment of derived quantities. While force is unequivocally a vector, derived measures such as pressure require more careful consideration regarding their physical meaning. Treating it as a simple scalar or vector results in major conceptual and

Moreover, modern computational mechanics, such as finite element methods and computational fluid dynamics, relies fundamentally on the correct specification of variable types to avoid numerical instability, dimensional inconsistency, and improper application of boundary conditions [16].

Furthermore, the ability to accurately classify physical quantities enhances clarity in communication and interpretation of mechanical behavior. Research papers, engineering manuals, and technical reports commonly assume that readers can distinguish between vectorial and scalar information. Ambiguities in this classification can hinder the interpretation of experimental data, misguide analytical derivations, or introduce unnecessary complexities. Thus, a systematic educational treatment of this topic holds significant value for students, educators, researchers, and practitioners alike.

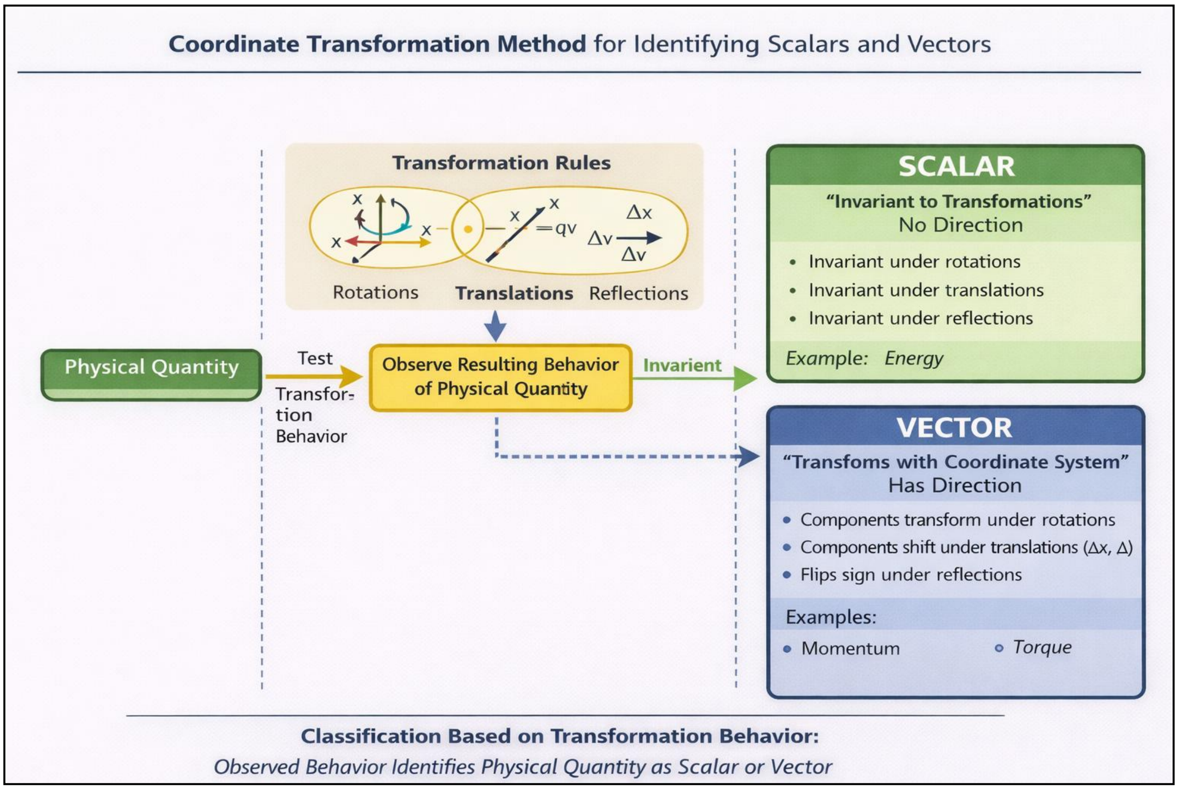

Fig. 3. Details of the coordinate transformation method for identifying scalar and vectors. The coordinate transformation provides how quantities transform under rotations, translations, and reflections within the coordinate system. A scalar remains invariant under these transfomations, while a vector transforms linearly, maintaining its geometric properties across frames.

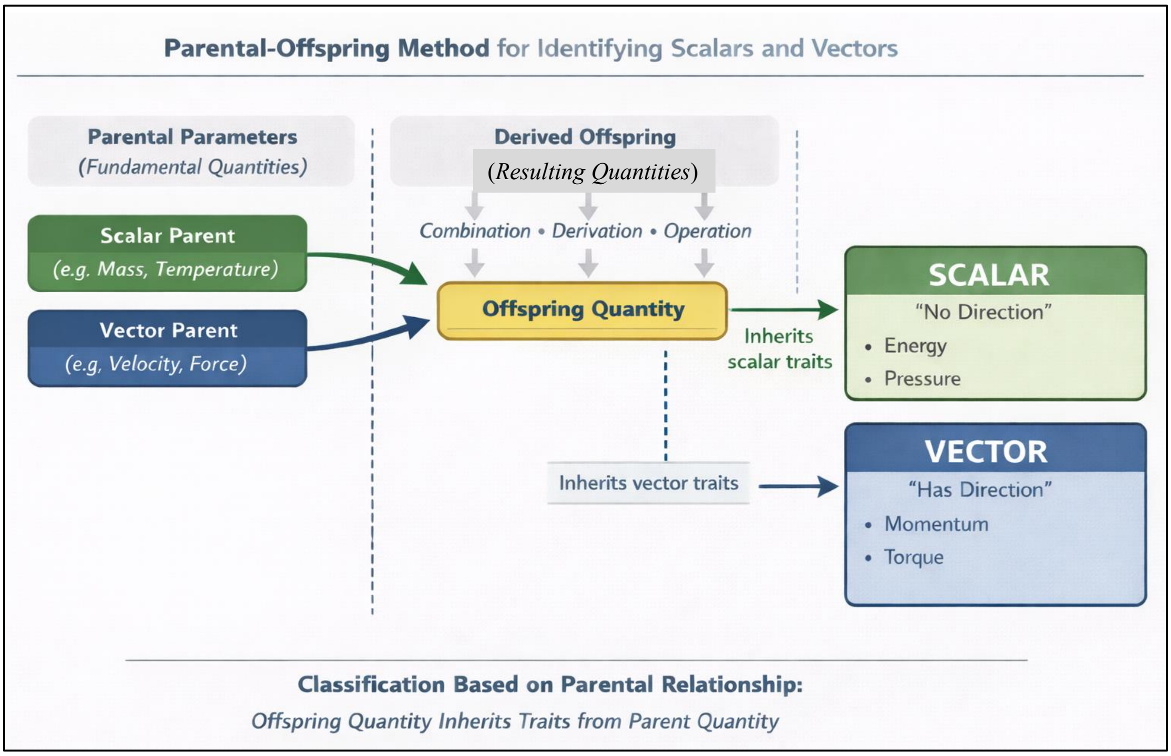

Fig. 4. Flow chart of parental-offspring method for identifying scalar and vectors. In this parental-offspring approach, quantities are classified as vectors, scalars, or generalized entities based on the relationship between derived quantities and their parent parameters. This method traces a derived quantity's formation back to its parent, providing a clear, hierarchical framework grounded in physical principles.

The study of vectors and scalars is fundamental not only to theoretical mechanics but also to practical applications in fields such as fluid dynamics, thermodynamics, and electromagnetism. Accurate classification is necessary for the proper formulation and solution of problems in these areas. Additionally, it plays a key role in modern computational mechanics, including finite element methods and computational fluid dynamics, where the correct specification of variables is essential.

Given these motivations, this article aims to present a comprehensive and pedagogically structured framework for identifying vectors and scalars within the domain of classical mechanics. The paper synthesizes conceptual definitions, geometric interpretations, formation properties, and common examples to provide a rigorous yet accessible classification methodology. We examine both fundamental quantities (i.e., displacement, mass, and time) and derived quantities, such as work, torque, angular momentum, momentum flux, and energy density. Particular emphasis is placed on quantities that are frequently misidentified, with explanations rooted in evolutionary behavior rather than intuitive notions alone. We also highlight the distinctions between vectors (true or polar vectors), pseudovectors, scalars (true scalars), and pseudoscalars, and demonstrate how these classifications arise from the mathematical structure of physical laws. Using this systematic approach, the article aims to strengthen conceptual understanding, reduce misconceptions, and provide a reference framework that improves the formulation and solution of problems in classical mechanics. By deepening the foundational understanding of scalar and vector classification, the work contributes to more robust learning, clearer communication, and more accurate application of mechanical principles across a wide range of physical and engineering systems.

II. Parameters In Classical Mechanics

In classical mechanics, physical parameters are generally classified into fundamental parameters and derived parameters, each serving a distinct role in describing mechanical systems (Fig. 5). Fundamental parameters, namely length, mass, and time, serve as the foundational quantities from which all other mechanical variables are constructed. The fundamental parameters are independent, cannot be expressed in terms of one another, and define the basis of dimensional analysis in the MKS (meter-kilogram-second) unit system. In contrast, derived parameters arise from combinations of these fundamental quantities and describe specific aspects of motion and interaction. One potential point of confusion is temperature, as it might appear to fall into either category. However, temperature is a derived physical quantity, not a fundamental one. It represents a measure of the average kinetic energy of the particles in a substance and is influenced by the fundamental parameters, such as energy. Temperature is therefore a scalar.

As indicated in Fig. 5, both mass and time are scalar quantities, which are straightforward to understand. In contrast, whether length is treated as a scalar or a vector depends on its definition and the context in which it is used. In classical mechanics, displacement (a form of length) is considered a vector quantity, while distance (another form of length) is treated as a scalar. A natural question arises: what determines whether length should be regarded as a scalar or a vector? To understand the difference between scalar and vector quantities, consider the following story. Faheem, a mechanical engineering student at HITsz, visited a doctor at SUSTech Hospital in Shenzhen for a routine health checkup. After reviewing his health report, the doctor smiled and said, "Faheem, you need more exercise. Starting tomorrow, walk four kilometers every morning." The next morning, Faheem put on his running shoes and prepared for his walk. Just as he was about to leave, a thought crossed his mind. "Wait a minute," he said to himself. "I study mechanics. The doctor told me to walk four kilometers, but he never told me in which direction. *Should I walk north, south, east, or west*?"

Being a curious and cautious engineering student, he began to analyze the problem. If he walked four kilometers east, would that improve his health more than walking four kilometers west? Of course not. The health benefit depended only on the total distance walked, not on the direction. "Ah, I see!" he exclaimed. "The doctor's instruction only specifies a magnitude—four kilometers. Direction does not matter." Satisfied with his reasoning, Faheem chose his favorite route

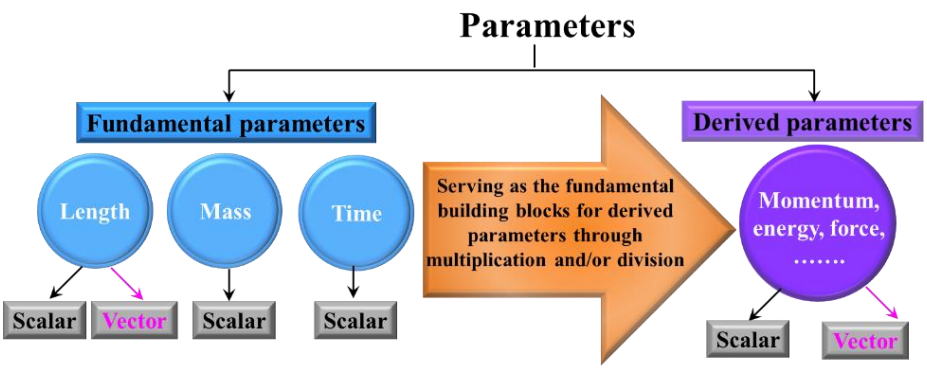

Fig. 5. Properties of fundamental and derived parameters. Length, mass, and time are fundamental parameters that form derived quantities through multiplication and division. Mass and time are scalars, defined by magnitude, while length's classification depends on context. In classical mechanics, displacement is a vector (with magnitude and direction), while distance is a scalar (with magnitude only). This distinction depends on the quantity's properties and context.

and completed a four-kilometer walk. In this situation, the quantity four kilometers represents a scalar quantity because only the magnitude is important.

A week later, during a follow-up visit, the doctor refers Faheem to Shenzhen University Hospital, located four kilometers from SUSTech Hospital, and advises him to walk there. This time, however, Faheem faces a different challenge. *Can he simply walk four kilometers in any direction*? Suppose he walks four kilometers west. Although he would successfully cover the prescribed distance, he would end up farther away from Shenzhen University Hospital rather than reaching it. To arrive at the destination, he must walk four kilometers in the specific northeast direction. In this case, both the magnitude and the direction are essential.

The quantity describing the location of Shenzhen University Hospital relative to SUSTech Hospital is therefore a vector quantity—specifically, a displacement vector of four kilometers directed northeast. Unlike the previous example, where only the distance traveled mattered, reaching a particular destination requires information about both how far to travel and in which direction to move. Thus, Faheem's first task involved a scalar quantity—the distance he needed to walk. His second task involved a vector quantity—the displacement required to reach a specific destination. The magnitude was the same in both cases (four kilometers), but only the second situation required a direction.

III. Fundamental Parameters and Their Significance

Classical mechanics is founded on three fundamental physical parameters—length, mass, and time—which form the basis of the MKS (meter-kilogram-second) unit system. These quantities serve as the essential building blocks from which all mechanical variables are constructed, including velocity, acceleration, force, energy, and momentum. Their designation as fundamental is not arbitrary; rather, it reflects the intrinsic ways in which natural phenomena manifest and are quantified. Length characterizes spatial dimensions, mass represents inertia or the amount of matter, and time governs the progression of motion. Together, they provide a minimal and complete framework for describing, measuring, and analyzing all mechanical processes in a consistent and physically meaningful manner.

The three fundamental parameters (mass, length, and time) in classical mechanics play a role analogous to prime numbers (2, 3, 5…) in mathematics and vowels (a, e, i, o, u) in language (Fig. 6). Just as prime numbers are indivisible (except by 1 and themselves) and serve as the basic building blocks from which all composite numbers are constructed, these fundamental parameters constitute the indivisible basis from which all mechanical quantities are derived. A similar analogy arises in language: vowels possess distinct, irreducible sounds, whereas consonants must combine with vowels to form complete syllables and meaningful words. In much the same way, derived physical quantities, such as force, energy, velocity, acceleration, and pressure, emerge from various combinations of mass, length, and time. Without these three fundamental parameters, the entire quantitative 'language' of mechanics would lose its structure and coherence. This analogy underscores the indispensable and foundational nature of the fundamental parameters. They function as the core elements that enable us to express and understand all mechanical phenomena in a systematic and

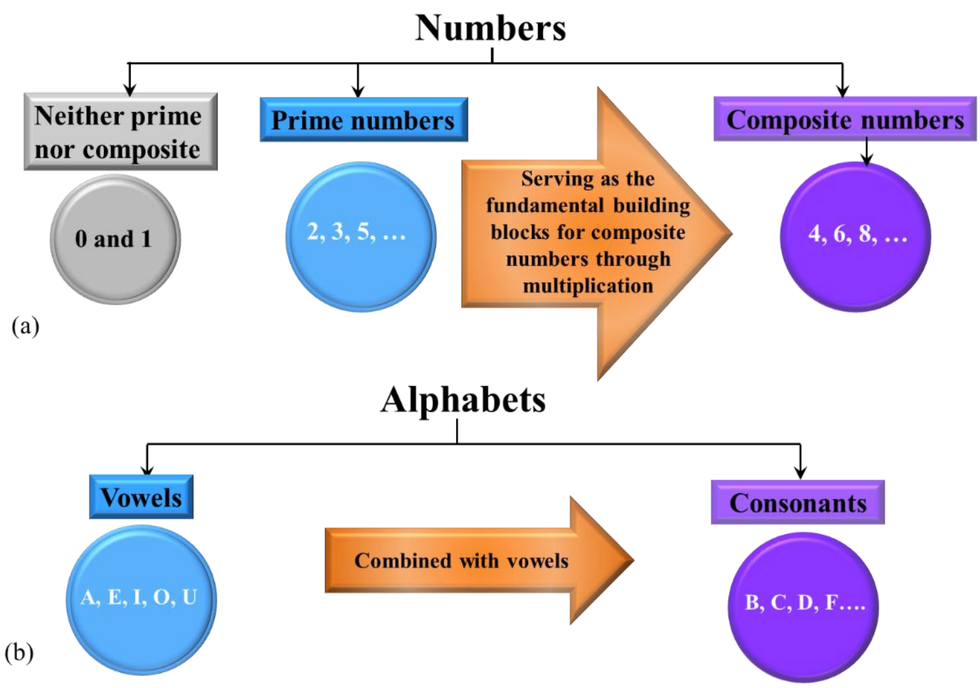

Fig. 6. Properties of (a) prime and composite numbers, and (b) vowels and consonants. Mass, length, and time, being the fundamental parameters, are much like prime numbers in mathematics or vowels in language. Just as prime numbers are the building blocks of all composite numbers, these three fundamental parameters form the basis from which all mechanical quantities are derived. It is similar to how vowels combine with consonants to form syllables and words and how force, energy, velocity, and other derived quantities emerge from combinations of mass, length, and time.

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consistent way. It is sometimes noted that every number is divisible by 1 and itself; however, this does not make 0 or 1 prime, as neither satisfies the formal definition of a prime number. Likewise, both fundamental and derived parameters can be divided by 1 or by another quantity of the same unit to produce nondimensional parameters. This does not diminish their foundational status; instead, it reflects the flexibility of dimensional analysis while preserving the unique role of mass, length, and time as the true building blocks of mechanics.

IV. Mathematical Operations Of Scalars

Scalars are physical quantities that are described solely by their magnitude and units, without any consideration of direction. Due to this lack of directional dependence, the value of a scalar remains invariant regardless of the chosen frame of reference or coordinate system, as illustrated in Fig. 7a. Operations involving scalars, such as addition, subtraction, multiplication, and division, follow the same rules as ordinary real numbers and are independent of any spatial considerations. However, there are important distinctions in how these operations apply to different scalar quantities. Specifically, the first two operations, addition and subtraction, are only valid for scalars that are of the same type and unit. In contrast, multiplication and division can occur with either the same or different scalar types and units, depending on the context. For example, we can add two quantities of the same type and unit: 2 m^3 + 7 m^3 = 9 m^3 and 6 kg + 12 kg = 18 kg, because both added quantities are of the same type and unit. However, adding quantities of different types, such as 2 m^3 + 6 kg, is not meaningful because they represent different physical quantities—volume and mass—and therefore cannot be directly combined. On the other hand, multiplication and division can yield meaningful results even when the scalar quantities have different types or units. For example, dividing mass by volume gives a new physical quantity: density. If you divide 6 kg by 2 m^3, the result is 3 kg/m^3, which represents the density of the material.

V. Mathematical Operations of Vectors

Since vectors are defined by both their magnitude and their direction in space, this dual nature makes vectors sensitive to the choice of coordinate system or frame of reference (Fig. 7b). While the vector itself represents the same physical quantity, its components (such as its x, y, and z components) change when observed from different frames or coordinate systems. This sensitivity arises because vectors are not just abstract quantities; they are inherently tied to the space in which they exist and the chosen reference frame. As a result, the mathematical treatment of vectors involves operations such as vector addition, subtraction, and multiplication, which adhere to specific rules. The addition or subtraction of two vectors (again of the same type or unit) results in a new vector of the same type or unit. Multiplication can occur with vectors of the same or different types, or even with scalars. For example, momentum is the scalar product of mass (a scalar, unit kg) and velocity (a vector, unit m/s). Conversely, the moment is the cross product of displacement (a vector, unit m/s) and force (a vector, unit N).

A. *Ambiguity of vector division*

For instance, the dot product and cross product provide scalar and vector results, respectively, depending on the nature of the interaction between the two vectors. These operations enable vectors to represent not only magnitudes and directions but also the relationships between different vector quantities in space. The division of two vectors is not defined in vector algebra because vectors have both magnitude and direction, and there is no straightforward way to 'divide' one vector by another in the same sense as scalar division. Vector division, such as $\vec{C}=\frac{\vec{A}}{\vec{B}}$, implies finding a number or entity $\vec{C}$ that, when multiplied by the second vector $\vec{B}$, results in the first vector $\vec{A}$ (Fig. 8). However, in a vector space, there is no well-defined operation that you can multiply one vector by another to produce a vector in the same direction.

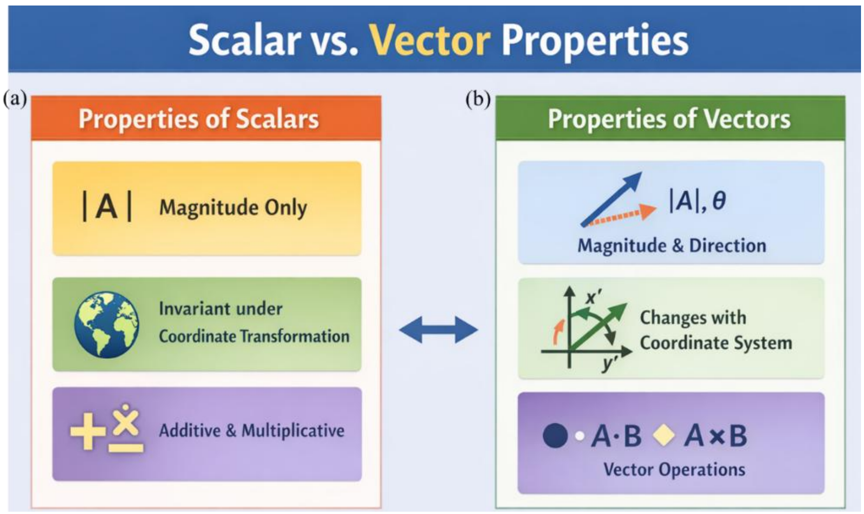

Fig. 7. Properties of (a) scalar and (b) vectors. Addition and subtraction occur within the same scalar or vector type, while multiplication and division can involve different types or units. Vector operations, including addition, subtraction, and multiplication, follow specific rules and may interact with scalars or other vectors, as demonstrated by momentum (scalar product) and moment (cross product).

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Multiplication can, in fact, lead to a scaled replica of the original vector (multiplication of a scalar by a vector) or a vector in a new direction through processes such as the cross product.

In other words, while multiplication of vectors is defined through the dot product for scalar results and the cross product for vector results, no analogous operation exists for division. The cross and dot products deal with specific geometric relationships, such as the angle between vectors or perpendicularity. In contrast, division requires an operation that reverses these relationships, which is not mathematically feasible in the same way. For example, consider two vectors $\vec{A}$ and $\vec{B}$, lying on *x-y* plane with an included angle θ (Fig. 8). Their cross product is $\vec{C}=\vec{A}\times\vec{B}$, where the direction of $\vec{C}$ is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ (i.e., *x-y* plane). If vector division were feasible (indeed not), one could hypothetically find $\vec{A}$ by performing the operation $\vec{A}=\frac{\vec{C}}{\vec{B}}$. However, this would lead to three potential solutions: $C_d=\vec{A}.\vec{B}$ or $\vec{C}_c=\vec{A}\times\vec{B}$ or $\vec{C}_{c'}=\vec{B}\times\vec{A}$. The first solution is a scalar, whereas the result is expected to be a vector; the second reproduces the original vector; and the third yields a vector opposite in direction to the original one. Thus, the concept of 'reversing' the effect of one vector on another becomes ambiguous. In other words, the result of $\frac{\vec{C}}{\vec{B}}$ is indeterminate. For these reasons, vector division is not a valid operation in vector algebra. Instead, relationships between vectors are described using well-defined operations such as the dot product, cross product, or vector projection. This discussion is useful for clarifying the distinction between vectors and scalars and for identifying appropriate vector operations.

B. *Power and beauty of vector multiplication*

Vector multiplication extends simple geometric ideas into powerful mathematical tools for describing physical reality. Unlike scalar multiplication, vector multiplication is not a single operation but a family of dot and cross products, each capturing a distinct aspect of the interaction between vectors. For two vectors $\vec{A}$ and $\vec{B}$, the dot product is defined as

$$C_d=\vec{A}.\vec{B} \quad (1)$$

while the cross product of them is expressed as

$$\vec{C}_c=\vec{A}\times\vec{B} \quad (2)$$

Considering $\vec{A}=a_x i+a_y j+a_z k$ and $\vec{B}=b_x i+b_y j+b_z k$, one can find the dot product of $\vec{A}$ and $\vec{B}$ as

$$C_d=\vec{A}.\vec{B}=a_x b_x+a_y b_y+a_z b_z \quad (3)$$

Notably, although the components of $\vec{A}$ and $\vec{B}$ are associated with the coordinate direction (*i*, *j*, *k*), the unit vectors do not appear in the dot product expression C_d (Eq. 3), implying that $\vec{A}.\vec{B}$ has no direction. The C_d or $\vec{A}.\vec{B}$ is, therefore, a scalar quantity, independent of the choice of coordinate system, much like the mass of an object. The dot product simply represents the sum of the products of the corresponding components of the two vectors.

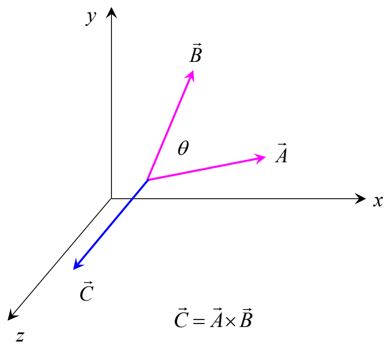

Fig. 8. Vector operations.

Since the magnitude and direction of a vector are independent of the coordinate system, one may choose a coordinate axis aligned with either $\vec{A}$ or $\vec{B}$, leading to the well-known geometric form

$$\vec{A}.\vec{B} = \text{AB}\cos\theta, \quad (4)$$

where A and B denote the magnitudes of vectors $\vec{A}$ and $\vec{B}$, respectively, and θ is the included angle between them. When $\vec{A}$ and $\vec{B}$ are orthogonal, their dot product is zero.

Now the cross product of $\vec{A}$ and $\vec{B}$ can be written as

$$\vec{C}_c=\vec{A}\times\vec{B}=(a_y b_z-a_z b_y)i+(a_z b_x-a_x b_z)j+(a_x b_y-a_y b_x)k \quad (5)$$

The $\vec{A}\times\vec{B}$ stays as a vector, whose direction is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ (Fig. 8). Equivalently, the vectors $\vec{A}$ and $\vec{B}$ are each perpendicular to $\vec{A}\times\vec{B}$, which allows the relationship to be expressed compactly as

$$(\vec{A}\times\vec{B}).\vec{A}=(\vec{A}\times\vec{B}).\vec{B}=0 \quad (6)$$

This orthogonality property is a fundamental principle in vector mechanics.

The outcomes of different combinations of scalar and vector operations may now be summarized in Table 2. Any scalar–scalar operation involving addition, subtraction, multiplication, or division yields a scalar; for example, density is defined as mass divided by volume, where all quantities are scalars. Scalar–vector operations, including multiplication, or division by a scalar, result in a vector; a familiar example is force, defined as the product of mass and acceleration. In contrast, vector–vector operations may yield either a scalar or a vector, depending on the operation employed. As discussed above, the dot product of two vectors produces a scalar quantity; for example, work is defined as the dot product of force and displacement. Conversely, the cross product of two vectors results in a vector quantity; for instance, torque is defined as the cross product of displacement and force. In summary, new vectors may be generated through several fundamental operations: (i) multiplication by a scalar, (ii) differentiation with respect to a scalar, (iii) vector addition or subtraction, and (iv) the cross product of two vectors.

VI. Identification of Vectors and Scalars

As stated above, length, mass, and time are the three fundamental physical parameters from which all other mechanical quantities are derived. A natural question then arises: can vectors and scalars be identified simply by relying on human intuition or 'sense' of magnitude and direction?

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Such an approach, however, is inherently unreliable. Human perception is subjective, varies from person to person, and may lead to inconsistent or even contradictory conclusions. Consider, for example, the concept of work or energy transfer between a flowing fluid and an oscillating structure submerged in the fluid. One may intuitively sense a direction of transfer—from the fluid to the structure or vice versa—yet work is defined as a scalar quantity. Conversely, when considering 'area', most people do not perceive an inherent direction, and yet 'area' is formally defined as a vector quantity, as is proved later. Pressure provides another instructive example: although it acts perpendicular to a surface and thus appears to possess a direction, it is nevertheless a scalar quantity. Similarly, speed and velocity are both defined as length per unit time, but speed is a scalar, whereas velocity is a vector. These examples clearly demonstrate that intuition alone is insufficient and often misleading when identifying vectors and scalars.

Consider another illustrative example. If a person wearing clothing typically associated with the opposite gender is shown to a group of observers, some may identify the person one way while others may reach a different conclusion. Even if all observers agree on a particular identification based on appearance alone, that conclusion may still be incorrect. This highlights the unreliability of classification based solely on outward perception. In contexts where an unambiguous determination is required—such as in legal or medical settings—subjective judgment must be replaced by an objective and well-defined criterion. Biological sex, for example, is determined not by appearance but by genetic origin, where chromosomes (XX or XY) provide a definitive classification. The key point is that correct identification depends on how an entity originates, not on how it is perceived.

By analogy, whether a derived physical quantity is a vector or a scalar cannot be decided by intuition or perceived directionality alone. Instead, it must be rigorously inferred from its origin; specifically, how it is constructed or derived from the fundamental parameters of length, mass, and time. Relying on perception or 'common sense' to classify physical quantities inevitably leads to ambiguity, inconsistencies, and disagreement. A systematic approach grounded in their fundamental dimensional composition may ensure clarity and universality in mechanics. In physics, that objective criterion lies in the origin of a quantity. Here, we step-by-step show that the vector or scalar nature of a physical quantity can be unambiguously identified by examining how it originates from these fundamental parameters. The origin of a derived (definiendum) parameter is the mathematical operations of the definiens.

A. Velocity and speed

Velocity and speed both originate or are derived from the fundamental quantities of length and time. The interpretation of length, however, can vary depending on its definition and context, influencing whether it is treated as a scalar or a vector. Although we possess an intuitive understanding of what *speed* represents, a precise definition reveals several subtle and nontrivial conceptual issues. These subtleties arise when one attempts to distinguish rigorously between *speed* and *velocity*, a distinction that historically caused considerable confusion. To illustrate this subtlety, consider the following problem. A spherical balloon is inflated such that its *volume* increases at a rate of 10 m^3/s. At what rate is the *radius* increasing when the volume of the balloon is 100 cm^3? The immediate question that arises is whether the rate of increase of the radius should be interpreted as a *speed* (a scalar) or a *velocity* (a vector). Intuitively, one might argue that since the radius increases in the radial direction, the associated rate must be a velocity. However, this interpretation is misleading. The key lies in examining how the quantity originates. The

Table 2. Summary of outcomes of scalar and vector operations

Operands	Operation(s)	Resulting Quantity	Example
Scalar – Scalar	Addition, subtraction, multiplication, division	Scalar	Density = mass/volume
Scalar – Vector	Multiplication, division by a scalar	Vector	Force = mass × acceleration
Vector – Vector	Addition, subtraction	Vector	Relative velocity = $\vec{V}_2 - \vec{V}_1$
Vector – Vector	Dot product	Scalar	Work = $\vec{F} . \vec{S}$
Vector – Vector	Cross product	Vector	Torque = $\vec{r} \times \vec{F}$

M.M. Alam, "A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics," *International Research Journal of Advanced Engineering and Science*, Volume 11, Issue 2, pp. 101-116, 2026.

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process is initiated by a change in volume, which is a scalar quantity, as is demonstrated later. The rate of change of volume is therefore also a scalar. In fact, the volume of the balloon increases simultaneously in infinitely many radial directions. A quantity associated with infinitely many directions has no unique direction and hence cannot be a vector. Consequently, the rate of increase of the radius is properly interpreted as a scalar speed, not a velocity.

A closely related example arises in fluid mechanics. Consider the pressure at a point in a fluid. Regardless of whether the fluid is at rest or in motion. Pressure at a point acts equally in all directions, forming what may be visualized as a 'zero-radius sphere' of action. Because it is associated with infinitely many directions and no preferred orientation, pressure is not a vector but a scalar—a result that can be demonstrated rigorously using continuum mechanics.

The distinction between speed and velocity may now be clarified through their definitions. *Speed* is defined as the rate of change of distance with time, whereas *velocity* is defined as the rate of change of displacement with time. To understand the difference between distance and displacement, consider an object moving from point O to point A along a curved path, as shown in Fig. 9. The *distance* L is the total length of the path traveled by the object, while the *displacement* is the vector ***OA***, whose magnitude S is the straight-line distance between O and A and whose direction is from O to A. If the total travel time is t seconds, the average speed is L/t, whereas the average velocity has a magnitude S/t. Since $L \geq S$, the magnitude of the average velocity is always less than or equal to the average speed. In the extreme case where the object returns to its starting point, the displacement is zero, and hence the average velocity is zero, even though the distance traveled and, therefore, the average speed, is nonzero. As t is a scalar, scalar division preserves vector direction, and therefore the velocity vector ***OA****/t* is collinear with the displacement vector ***OA***. In other words, speed = distance/time → scalar/scalar → scalar, while velocity = displacement/time → vector/scalar → vector (Table 2).

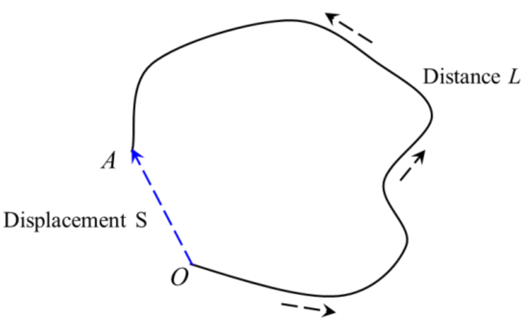

Fig. 9. Motion of an object showing distance and displacement.

One evening, Faheem took a taxi across campus to deliver a book to a friend. After handing over the book, he returned in the same taxi and arrived back at his starting point exactly 20 minutes later. "How far did we travel?", he asked the driver. "About seven kilometers," the driver replied. "And what was my displacement?" The driver looked puzzled. "Zero," Faheem answered proudly. The driver laughed. "Then why am I charging you for seven kilometers instead of zero?" "Because you are paid for distance, not displacement," Faheem replied. "Excellent," said the driver. "If taxi fares were based on displacement, I would have spent 20 minutes driving you around for free."

B. Area

The notion of area is often introduced as a purely geometric measure. While this interpretation is adequate in elementary geometry, it is incomplete in the context of mechanics and field theory. In many physical applications, a surface is characterized not only by its size but also by its orientation in space. This naturally leads to the concept of the area vector, whose magnitude equals the surface area and whose direction is defined by the unit normal to the surface. The vector area is fundamental in the formulation of surface integrals, fluxes of vector fields, and the transmission of stress across surfaces, and it plays a central role in continuum mechanics and fluid dynamics. Yet, when we ask this question in class, students almost always answer without hesitation: "*Area is simple: a number, like four square meters. Area is a scalar.*" It is one of the few results in physics where the answer is given so confidently that one might suspect the question was never asked at all.

In fluid mechanics, the volume flow rate (flux) through a surface of area A is defined as $Q = \vec{V}.\vec{A}$ where the direction of $\vec{A}$ is along the outward unit normal, and $\vec{V}$ is the fluid velocity. Here, Q, representing volume per unit time, results from the dot product of two vectors, and is therefore a scalar. Similarly, quantities such as mass flux, momentum flux, and force flux, which are obtained through dot products involving an area vector, yield scalar quantities. This interpretation is also consistent with Gauss' divergence theorem, which states: $\iiint_{\Omega} \vec{\nabla}.\vec{F} d\Omega = \iint_{A} \vec{F}.d\vec{A}$, where Ω represents a control volume, $\vec{F}$ is a vector field, and $\vec{A}$ is the surface area vector. Here, area explicitly appears as a vector quantity. This naturally raises the question: *what is the mathematical basis for treating area as a vector?* To answer this, we examine how the area vector originates from the three (mass, length, time) fundamental parameters.

It is natural to recognize that area is derived from length measures. Consider two displacement vectors $\vec{B}$ and $\vec{C}$ enclosing an angle θ, as shown in Fig. 10a. Their cross product is defined as $\vec{A} = \vec{B} \times \vec{C}$. From vector algebra, the direction of $\vec{A}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{C}$, following the right-hand rule. Yet, at this stage, we do not know the physical significance of $\vec{A}$. The magnitude of $\vec{A}$ is A = BCsinθ, where B and C are the lengths of the two sides of the parallelogram formed by $\vec{B}$ and $\vec{C}$ (Fig. 10b). Rearranging the definition A = B|Csinθ| = C|Bsinθ|, we can find that A is precisely the area of the parallelogram formed by $\vec{B}$ and $\vec{C}$ (Fig. 10b). Thus, the cross product yields a vector whose magnitude equals the surface area and whose direction uniquely defines the surface orientation. This construction provides the mathematical foundation for the concept of

M.M. Alam, "A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics," *International Research Journal of Advanced Engineering and Science*, Volume 11, Issue 2, pp. 101-116, 2026.

vector area and justifies its use in flux integrals and continuum mechanics.

In the context of coordinate transformations, identifying whether an *area* is a scalar or a vector can be a complex, confusing, and sometimes contentious task. The classification depends on how the area behaves under transformations such as rotation, translation, and reflection. On one hand, an area has magnitude (which is easy to sense), but whether it has direction is not immediately apparent without further consideration. In fact, without considering the directional component, the transformation technique alone cannot identify whether a quantity has direction or not, as the transformation method itself focuses only on the magnitude. Therefore, under the basic principles of coordinate transformations, area is a scalar because it satisfies the fundamental requirement for scalars: invariance under rotation and translation. However, this classification becomes misleading or inadequate when we account for the directionality of area in more complex contexts, such as in vector calculus or surface integrals. In these cases, area is a directional component, i.e. vector. Thus, while area is typically classified as a scalar in basic coordinate transformations, this classification fails to capture the full complexity of its directional nature in more advanced applications, where its vector-like properties become evident.

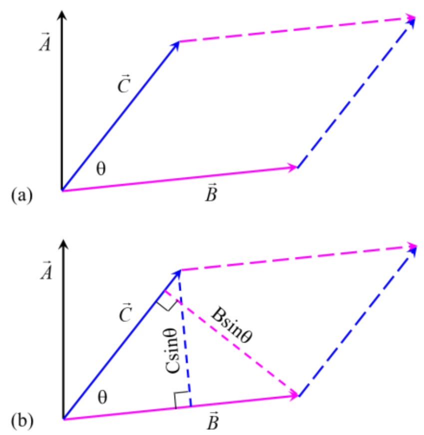

Fig. 10. Representation of area as a vector obtained from the cross product of two vectors.

One windy afternoon, Faheem was flying a rectangular kite in an open field. He noticed that the kite pulled strongly on the string when its broad face was oriented perpendicular to the wind. However, whenever the kite turned edge-on to the wind, the pulling force almost disappeared, and the kite seemed to slip effortlessly through the air. Puzzled, he wondered, "How can the same area behave so differently when its size never changes?" He paused for a moment and recalled a lecture from his fluid mechanics class. *Area is more than just a number; every surface has a hidden direction represented by a vector normal to the surface.* He imagined the wind velocity as a vector $\vec{V}$ and the kite surface as an area vector $\vec{A}$. The amount of air approaching the surface depends on the dot product $\vec{V}.\vec{A}$. When the kite faces the wind, the area vector is nearly aligned with the wind velocity, making the dot product large. As a result, a large volume of air encounters the kite surface every second. The airflow is forced to change its velocity and direction as it passes around the kite, producing a substantial change in momentum and, consequently, a large aerodynamic force. When the kite turns edge-on, however, the area vector becomes perpendicular to the wind velocity, causing the dot product to approach zero. In this orientation, almost no air approaches the surface in the normal direction. The disturbance to the airflow and the associated momentum change become minimal, and the aerodynamic force is greatly reduced.

C. Volume

After students begin to accept the idea that area can be treated as a vector in the context of surface integrals, they often become noticeably more cautious when a similar question is posed about volume. When asked whether volume is a vector or a scalar, the students pause, exchange thoughtful glances, and—clearly determined not to fall into another conceptual trap—respond almost unanimously: "Volume must be a vector, since it is enclosed by surface areas, which are vectors." At this point, their reasoning becomes impressively structured, if slightly overconfident. Some even begin to gesture in three dimensions, as if trying to convince the air itself that volume must have a direction hidden somewhere inside it.

Volume is a fundamental geometric measure that quantifies the three-dimensional extent of a body or region in space and is inherently a scalar quantity. Unlike displacement or area, volume possesses magnitude only and no associated direction or orientation. In physical and mathematical formulations, volume appears as a positive real number representing size, independent of how the body is positioned or oriented in space. This scalar nature of volume is essential in continuum mechanics and field theory, where conservation laws—such as the conservation of mass and energy—are expressed through volume integrals that depend solely on magnitude, not on direction. The scalar character of volume is also evident in applications such as the volume flow rate $Q = \vec{V}.\vec{A}$, which is unambiguously a scalar quantity. While this is easily perceived from its physical interpretation, such an observation alone does not explain the origin or fundamental nature of volume itself. To compute volume geometrically, three independent spatial dimensions are required: breadth (*x*-direction), width (*y*-direction), and height (*z*-direction). Each of these dimensions is fundamentally a length scale and may be represented by vectors. This naturally raises an important question: how can three vectors combine to produce a scalar quantity?

To address this, consider three vectors $\vec{B}$, $\vec{C}$ and $\vec{D}$ as shown in Fig. 11a. Let θ denote the angle between $\vec{B}$ and $\vec{C}$, and let ϕ be the angle between $\vec{D}$ and the normal to the plane containing $\vec{B}$ and $\vec{C}$. As previously established, the cross product $\vec{B}\times\vec{C}=\vec{A}$ represents the vector area of the parallelogram formed by $\vec{B}$ and $\vec{C}$ with magnitude A = BCsinθ (Fig. 11b). This area serves as the base of a parallelepiped constructed using the three vectors $\vec{B}$, $\vec{C}$ and $\vec{D}$. The height of the parallelepiped can be obtained by projecting $\vec{D}$ on the normal direction of the base plane. Since $\vec{D}$ makes an angle ϕ with this normal $\vec{A}$, the effective height is Dcosϕ. Therefore, the volume of the parallelepiped can be written as

$$\text{Volume} = \text{AD}\cos\phi = \vec{A}.\vec{D} = (\vec{B}\times\vec{C}).\vec{D} \quad (7)$$

The volume is thus just a scalar triple product of $\vec{B}$, $\vec{C}$ and $\vec{D}$. This could also be written as

$$\text{Volume} = (\vec{B}\times\vec{C}).\vec{D} = (\vec{D}\times\vec{B}).\vec{C} = (\vec{C}\times\vec{D}).\vec{B} \quad (8)$$

The scalar triple product yields a signed scalar whose magnitude equals the volume of the parallelepiped and whose sign depends on the orientation (handedness) of the vector set. Importantly, the final result contains no directional information—only magnitude—thereby demonstrating that volume, despite originating from three vector quantities, is fundamentally a scalar. This origin-based analysis provides a rigorous justification for the scalar nature of volume in geometry, mechanics, and field theory.

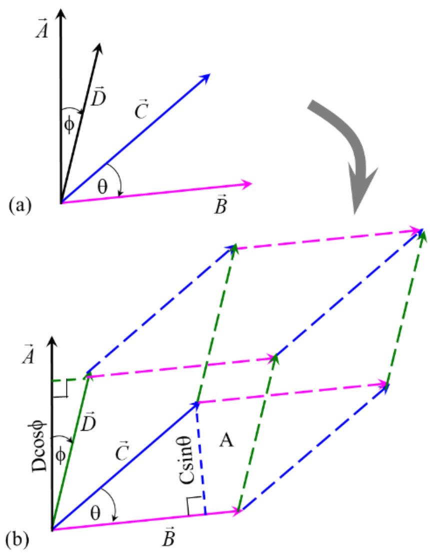

Fig. 11. Representation of volume as a scalar obtained from the cross and dot product of three vectors.

D. Pressure and force

Pressure and force are fundamental physical quantities in mechanics, yet they differ essentially in their mathematical nature. Pressure is defined as the normal force exerted per unit area. When pressure acts on a surface, it produces a force whose direction is dictated by the surface normal. While the vector nature of force is readily perceived, the nature of pressure, whether scalar or vector, requires more careful examination.

From classical mechanics, force is given by $\vec{F} = m\vec{a} = m\mathrm{d}^2\vec{s}/\mathrm{d}t^2$, where mass m is a fundamental scalar parameter, displacement $\vec{s}$ is a fundamental vector parameter, and time t is a fundamental scalar parameter (Fig. 5). Since acceleration $\vec{a} = \mathrm{d}^2\vec{s}/\mathrm{d}t^2$ is the second derivative of a vector with respect to a scalar, it is itself a vector. Consequently, force is also a vector, with its direction coinciding with that of the acceleration. In other words, $\vec{F} = m\vec{a} \rightarrow \textit{scalar times vector} \rightarrow \textit{vector}$ (Table 2), which implies that the force is just the acceleration vector magnified by *m*.

We now turn to pressure. Unlike force, its vector or scalar nature is not immediately obvious. After mastering the distinction between vectors and scalars during his morning walks, Faheem became curious about another quantity he frequently encountered in fluid mechanics: *pressure*. One afternoon, while visiting a laboratory at HITsz, he noticed a large transparent water tank. A pressure sensor was mounted inside the tank at a point several meters below the water surface. Looking at the setup, he wondered: "*Is pressure a vector or a scalar*?" At first glance, he thought pressure might be a vector, just as many students do. After all, the water in the tank has weight, and weight is caused by gravity, which acts downward. Since gravity has a direction, perhaps the pressure should also point downward.

To test the idea, Faheem imagined placing three identical pressure sensors at the same depth: one facing upward, another sideways, and the third downward. If pressure were a vector, the readings should depend on the orientation of the sensor. A downward-facing sensor, he reasoned, ought to measure something different from an upward-facing one. "*That is surprising*," Faheem said. "*The pressure is the same regardless of which way the sensor faces*!" He paused for a moment, trying to make sense of it. A quantity that behaves identically in every direction cannot be assigned a single preferred direction.

His thoughts drifted to an analogy. It is like a student who wants to be a professor, entrepreneur, astronaut, football player, actor, and musician all at the same time—perhaps they have not really chosen a direction at all. He then chuckled, recalling his professor's remark: "And if a PhD student says, 'I will do this, this, and this, and also a bit of everything,' then the research itself may end up lacking a clear direction."

Up to this point, physical intuition may suggest that pressure exhibits vector-like characteristics. However, the preceding discussion and experimental observations indicate that pressure is independent of direction, manifesting identically in all orientations at a given point in a fluid at rest. This observation naturally raises a fundamental question: *how can one rigorously demonstrate that pressure is a scalar quantity*?

To address this, consider a fluid at rest in a tank (Fig. 12). At a point located at a depth $\vec{h}$ below the free surface, the pressure P depends on the gravitational acceleration $\vec{g}$, the depth vector $\vec{h}$, and the fluid density ρ. Since density is defined as mass per unit volume, and both mass and volume are scalar quantities, the density ρ is explicitly a scalar. Then $\rho\vec{g}$ remains a vector quantity, representing the gravitational force per unit volume of the fluid. The question then arises: how do the two vector quantities $\rho\vec{g}$ and $\vec{h}$ combine to produce pressure? The interaction between two vectors can occur through either a cross product or a dot product. Because $\rho\vec{g}$ and $\vec{h}$ are collinear, their cross product vanishes, yielding no physically meaningful result. The only viable interaction is

M.M. Alam, "A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics," *International Research Journal of Advanced Engineering and Science*, Volume 11, Issue 2, pp. 101-116, 2026.

therefore the dot product, leading to $P = \rho\vec{h}.\vec{g} = \rho gh$. This expression explicitly demonstrates that *pressure* is a *scalar quantity,* as the dot product of two vectors produces a scalar.

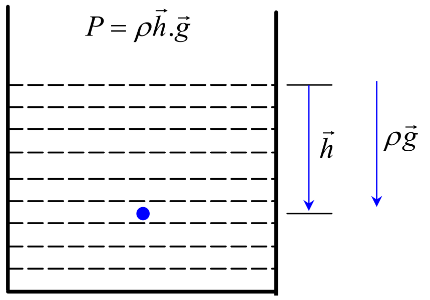

Fig. 12. Representation of pressure P as a scalar quantity, obtained from the dot product of height and gravitational acceleration

Finally, consider the *pressure force* acting on a surface. The pressure force is obtained by multiplying the scalar pressure by the vector area of the surface, $\vec{F}_P = P\vec{A} = \int P\mathrm{d}\vec{A}$. Since the area vector $\vec{A}$ has both magnitude and direction (defined by the surface normal), the resulting pressure force $\vec{F}_P$ is necessarily a vector, and its direction is normal to the surface. Thus, although pressure itself is a scalar, it gives rise to a vector force when acting on an oriented surface. It should be noted that, without treating the area explicitly as a vector, rigorously demonstrating that the pressure-induced force is a vector becomes challenging.

E. Work and energy

Work W, being associated with force and displacement, is defined as the product of a force and the component of displacement in the direction of that force. Equivalently, it may be expressed as the product of the displacement and the component of force along the displacement. This definition is naturally written as $W = \vec{F}.\vec{S} = \int \vec{F}.\mathrm{d}\vec{S}$. Since work is defined as the dot product of two vectors, W is a scalar quantity. In some cases, the work done by a force depends only on the initial and final positions of the system and is independent of the path taken. Such forces are called conservative forces. Typical examples include the gravitational force and the elastic (spring) force. For a conservative force, the work done over any closed path is zero, i.e., $W = \oint \vec{F}.\mathrm{d}\vec{S} = 0$. In contrast, when the work done by a force depends on the specific path followed $(W = \oint \vec{F}.\mathrm{d}\vec{S} \neq 0)$, the force is termed non-conservative. Common examples of non-conservative forces are friction and viscous forces. Work represents a process of energy transfer from one form or system to another. Consequently, work and energy share the same unit and dimensions and are both scalar quantities.

Consider kinetic energy, a particular form of energy associated with a moving object of velocity $\vec{V}$. One might initially suspect that, since the object moves in a definite direction, kinetic energy could also possess direction and thus be a vector. To clarify this, let us examine how kinetic energy originates. Suppose a body is initially at rest at time $t = 0$ and reaches a velocity V at time t under the action of a constant acceleration a. The acceleration results from an applied force $F = ma$, and during this time interval, the body undergoes a displacement $S = V^2/2a$. What is the kinetic energy at time t? How does this kinetic energy originate? Since F and S have the same direction, the work done by the force is $W = \vec{F}.\vec{S} = FS = maV^2/2a = mV^2/2 = m\vec{V}.\vec{V}/2$. This derivation demonstrates that although kinetic energy originates from the dot product of velocity vectors. Kinetic energy is thus unequivocally a scalar quantity, possessing magnitude only and no direction.

F. Moment and torque

Moment, or torque, measures the rotational effect of a force about a point or axis. Mathematically, it is defined as the cross product of the position vector and the applied force,

$$\vec{M} = \vec{r} \times \vec{F} \tag{9}$$

Because it arises from a cross product, torque is a pseudovector: it has a direction perpendicular to the plane of rotation (determined by the right-hand rule) and a magnitude proportional to the lever arm and force. Unlike a scalar, torque encodes both the magnitude of rotation and the axis about which rotation occurs, making it essential for describing rotational dynamics in three dimensions.

Why are angular quantities vectors despite angle being a scalar?

Let us begin by examining the nature of the angle itself. Is an angle a vector or a scalar? If the angle is a scalar, how can related quantities such as angular displacement, angular velocity, and angular acceleration be vectors? An angle measures the *magnitude of rotation* between two directions or vectors and, by itself, has no independent spatial direction. In mathematics and physics, an angle is commonly defined as the ratio of arc length to radius,

$$\theta = \frac{s}{r} \tag{10}$$

From this definition, the angle is dimensionless and is usually regarded as a scalar since both s and r are treated as scalar lengths. This interpretation is generally acceptable, as length can be viewed either as a scalar magnitude or as the magnitude of a vector (Fig. 5).

However, this scalar interpretation of angle leads to an apparent paradox. If θ is purely scalar, then its time derivative—angular velocity $\vec{\omega} = d\theta/dt$ —would also be scalar. This contradicts physical experience, since angular velocity clearly possesses a direction, namely one perpendicular to the plane of rotation.

In two-dimensional problems, this issue is often bypassed by assigning a *sign* to the angle (positive or negative) to indicate clockwise or counterclockwise rotation. In three-dimensional space, the situation becomes more subtle: a rotation must be associated with an *axis*, and the direction of this axis is conventionally determined by the right-hand rule. These conventions introduce directional information into rotational descriptions, making it increasingly subtle to maintain the interpretation of angle as a purely scalar quantity.

M.M. Alam, "A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics," *International Research Journal of Advanced Engineering and Science*, Volume 11, Issue 2, pp. 101-116, 2026.

A closer examination of the definition $\theta = s / r$ reveals the source of the difficulty. The expression is meaningful only if both s and r are treated as scalars. Yet, for motion along a curved path, the radius vector generally changes in both magnitude and direction, and the arc length is traced along a specific direction on the curve. In this sense, neither r nor the infinitesimal arc ds can be regarded as purely scalar quantities. Treating both as vectors makes the ratio $\vec{s}/\vec{r}$ ill-defined, since division of vectors is not a valid operation.

To resolve this, the angle itself must be interpreted not as a true vector, but as a *pseudovector* (axial quantity) whose magnitude is the rotation angle and whose direction is along the axis of rotation. In this interpretation, the angular displacement can be written in vector form as

$$\vec{\theta} = \frac{\vec{r} \times \vec{s}}{\vec{r}.\vec{r}} = \frac{\vec{r} \times \vec{s}}{r^2} \quad (11)$$

That is now well-defined. As $\vec{r}$ and $\vec{s}$ are locally perpendicular, $\vec{\theta}$ is normal to the plane formed by them, giving the rotation a definite directional sense.

Without this interpretation, treating angular velocity as a vector would be inconsistent, since none of the quantities on the right-hand side of $\vec{\omega} = d\theta / dt$ is a vector. With this interpretation, however, the vector nature of angular velocity follows naturally and consistently. The angular velocity vector is defined as

$$\vec{\omega} = \frac{d\vec{\theta}}{dt} \quad (12)$$

Or equivalently,

$$\vec{\omega} = \frac{d(\vec{r} \times \vec{s})}{r^2 dt} = \frac{1}{r^2}\left((d\vec{r} / dt \times \vec{s}) + (\vec{r} \times d\vec{s} / dt)\right) \quad (13)$$

On the right-hand side, $d\vec{r} / dt$ and $\vec{s}$ are parallel, and hence their cross product vanishes. The equation can thus be reduced to

$$\vec{\omega} = \frac{\vec{r} \times \vec{V}}{r^2} \quad (14)$$

which clearly means a well-defined direction. This formulation implies that $\vec{\theta}$, $\vec{\omega}$, and $\vec{\alpha}$ $(= d^2\vec{\theta} / dt^2)$ all share the same axial direction.

Finally, the physical meaning of angular velocity is evident from its relationship with linear motion. For a point rotating with $\vec{\omega}$, the linear (transverse) velocity is given by

$$\vec{V} = \vec{\omega} \times \vec{r} \quad (15)$$

Thus, the linear velocity is perpendicular to both $\vec{r}$ and $\vec{\omega}$, lying in the plane of motion

Overall, although the angle itself is a scalar, rotational motion in three-dimensional space necessarily involves an axis. Angular displacement, angular velocity, and angular acceleration therefore emerge as pseudovectors, encoding both the magnitude of rotation and its directional sense.

One day, Faheem tried to open a heavy laboratory door by pushing near the hinges. Nothing happened. He pushed harder, but the door still refused to move. A janitor passing by noticed his struggle and gently pushed the same door near the handle. To Faheem's surprise, the door swung open effortlessly. "How did you do that?" he asked. "I used experience," the janitor replied. "But I used much more force!" Faheem protested. "Yes," said the janitor, "but the door cares about torque, not your enthusiasm." Faheem paused and reflected on the lesson. Rotational motion depends not only on the magnitude of the force but also on the lever arm. A small force applied far from the hinge can generate more torque than a much larger force applied close to it. "Interesting," he remarked. "Indeed," replied the janitor. "In mechanics, where you apply the force is often more important than how much force you apply." He then pointed toward the research laboratory and added, "Research is not very different. A student may work day and night, investing enormous amounts of time, effort, and resources, yet achieve little progress if the work is directed along the wrong path. Hard work alone does not guarantee success. There are infinitely many wrong ways to approach a problem, but only a few correct ones. Finding the right direction is often more valuable than simply pushing harder." As Faheem walked away, he realized the lesson: whether in mechanics or research, success comes not just from effort, but from effort aimed in the right direction.

VII. Identification of True Vectors, Pseudo Vectors, True Scalars, and Pseudo Scalars

Vectors may be classified into true (polar) vectors and pseudovectors (axial vectors), while scalars may be categorized into true scalars and pseudoscalars. Although these quantities are often introduced in terms of magnitude and direction, such geometric intuition alone is generally insufficient to distinguish between true and pseudo quantities. A rigorous distinction is usually made based on their transformation behavior under coordinate reflection (parity transformation). An alternative and practically convenient approach is the parent–offspring technique, in which the nature (true or pseudo) of a resulting quantity is identified from the types of its operands and the mathematical operation involved. This approach is particularly useful in continuum mechanics and fluid dynamics, where vector and scalar quantities frequently arise from combinations of other physical variables.

True scalars can be obtained from two types of parent operations (Table 3):

(i) *true scalar – true scalar* operation, including addition, subtraction, multiplication, and division, which naturally preserve scalar character (e.g., density).

(ii) *true vector – true vector* operation through the dot product, yielding scalar quantities such as work and kinetic energy.

In contrast, *pseudoscalars* arise from operations that encode spatial handedness. Specifically, a pseudoscalar is produced by the dot product of a *true vector* and a *pseudovector*, as summarized in Table 3. Typical examples include quantities related to orientation or chirality, such as volume expressed as the product of height and oriented area, or divergence-type quantities associated with axial fields.

True vectors, also referred to as polar vectors, may be produced through three distinct parent–offspring pathways (Table 3):

M.M. Alam, "A Novel Parent-Offspring Framework for Identifying Vectors and Scalars in Mechanics," *International Research Journal of Advanced Engineering and Science*, Volume 11, Issue 2, pp. 101-116, 2026.

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(i) *True vector – true vector* operations involving only addition or subtraction, resulting in vectors such as relative velocity or resultant velocity.
(ii) *True scalar – true vector* operations, including multiplication, division, or the gradient of a scalar field, which yield vectors such as linear momentum and acceleration.
(iii) *True vector – pseudovector* operations through the cross product also yield a true vector (e.g., velocity induced by rotational effects).

A pseudovector, or axial vector, is always an offspring quantity whose direction is perpendicular to the plane formed by its parent vectors. For example, angular momentum, defined as the moment of momentum, results from the cross product of the position vector and linear momentum, with its axis perpendicular to the plane formed by the moment arm and the momentum vector. According to the parent–offspring classification, pseudovectors can be generated in two ways (Table 3):
(i) *true scalar – pseudovector* operations where multiplication or division by a scalar preserves the pseudovector character.
(ii) *true vector – true vector* operations through the cross product, yielding pseudovectors such as moment, angular momentum, and vorticity.

Overall, Table 3 provides a systematic summary of how true vectors, pseudovectors, true scalars, and pseudoscalars emerge from different combinations of operand types and mathematical operations. This classification offers a clear and physically consistent framework for identifying the nature of derived quantities in fluid mechanics, engineering mechanics, and related fields.

Table 3. Relationship between resulting vector and scalar types and the corresponding mathematical operations.

Operand type	Operation	Results (true/pseudo)	Example
True scalar – True scalar	Addition, subtraction, multiplication, division	True scalar	Power $\rho = m/\|V_{vol}\|$
True vector – True vector	Dot product $(True\ vector).(True\ vector)$	True scalar	Work $W = \vec{F}.\vec{S}$ Kinetic energy $E = m\vec{V}.\vec{V}/2$
True vector – Pseudovector	Dot product $(True\ vector).(Pseudovector)$	Pseudoscalar	Volume (height×area) $V_{vol} = \vec{C}.\vec{A}$ Divergence $Q = \vec{\nabla}.\vec{V}$ Helicity $\lambda = \vec{V}.\vec{\omega}$
True vector – True vector	Addition, subtraction $(True\ vector) \pm (True\ vector)$	True vector	Relative velocity $\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$ Resultant velocity $\vec{V}_R = \vec{V}_A + \vec{V}_B$
True scalar – True vector	Multiplication, division, gradient $(Scalar)(True\ vector)$ $(True\ vector) \div (Scalar)$	True vector	Linear momentum $\vec{L} = m\vec{v}$ Acceleration $\vec{a} = \vec{F}/m$
True vector – Pseudovector	Cross product $(True\ vector) \times (Pseudovector)$	True vector	Velocity $\vec{V} = \vec{\omega} \times \vec{r}$
True scalar – Pseudovector	Multiplication, division by a scalar $(Scalar)(Pseudovector)$ $(Pseudovector) \div (Scalar)$	Pseudovector	Linear momentum $\vec{L} = m\vec{v}$ Acceleration $\vec{a} = \vec{F}/m$
True vector – True vector	Cross product $(True\ vector) \times (True\ vector)$	Pseudovector	Moment $\vec{M}_o = \vec{r} \times \vec{F}$ Angular momentum $\vec{H}_o = \vec{r} \times m\vec{v}$ Vorticity $\vec{\omega} = \vec{\nabla} \times \vec{V}$

VIII. CONCLUSION

This paper offers a comprehensive framework for identifying and classifying physical quantities as vectors, scalars, and generalized quantities within classical mechanics. The rigorous distinction between these categories, explored through both intuitive and formal approaches, is essential for accurately understanding and solving mechanical problems. We propose the *parental-offspring* technique, which provides clear and consistent guidelines for recognizing true vectors, pseudovectors, true scalars, and pseudoscalars. A derived parameter may be classified as a vector or scalar depending on the manner in which it is generated from its parent parameters.

This innovative parent-offspring approach provides a structured, hierarchical method for classifying derived physical quantities as scalars or vectors, based on their relationship to fundamental (parent) parameters. By directly linking derived parameters to their parent quantities, this technique offers a deeper, more comprehensive understanding of their nature. It traces how offspring quantities emerge from fundamental parameters such as length, mass, and time, while elucidating the connection between mathematical operations and the classification of scalars and vectors. This framework proves particularly valuable in complex systems where derived quantities are combinations of fundamental

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parameters, enabling clear and consistent identification of their true nature. In doing so, the parent-offspring method becomes an essential tool for accurately categorizing quantities in fields like mechanics and fluid dynamics, ensuring that physical laws and equations are applied correctly and consistently across a range of applications.

Our proposed technique, the parent-offspring approach, provides a systematic framework for addressing common misconceptions in the classification of physical quantities. The parent-offspring approach also serves to mathematically resolve ambiguities that arise in cases like the classification of area as a vector or the treatment of pressure as a scalar. By tracing how these quantities emerge from fundamental parameters—such as length, mass, and time—this method provides a clear rationale for their classification, ensuring that the mathematical operations involved are consistent with their physical properties. Ultimately, the parent-offspring technique enhances both conceptual understanding and practical application, offering a robust tool for resolving complex classification issues that are critical for the accurate formulation and solution of problems in various scientific and engineering fields.

The paper identifies how various mathematical operations lead to the classification of derived parameters. Fundamental quantities such as length, mass, and time serve as the building blocks from which derived parameters are formed. Mathematical operations on these parameters, such as addition, multiplication, dot products, and cross products, govern the emergence of true vectors, true scalars, pseudovectors and pseudoscalars. Specifically, *true scalars* result from scalar-scalar operations or the dot product of vectors, while *pseudoscalars* arise from operations involving the dot product of a true vector and a pseudovector, such as volume or divergence-type quantities. *True vectors* emerge from operations such as vector addition and subtraction, the gradient of a scalar field, or the multiplication of a true vector by a true scalar, yielding quantities like linear momentum and acceleration. *Pseudovectors*, on the other hand, arise from the cross product of two true vectors or the multiplication of a true scalar by a pseudovector, giving quantities like angular momentum and vorticity. The hierarchical relationship between fundamental and derived parameters helps in systematically classifying quantities based on their operations and transformation behavior under coordinate changes. This classification framework enhances clarity in the application and interpretation of mechanical principles, providing a rigorous foundation for further exploration in fluid dynamics and engineering mechanics.

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