

Investigation of Machining Parameters and Surface Roughness Optimization in Ultra-Precision Diamond Turning of Ge aspheres Using the GWO Algorithm

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Abstract— Germanium (Ge) is a pivotal optical material extensively utilized in infrared systems, wherein stringent requirements for geometric precision and surface quality are paramount. In ultra-precision diamond turning (UPDT), surface roughness (R_a) is a critical determinant of the optical performance of Ge components aspheres. However, the quantitative correlation between machining parameters and surface roughness in ultra-precision diamond turning (UPDT) of Ge aspheres remains insufficiently characterized. Therefore, this study investigates the influence of cutting parameters - spindle speed (n), feed rate (f), and depth of cut (a_p)- to identify optimal machining conditions for Ge aspheres. A second-order regression model was established through a Box–Behnken experimental design comprising 15 runs and validated via Analysis of Variance (ANOVA). The predictive model was subsequently integrated with the Grey Wolf Optimizer (GWO) algorithm model to minimize surface roughness. The results indicate that feed rate is the most significant factor influencing R_a , followed by the depth of cut, while spindle speed exerts a relatively marginal effect. The derived optimal parameter combination yielded a minimum surface roughness within the investigated domain, which was experimentally validated with negligible deviation. This research provides a robust scientific framework for parameter selection in the ultra-precision manufacturing of Germanium infrared optics.

Keywords— Aspheric surface: Germanium: Grey Wolf Optimizer: Surface roughness: Ultra-precision turning.

I. INTRODUCTION

Infrared optical systems require optical components with high geometric accuracy and superior surface quality to maximize transmission efficiency and minimize parasitic scattering. Specifically, the surface roughness and geometric accuracy are especially critical for aspheric surfaces, as their continuously varying geometries present more rigorous manufacturing challenges compared to planar or spherical counterparts.

Ge is an important optical material widely used in infrared systems due to its high transparency in the infrared region (2- 14 μ m wavelength). To ensure functional reliability, surface finishes must be maintained at the nanometer scale [1], [2]. In ultra-precision diamond turning (UPDT), R_a is mainly influenced by 3 variables, including spindle speed, feed rate, and depth of cut; even small fluctuations in these variables can significantly alter cutting mechanisms and surface integrity [3], [4]. Furthermore, the inherent brittleness and low thermal diffusivity of Ge render it highly susceptible to micro-scale

fracture and subsurface damage under suboptimal machining regimes [5].

UPDT using single-point tools has proven to be an effective technique for producing optical surfaces with nanometric surface finishes and sub-micron form accuracy on crystalline substrates, including Ge, Si, ZnS, and ZnSe [6], [7]. However, most of existing studies focuses on planar or spherical surfaces. The fabrication of Ge aspheres, which entails more complex tool trajectories and necessitates superior machine dynamic stability, remains insufficiently explored [8].

To determine optimal cutting parameters, numerous studies integrated Design of Experiments (DoE) with advanced optimization metaheuristics. Among these, the Grey Wolf Optimizer (GWO) has gained prominence due to its structural simplicity, minimal parameter tuning, and robust convergence characteristics, particularly in bypassing local optima [9], [10]. Several investigations have demonstrated the effectiveness of GWO in surface roughness optimization based on empirical models [11], [12].

Building upon these foundations, this study systematically evaluates the influence of machining parameters on the surface roughness during UPDT of Ge aspheres on a Nanoform-X ultra-precision turning machine. A second-order regression model was established using a Box–Behnken design (BBD) to serve as the objective function for the GWO algorithm. This integrated approach aims to identify the optimal parameter configuration, providing a rigorous scientific framework for the ultra-precision manufacturing of aspheric Germanium optics.

II. EXPERIMENT

A. Research Procedure

The research methodology was structured into four sequential steps.

Step 1: Three machining parameters: spindle speed n , feed rate f , and depth of cut a_p were designated as input variables. Their operational ranges were determined based on the kinematic specifications of the Nanoform-X ultra-precision turning machine as well as the prior studies to ensure ductile regime cutting of Germanium.

Step 2: BBD was utilized to develop a matrix comprising 15 experimental runs. Based on the obtained data, a second-order regression model was established to correlate the machining parameters with the surface roughness R_a , followed by statistical validation through ANOVA.

Step 3: The Grey Wolf Optimizer (GWO) algorithm was implemented to minimize surface roughness R_a based on the validated regression model.

Step 4: Independent confirmation experiments were conducted under these optimized conditions to verify the accuracy of the model and to assess the practical feasibility of the solution in real-world machining environments.

B. Experimental Model, Machining System, and Workpiece

1) Experimental Model and Design Matrix

In this study, the Box–Behnken Design (BBD)—a Response Surface Methodology (RSM) approach—was utilized to develop a second-order regression model correlating machining parameters with surface roughness R_a [10]. A 15-run BBD was selected to optimize the experimental efficiency while avoiding extreme conditions that could compromise the integrity of the Germanium material.

During the experiments, although the Nanoform-X ultra-precision turning machine was programmed via a linear feed rate F ($\mu\text{m}/\text{min}$) as guided by the control system, the feed per revolution f ($\mu\text{m}/\text{rev}$), which calculated as $f = F/n$, was adopted as the primary design variable. This selection is justified by the fact that f directly governs the surface generation mechanism and tool mark geometry in UPDT.

The operational ranges were determined based on the kinematic specifications of the Nanoform-X machine, manufacturer guidelines for diamond tooling, and established literature.

Specifically, the operational ranges for $n = 2000\text{--}3000$ rev/min, $f = 0.5\text{--}2.5$ $\mu\text{m}/\text{rev}$, and $a_p = 2\text{--}10$ μm to ensure stable ductile-regime machining and achieve nanometric surface finishes, as supported by previous investigations [1], [6], [13].

TABLE I. Range of machining parameters

Cutting Parameter	Coded Factor	Low Setting (-1)	Central Setting (0)	High Setting (+1)
n (rev/min)	A	2000	2500	3000
f ($\mu\text{m}/\text{rev}$)	B	0.5	1.5	2.5
a_p (μm)	C	2	6	10

2) Machining System, Fixturing, and Cutting Tool

UPDT experiments were performed on a Nanoform-X diamond turning machine, which provides nanometer-level control resolution and meets the requirements for machining optical-quality surfaces. To ensure the integrity of the experimental results and isolate the effects of the investigated machining parameters, a single-crystal diamond tool was utilized with its geometry and mounting configuration held constant throughout the study.

The workpiece material was infrared-grade Germanium (Ge), a material known for its inherent brittleness and high sensitivity to thermo-mechanical loading during cutting. The specimens were prepared with an aspheric profile to represent high-precision optical applications. Machining conditions were selected to maintain the ductile-regime cutting mechanism and ensure stable UPDT performance.

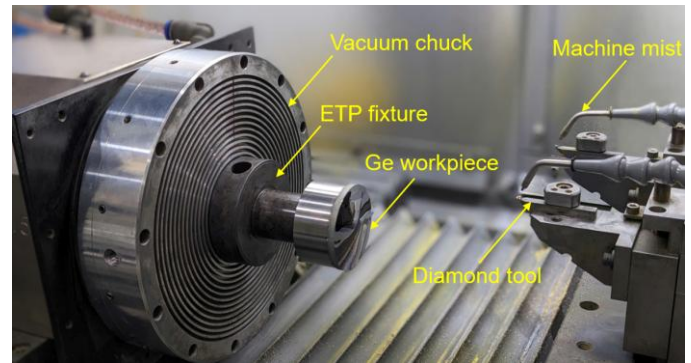


Fig. 1. Lathe system, fixtures, tools, and Ge workpiece

3) Surface roughness measurement system

Surface roughness R_a was characterized using a Zegage Pro HR 3D optical profilometer. This non-contact metrology system, based on Coherence Scanning Interferometry (CSI), provides the nanometric resolution essential for evaluating ultra-precision surfaces.

Since the surface quality of aspheric components may vary with position, a systematic measurement procedure was adopted to ensure representative roughness evaluation:

Measurement locations and number of points: Twelve fixed measurement points were selected on each workpiece, divided into two radial zones: six points at the apex region and six points in the near-apex region (with the center of the near-apex zone located 7 mm from the apex center). Within each zone, measurements were conducted along two orthogonal directions through the center and four additional directions at quadrant positions, located 100 μm from the apex.

Measured value and evaluation length: The surface roughness value used for analysis was the average of the 12 measured points. The standardized evaluation length L_n was set to 80 μm .

Measurement direction: Measurements were performed parallel to the tool marks to effectively assess micro-scale surface defects and cutting-induced features while minimizing the influence of feed-per-revolution geometry.

Data processing standard: Surface data were analyzed using MX software with a Gaussian filter, in accordance with ISO 25178 standards.

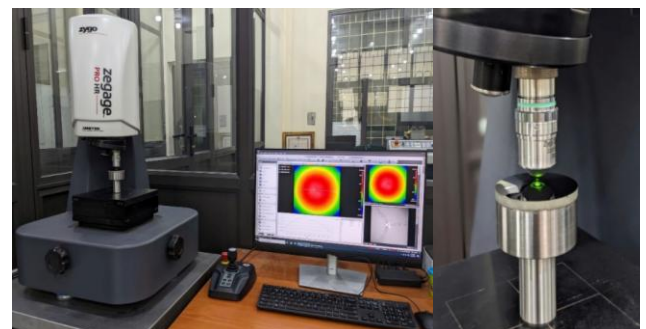


Fig. 2. Zegage Pro HR 3D optical surface roughness measurement system

C. Experimental Design Using the Box–Behnken Method (BBD)

In this study, BBD was applied to three machining parameters: Spindle speed n , feed rate f , and depth of cut a_p resulting in a total of 15 experiments to model their effects on surface roughness. The resulting BBD matrix served as the basis for developing the regression equation, evaluating statistical significance via ANOVA, and supporting the subsequent optimization stage using the GWO algorithm.

TABLE II. Experimental design matrix and measured results

No.	Cutting Parameter			R_a (nm)
	n (rev/min)	f ($\mu\text{m}/\text{rev}$)	a_p (μm)	
1	2000	0.5	6	5.586
2	3000	0.5	6	4.455
3	2000	2.5	6	8.678
4	3000	2.5	6	6.205
5	2000	1.5	2	5.456
6	3000	1.5	2	4.121
7	2000	1.5	10	7.125
8	3000	1.5	10	6.157
9	2500	0.5	2	3.921
10	2500	2.5	2	6.256
11	2500	0.5	10	5.869
12	2500	2.5	10	8.455
13	2500	1.5	6	5.432
14	2500	1.5	6	5.324
15	2500	1.5	6	5.186

III. RESULTS AND DISCUSSION

A. Results of the ANOVA analysis

Design-Expert software was employed to generate the experimental design based on the BBD methodology. Using the experimental data, a second-order multivariable regression model was established. The regression coefficients were determined through ANOVA analysis, as illustrated in Fig. 3, and are explicitly presented in Equation (1).

$$Ra(\text{nm}) = 5.31 - 0.7384A + 1.22B + 0.9815C - 0.3355AB + 0.6638B^2 \quad (1)$$

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	26.29	9	2.92	54.66	0.0002	significant
A-n	4.36	1	4.36	81.62	0.0003	
B-F	11.91	1	11.91	222.96	< 0.0001	
C- a_p	7.71	1	7.71	144.22	< 0.0001	
AB	0.4502	1	0.4502	8.43	0.0337	
AC	0.0337	1	0.0337	0.6301	0.4633	
BC	0.0158	1	0.0158	0.2947	0.6105	
A ²	0.2368	1	0.2368	4.43	0.0892	
B ²	1.63	1	1.63	30.44	0.0027	
C ²	0.0803	1	0.0803	1.50	0.2748	
Residual	0.2672	5	0.0534			
Lack of Fit	0.2368	3	0.0789	5.19	0.1658	not significant
Pure Error	0.0304	2	0.0152			
Cor Total	26.55	14				

R_a	=
+5.31	
-0.7384 * A	
+1.22 * B	
+0.9815 * C	
-0.3355 * AB	
+0.0918 * AC	
+0.0628 * BC	
+0.2533 * A ²	
+0.6638 * B ²	
+0.1475 * C ²	

Fig. 3. Results of ANOVA analysis and regression coefficients

1) ANOVA Analysis of the Regression Model

The ANOVA results indicate that the regression model is highly statistically significant ($p = 0.0002$), demonstrating that the model adequately describes the relationship between the independent variables and the response.

Among the main effects, factors B and C ($p < 0.0001$) exhibit strong and statistically significant influences on the response, while factor A also shows a significant effect ($p = 0.0003$). The interaction term AB is statistically significant ($p = 0.0337$), whereas the interactions AC and BC are not significant ($p > 0.05$).

Regarding the quadratic terms, B² has a pronounced and statistically significant effect ($p = 0.0027$), while A² and C² show limited statistical significance.

The Lack-of-Fit test is not significant ($p = 0.1658$), confirming that the regression model provides an adequate fit to the experimental data.

2) Model Adequacy

Std. Dev.	0.2312	R ²	0.9899
Mean	5.88	Adjusted R ²	0.9718
C.V. %	3.93	Predicted R ²	0.8548
		Adeq Precision	24.4888

Fig. 4. Model adequacy assessment

The regression model exhibits a high coefficient of determination $R_2 = 0.9899$ and Adjusted $R_2 = 0.9718$ indicating excellent capability in explaining data variability. The Predicted $R_2 = 0.8548$ is in good agreement with the Adjusted R_2 , confirming the strong predictive performance of the model. In addition, an Adeq Precision value of 24.488 (greater than the recommended threshold of 4) demonstrates a strong and reliable signal-to-noise ratio.

The resulting regression equation indicates that factors B and C are the dominant contributors to the response, which is consistent with the ANOVA results.

B. Optimization of Machining Parameters

1) Description of the GWO Algorithm

The Grey Wolf Optimizer (GWO) is a metaheuristic optimization algorithm inspired by the hunting behavior and social hierarchy of grey wolves, originally proposed by Mirjalili *et al.* [6]. The optimization mechanism of GWO relies on updating the positions of candidate solutions based on the three best solutions identified at each iteration, thereby achieving a balance between global exploration and local exploitation [4].

To ensure reproducibility and reliability of the optimization results, the algorithm was implemented in the MATLAB environment with the following configuration:

- Objective function: The coded second-order regression equation (1) derived from the BBD model.

- Search space: Design variables normalized within the coded range of $[-1, 1]$.

- Algorithm settings: Number of wolves: 25; maximum number of iterations: 100; number of design variables: 3 (n , f , and a_p).

2) The optimization value

The simulation results (Fig. 6) indicate that the objective function converges rapidly and reaches a stable state after approximately 20 iterations.

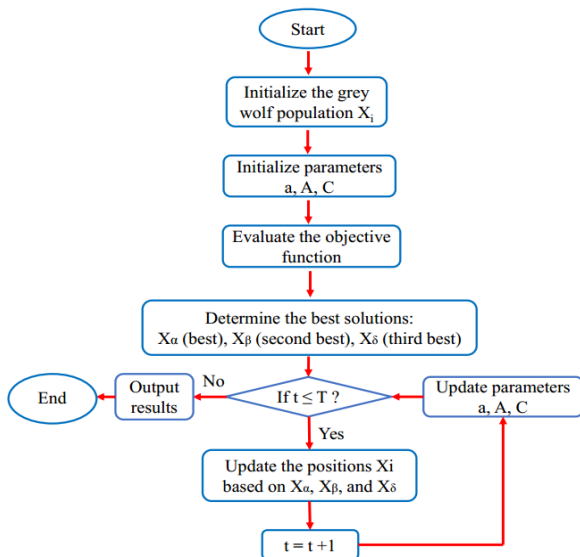


Fig. 5. Algorithm implementation steps in GWO

Optimal solution:

The optimal solution was obtained at the boundary of the design space with coded values $x^* = [1, -0.619, -1]$, corresponding to a spindle speed of $n = 3000$ [rev/min], a feed rate of $f = 0.88$ [$\mu\text{m}/\text{rev}$], and a depth of cut $a_p = 2$ [μm]. Under these conditions, the minimum predicted surface roughness is $R_a = 3.6448$ nm.

This result is fully consistent with the physical interpretation of the effects of feed rate and depth of cut on surface quality in UPDT of Germanium.

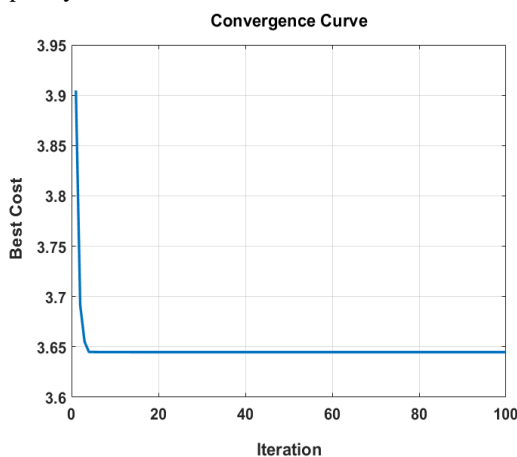


Fig. 6. Convergence curve of the algorithm implemented in MATLAB

C. Evaluation of the relationship between machining parameters and surface roughness

1) Relationship between f and a_p at fixed n

The response surface $R_a(f, a_p)$ shows that surface roughness increases with increasing feed rate f and depth of cut a_p . Among these parameters, f plays a dominant role and exhibits a pronounced nonlinear relationship with R_a , whereas the effect of a_p is approximately linear within the investigated range. The interaction between f and a_p is weak; therefore, no distinct

optimal point with respect to a_p is observed. The lowest surface roughness is achieved in the region of low f and low a_p .

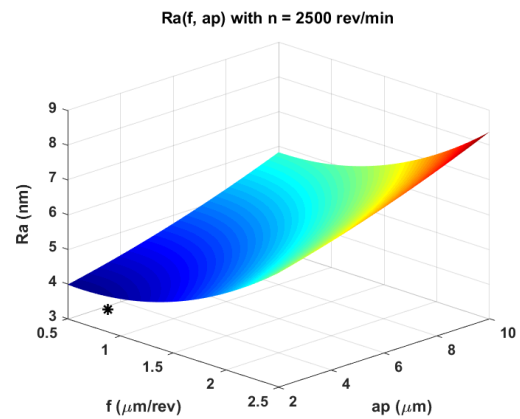


Fig. 7. Graph of the objective function $R_a(f, a_p)$

2) Relationship between n and a_p at fixed f

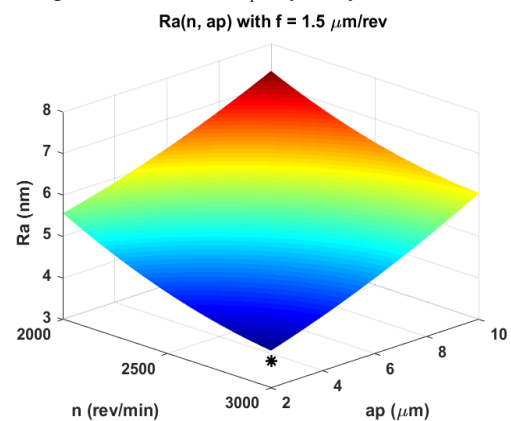


Fig. 8. Graph of the objective function $R_a(n, a_p)$

The response surface $R_a(n, a_p)$ exhibits an approximately linear trend, where R_a decreases with increasing spindle speed n and increases with increasing depth of cut a_p . Both main effects of n and a_p are statistically significant, whereas the interaction and quadratic terms are not dominant within the investigated range. The surface shows no pronounced curvature, indicating the absence of a clear internal optimum. Therefore, improvement in surface roughness is primarily achieved by increasing n and reducing a_p .

3) Relationship between n and f at fixed a_p

The response surface $R_a(f, n)$ reveals a pronounced nonlinear relationship between surface roughness and feed rate f . The surface roughness increases rapidly with increasing f and decreases with increasing spindle speed n . The quadratic term of f is statistically significant, indicating that excessive reduction in f no longer results in substantial improvement in R_a . The interaction between n and f is moderate, suggesting that the effect of n depends on the value of f . The lowest surface roughness is achieved in the region of low f and high n .

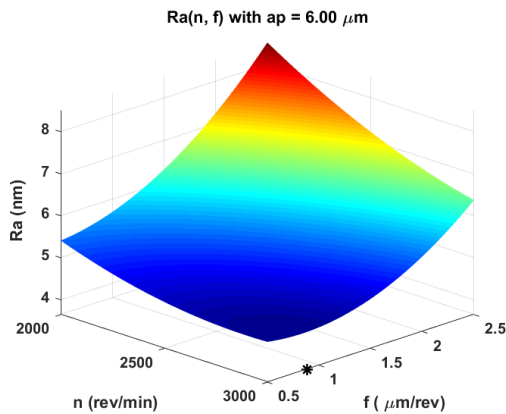


Fig. 9. Graph of the objective function $R_a(f, n)$

D. Confirmation Experiment for the Optimal Solution

To validate the reliability of the second-order regression model and the effectiveness of the Grey Wolf Optimizer (GWO), an independent confirmation experiment was conducted on the Nanoform-X ultra-precision turning machine. The optimal machining parameters determined by the GWO algorithm were a spindle speed of $n = 3000$ rev/min, a feed rate of $f = 0.88$ $\mu\text{m}/\text{rev}$, and a depth of cut of $a_p = 2$ μm .

TABLE III. Experimental results of the optimal parameter set

No.exp	R_a^{exp} (nm)	R_a^{pred} (nm)	Error (%)
1.	3.459	3.645	5.1
2.	3.595	3.645	1.3
3.	3.526	3.645	3.2

Experimental results show that the measured surface roughness values range from 3.459 nm to 3.595 nm, which are in close agreement with the GWO-predicted value of 3.645 nm. The deviation between experimental and predicted results is only 1.3%–5.1%, which falls within the acceptable range for ultra-precision machining studies.

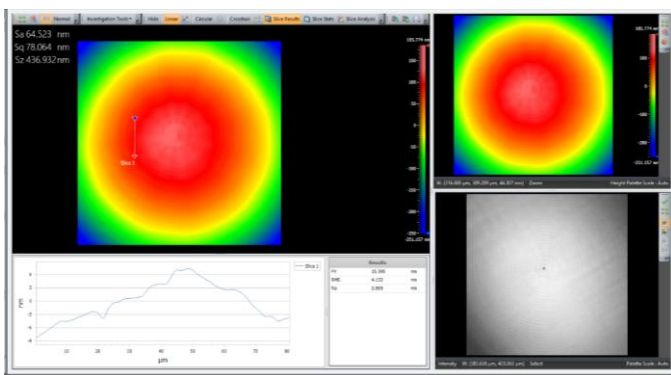


Fig. 10. R_a value with optimal parameter set

The optimization results indicate that the surface roughness R_a of Germanium is strongly influenced by the feed rate f and depth of cut a_p , whereas spindle speed n plays a secondary role within the investigated range. The optimal solution obtained using the GWO algorithm at the coded values

$x^* = [1, -0.619, -1]$, corresponding to ($n = 3000$ rev/min, $f = 0.88$ $\mu\text{m}/\text{rev}$, $a_p = 2$ μm), yields the minimum surface roughness of $R_a = 3.6448$ nm, which is consistent with the trends predicted by the regression model.

The results demonstrate a pronounced nonlinear effect of feed rate f on surface roughness, in agreement with previous studies on ductile-regime diamond turning of brittle materials [3], [11]. However, for aspheres, the continuously varying contact direction and tool trajectory make the effect of f more sensitive than for planar or spherical surfaces, leading to the existence of an internal optimal region. In contrast, the nearly linear influence of n and a_p within the studied domain suggests that these parameters mainly act to regulate cutting parameters and process stability rather than directly governing surface generation. This explains why the optimal values of n and a_p are located at the boundaries of the design space, while f reaches its optimum within the interior of the domain.

These findings indicate that conclusions and optimal cutting parameters derived from planar or spherical surface machining cannot be directly transferred to Ge aspheres. This constitutes a key contribution of the present study, providing experimental evidence for selecting machining parameters that account for the geometric characteristics of aspheres in UPDT of Germanium.

Nevertheless, several limitations should be noted. First, the investigated operational ranges are relatively limited; therefore, the obtained optimal solution is only valid within the studied domain. Second, the optimization problem considered a single objective surface roughness without simultaneously addressing other performance indicators such as form accuracy, productivity, or process stability. These aspects will be considered and extended in future studies.

IV. CONCLUSION

This study successfully developed a second-order regression model describing the relationship between machining parameters and surface roughness R_a in UPDT of Ge aspheres using the Box–Behnken experimental design. The model exhibits high goodness of fit and strong predictive capability, as validated by ANOVA and confirmation experiments.

Within the investigated operational ranges, feed rate f is the most influential factor affecting surface roughness, followed by depth of cut a_p , while spindle speed n plays a secondary role. The Grey Wolf Optimizer (GWO) effectively identified the optimal machining parameter combination yielding the minimum surface roughness, with only minor deviation from the experimentally validated results.

The findings provide a reliable scientific basis for selecting cutting parameters in ultra-precision machining of Ge aspheres and further demonstrate the potential of integrating DoE with metaheuristic optimization algorithms for optimizing high-precision optical manufacturing processes.

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