

Exergy–Energy Interaction in Solar-Based Air Conditioning Systems: A Simplified Analytical Approach

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Abstract: This study analyzes the interaction between exergy and energy in residential air conditioning. The objective is to ensure thermal comfort at lower cost by integrating solar air collectors and photovoltaic systems. The first part models and simulates a coupled solar air and photovoltaic system to supply the energy required for air conditioning. The second part develops an analytical method based on thermodynamic principles, including the Carnot cycle, to evaluate the exergy flow and cooling efficiency. The results show that the exergy method provides better performance in cooling during summer, while the energy method is more effective in heating during winter. The combination of both methods ensures stable and comfortable indoor conditions throughout the year.

Keywords: Carnot cycle, Energy efficiency, Exergy, photovoltaic system, Solar air system, thermal comfort.

I. INTRODUCTION

In Madagascar, domestic energy demand for air conditioning is increasing due to climate change. Winters are colder and summers are hotter, which creates a strong need for thermal comfort. Houses built for present and future generations must provide efficient and low-cost air conditioning, [1].

Energy analysis alone is not sufficient to evaluate the performance of air conditioning systems. Exergy analysis is also required to account for the quality of energy transformations. The combination of energy and exergy provides a better understanding of system efficiency and sustainability, [2].

This study investigates the heat required to maintain comfort in a conditioned space over a given period. The objective is to evaluate exergy flows through a simplified linear model and to propose a practical method for estimating the energy load of heating and cooling. The method is based on modeling and simulation of a solar air system coupled with a photovoltaic system. It also integrates thermodynamic principles, including the Carnot cycle, to optimize the production of cold air and to reduce operating costs.

II. ORGANIZATION HART

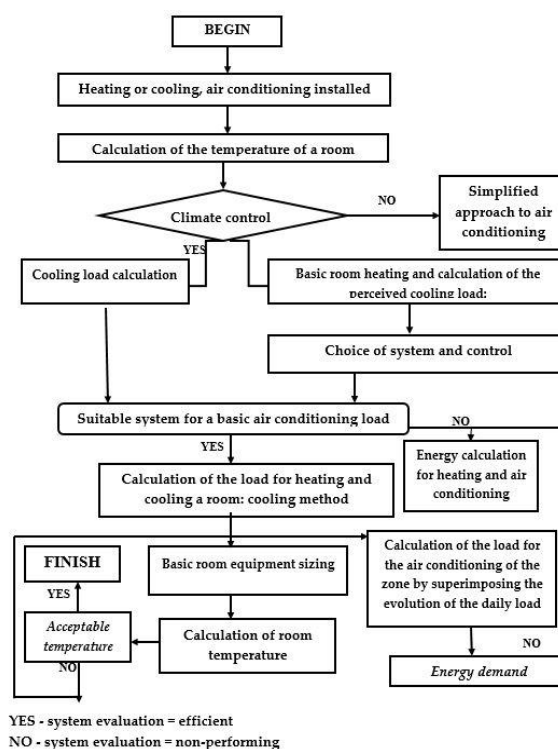


Figure 1: Flowchart of the general approach for heating or cooling, air conditioning of a room

III. MODELING AND SIMULATION OF AN AIR SOLAR SYSTEM COUPLED WITH A PHOTOVOLTAIC SYSTEM

A. System description and sizing

- Solar heated floor

The solar heated floor is a solar thermal energy exchanger for heating the home using the circulation of the heat transfer fluid heated in the solar collector in the copper pipe installed under the reticulated mat on the low floor. Its performance is directly linked to exterior temperatures, that of the fluid inside the coil and that inside the heated room. The mat has undergone cross-linking in order to improve its resistance to high temperatures, which allows the use of solar thermal collectors in a hot or normal air network, [3].

- *Thermal assessment of the sensor*

The resulting heat balance taking place at the level of the absorbing wall [2] is written:

$$Q_a = Q_u + Q_p + Q_{st} \quad (5)$$

Where:

- Q_a : solar flux absorbed (W/m^2),
 - Q_p : flux lost through insulation (W/m^2),
 - Q_u : useful flow transmitted to the heat transfer fluid (W/m^2),
 - Q_{st} : flow stored in the sensor (W/m^2),
- Let Q_{st} be the energy stored in the sensor or denoted by Q_u such that:

$$Q_{st} = M_a C_a \frac{dT}{dt} \quad (6)$$

Where:

- M_a : the air mass of the sensor,
- C_a : the air specific heat,
- $\frac{dT}{dt}$: the absorber temperature variation

By calculating the energy transferred to the heat transfer fluid called useful flow and denoted by Q_u , we have:

$$Q_u = \dot{m}_f c_f (T_{2f} - T_{1f}) \quad (7)$$

If \dot{m}_f denotes the mass flow rate of the fluid and c_f its specific heat.

Concerning the flux lost by the insulating wall of average temperature T_m , it is expressed by Newton's law:

$$Q_p = h_p A (T_m - T_a) \quad (8)$$

Where: T_a is the ambient temperature outside, T_m is the average sensor temperature.

For a flat sensor, its average temperature T_m is expressed:

$$T_m = \frac{T_v - T_c}{2} \quad (9)$$

Where: T_c is the temperature of the plate or the absorber.

The air thermal, exchanger metal coil and window conductivities are $0.026W/mK$, $386W/m K$ and $0.8W/mK$ respectively.

- *Sensor efficiency*

The evaluation of the sensor efficiency depends on the different influential parameters, which are sensor internal parameters (orientation, inclination and location, etc.) and sensor external parameters (irradiance due to the global radiation G , Sun position, etc.), [7].

The overall efficiency of the sensor is defined by:

$$\eta = \frac{Q_u}{G A} \quad (10)$$

Where A is the surface of the sensor.

- *Thermal inertia of the sensor*

In experimentation, we set the inlet temperature T_{1f} of the fluid and a flow rate \dot{m}_f . We collect the output temperature T_{2f} which allows evaluating:

$$Q_u = \dot{m}_f c_f (T_{2f} - T_{1f}) \quad (11)$$

c_f being the specific heat of the heat transfer fluid in Watt.

But when the temperature of the sensor varies, we consider the thermal diffusivity a' of the incident energy towards the heat transfer fluid.

$$a' = \frac{\lambda_a}{\rho_a c_a} \quad (12)$$

Where

λ_a : the thermal conductivity of the absorber,

ρ_a : the density,

C_u : the specific heat of the absorber.

The thermal inertia TI of the sensor is given by [2],

$$TI a' = \dot{m}_f c_f \quad (13)$$

- *Thermal balance of the heat transfer fluid*

The thermal balance of the heat transfer fluid inside the sensor is given by:

$$M_f C_f \frac{dT_f}{dt} = Q_{caf} - Q_{cfe} - Q_f \quad (14)$$

The heat exchanged by conduction-convection between the fluid and the outside Q_{cfe} is expressed by:

$$Q_{cfe} = A h_{cfe} (T_f - T_a) \quad (15)$$

The heat carried away by the heat transfer fluid Q_f is expressed by:

$$Q_f = A \dot{m}_{cfa} (T_f - T_a) \quad (16)$$

where,

h_{cfe} is the exchange coefficient by conductivity-convection between the fluid and the exterior and \dot{m}_{cfa} is the mass flow rate of the fluid used.

- *Thermal balance of the inner tube*

Neglecting the heat radiated outwards, we have the following expression:

$$\dot{m}_u C_u \frac{dT_u}{dt} = Q_f - Q_{cp} - Q_{cu} \quad (17)$$

The heat lost at the periphery of the inner tube by convection results in Q_{cp} and the heat used is practically taken from the inner tube with Q_{cu} .

$$Q_{cp} = A_r h_{cra} (T_r - T_a) \quad (18)$$

$$Q_{cu} = \dot{m}_u C_u (T_u - T_{ap}) \quad (19)$$

Where:

T_u : air temperature,

\dot{m}_u : mass flow,

C_u : specific heat at air pressure constant,

h_{cra} : coefficient of exchange by convection between air ambient and inner tube.

- *Thermal balance of air heated floor*

The floor has a heat supply by convection from the heat transfer fluid and another heat exchange in the form of heat losses by radiation and convection with the room. The equation is then written:

$$\frac{dT_p}{dt} = \frac{h_{fp} A_p}{M_p C_p} (T_f - T_p) - \frac{A_p}{M_p C_p} (Q_c + Q_r) \quad (20)$$

Where:

Q_c : heat exchanged by convection with the room such that:

$$Q_c = h_{ap} (T_p - T_a) \quad (21)$$

Q_r : heat exchanged by radiation with the room.

$$Q_r = \sigma \epsilon_p (T_p^4 - T_a^4) \quad (22)$$

T_p : floor temperature given by:

$$T_p = 19 - 5.42 \left(\frac{A_p}{U_p} \right) \quad (23)$$

U_p : heat loss coefficient whose value $5.42W/m^2$ corresponds to the energy gains per m^2 :

$$h_{fp} = 5.7 + 3.8 V_f \quad (24)$$

V_f : fluid speed between 0.13 and 0.25m/s with:

$$h_{ap} = 5.7 + 3.8 V_{air} \quad (25)$$

M_p : energy emittance from the floor,

C_p : Specific heat from the floor,

A_p : floor surface.

The use of solar energy as a heat source for local heating in winter [3].

$$h_{Cra} = 5.7 + 3.8 V_{air} \quad (26)$$

V_{air} : air speed which is between 6 and 7 m/s.

IV. ENERGY – EXERGY RELATIONSHIP

A. Exergetic analytical method

• Energy signature method

Exergetic analytical method is applied to the permanent regime of a habitat, which instantly results in the thermal balance between the exterior and interior exchanges of all the walls constituting the building.

The modeling is designed to express the heating load according to external demands (sunshine, outside temperature, etc.). Then, according to the heat balance equation, the simple energy signature method [8] is written:

$$Q' = a + b (T_{int} + T_{ext}) \quad (27)$$

where only heating (or air conditioning) periods are taken into account, i.e.:

$$(T_{int} + T_{ext})^+ = \max((T_{int} + T_{ext}), 0) \quad (28)$$

This heat balance equation can be simplified with the assumption of very small variations in the interior temperature of a home. When the heat balance of a home no longer requires measurements of the interior temperature, the equation becomes:

$$Q' = x_r + y_r T_{ext} \quad (29)$$

Where,

Q' : heating load,

T_{int} : home average interior temperature (°C),

T_{ext} : average exterior temperature leaving through the system on the roof (°C).

The coefficients x_r and y_r are estimated by linear regression from point clouds of daily, weekly or monthly data collected at the habitat level.

• Quadratic equation of the system

Exchanges in the system take place in three known modes: convective, radiative and by conduction. For the transfer calculation that occurs there, it needs to consider the main components of the sensor and the flow of the heat transfer fluid in the metal coil. For this, the quadratic equation of the system is defined as a linear function of coil, [9].

temperature in contact with the upper part of the absorber on the roof:

$$y = mx + p \quad (30)$$

Where:

x : Temperature of coil in contact with the upper part of the absorber on the roof (°C)

y : average exterior temperature leaving through the system on the roof (°C)

$$m = \frac{\sum_i^k (x_i + \bar{x})(y_i + \bar{y})}{\sum_i^k (x_i + \bar{x})} \quad (31)$$

$$p = \bar{y} - m\bar{x} \quad (32)$$

However, in the experiment

$\bar{x} = 42.8838$; $\bar{y} = 32.6238$; $m = 0.3655$ and $p = 16.9497$

Thus $y = 0,3655x + 16,9497$

B. Energy yield

Considering a heat transfer between the heat transfer fluid and the solar collector located above the temperature level of the ambient environment, the collector providing heat being called the hot source and the fluid (cold air) receiving heat being called cold well. This process is subject to irreversibility. We expect an equivalent system comprising a set of direct and inverse reversible machines: the sensor on the roof in question is called a driving machine, which supplies heat to the receiving machine (heat transfer fluid), [10].

- First law of thermodynamics:

$$|q| = |W_c| + |q_{oc}| \quad (33)$$

$$\frac{|W_c|}{|q|} = 1 - \frac{|q_{oc}|}{|q|} \quad (34)$$

- Second law of thermodynamics:

$$\frac{|q_{oc}|}{|q|} = \frac{T_a}{T_c} \quad (35)$$

$$\frac{|W_c|}{|q|} = 1 - \frac{T_a}{T_c} \quad (36)$$

The exergy associated with the hot spring:

$$E_{XC} = |W_c| = \left[1 - \frac{T_a}{T_c}\right] |q| \quad (37)$$

With: $|q| = TC$

$$E_{XC} = T_c - T_a$$

- Receiving machine consideration :

First law of thermodynamics:

$$|q| = |W_f| + |q_{of}| \quad (38)$$

$$\frac{|W_f|}{|q|} = 1 - \frac{|q_{of}|}{|q|} \quad (39)$$

Second law of thermodynamics:

$$\frac{|q_{of}|}{|q|} = \frac{T_a}{T_f} \quad (40)$$

$$\frac{|W_f|}{|q|} = 1 - \frac{T_a}{T_f} \quad (41)$$

The exergy associated with the cold well:

$$E_{Xf} = |W_f| = \left[1 - \frac{T_a}{T_f}\right] |q| \quad (42)$$

With: $|q| = T_f$

$$E_{Xf} = T_f - T_a$$

According to the system, the irreversibility and exergy yield:

$$I = |W_c| - |W_f| = E_{XC} - E_{Xf} \quad (43)$$

$$I = \max(E_{XC}, E_{Xf}) - \min(E_{XC}, E_{Xf}) \quad (44)$$

We introduce the dissipative factor which measures the ratio of irreversibility in relation to the motor work: $\xi = \frac{I}{|W_c|}$

$$\xi = \max(E_{XC}, E_{Xf}) - \frac{\min(E_{XC}, E_{Xf})}{\max(E_{XC}, E_{Xf})} \quad (45)$$

$$\xi = 1 - \frac{T_f}{T_c}$$

The exergy yield η_H is the complement of the dissipative factor:

$$\eta_H = 1 - \xi = \frac{T_f}{T_c} \quad (46)$$

In this study, η_H is approximately 76%.

C. System operation: heating and cooling

The system operation principle is based on the capture of solar radiation using the flat solar air collector to obtain heat from the heat transfer fluid which circulates in the coil and that in the PV panel to obtain electrical energy, [11]. Y and Z are two parameters without dimensions and Y is the ratio of

conventional capture losses to requirements, with possible corrections; Z: the ratio of solar energy absorbed to needs.

Coefficient values according to the type of storage:

Coefficient	Air storage	Heating floor
a	1.02	0.863
b	-0.065	-0.147
c	-0.245	-0.263
d	0.0018	0.008
e	0.0215	0.029
f	0	0.025

$$Y = AU_c \eta_p \Delta T \cdot t_{jour} C_{OS} / Q \quad (48)$$

$$Z = A \eta_o \eta_p \Delta T \cdot t_{jour} / Q \quad (49)$$

where,

t_{jour} : day length in hour;

A: sun azimuth;

η_o : solar collector optical yield;

η_p : capture loop yield;

U_c : the heat loss coefficient from the collection loop in relation to the surface area of the collectors;

ΔT : conventional temperature difference;

Q: heating or hot air needs;

C_{OS} : corrective storage coefficient;

I_{SC} : monthly average sunshine on the sensor

In this study, we work as well on a new mathematical analytical method which is as follows:

$$F = F_1 + F_2 \quad (50)$$

$$F_1 = \text{Cosh}(x) + \text{Sinh}(x) - u_1$$

$$F_2 = \text{Cosh}(y) + \text{Sinh}(y) - u_2$$

$$u_1 = u_2 = 1$$

where :

x: heat loss during the heating period and y the supply of hot air into the home given by:

$$x = D_p V_{hab} (T_a - T_{ex}) \quad (51)$$

$$y = 0.34 D_{ac} (T_{sa} - T_e) \quad (52)$$

where :

T_e : ambient temperature $\approx 23^\circ\text{C}$;

T_{ex} : external temperature;

T_{sa} : air blowing temperature;

D_p : loss coefficient;

D_{ac} : hot air flow in the home;

V_{hab} : home volume.

V. RESULTS

A. Result on exergy yield

Due to the gradual increase in the temperature received in an hour during the day, the temperature obtained inside the room is very high and higher than the comfort temperature. But, the application of the cooling method in this system (that means the passage of the heat transfer fluid in the iced water and the use of ventilation) gives the stabilization and obtaining of the temperature of the comfort used in this room. Then, Figures 5-

8 shows the curves of received temperature in the floor during a day.

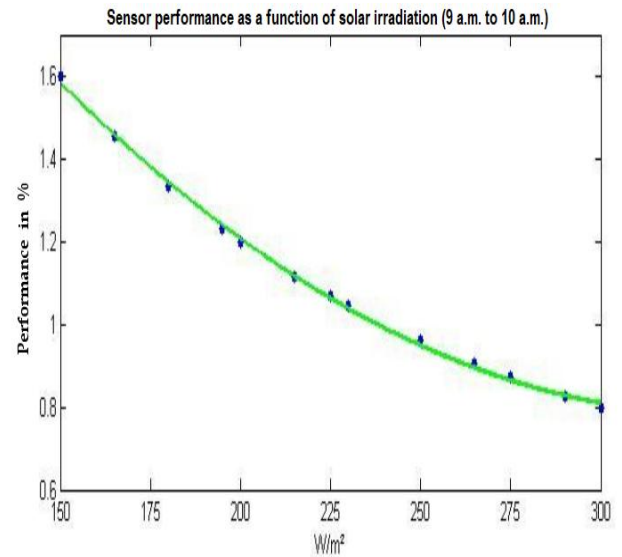


Figure 2: Overall performance of the sensor between 9 and 10 hours

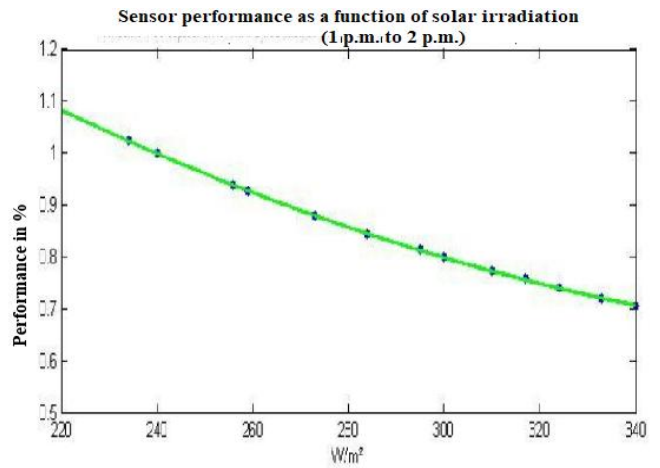


Figure 3: Overall power performance of the sensor between 1 and 2 p.m.

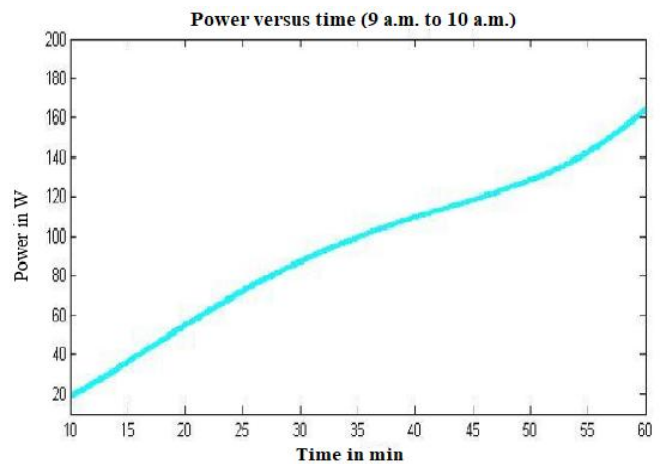


Figure 4: Overall power of the sensor between 9 and 10 a.m.

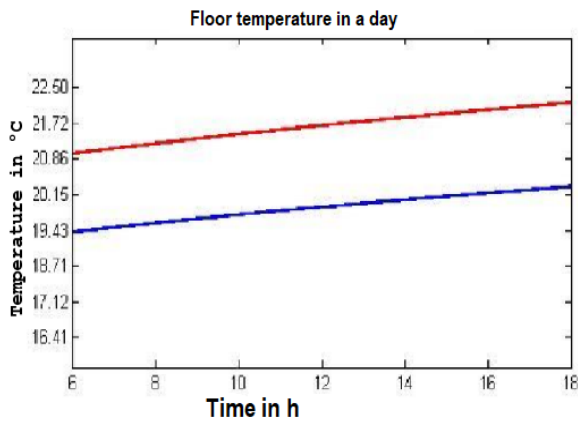


Figure 5: Floor temperature in a day.

B. Result on the overall thermal efficiency of the sensor

Calculating the overall heat balance on the flat air solar collector gives the results of the overall thermal efficiencies for this collector. They appear according to the pattern of the curve obtained by Matlab software. The figures display obtained results.

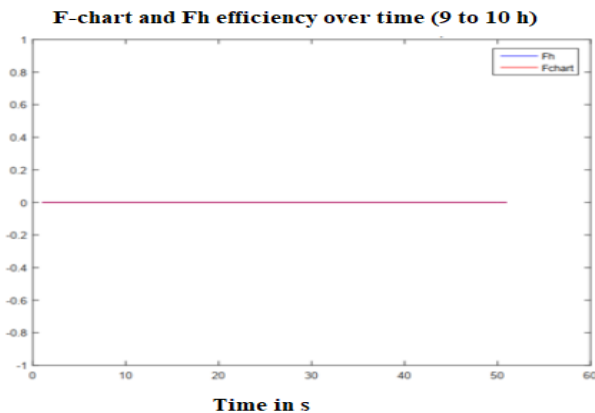


Figure 6: F-chart and Fh efficiency (analytical method and hyperbolic function) over time (9 to 10 h)

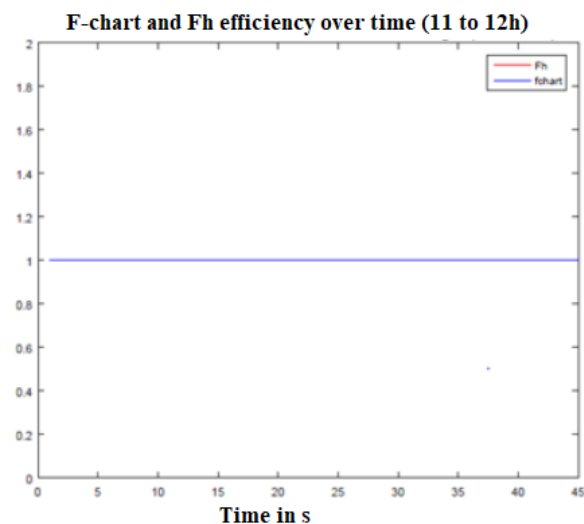


Figure 7: F-chart and Fh efficiency (analytical method and hyperbolic function) over time (11 to 12h)

The effectiveness of the system is achieved in clear skies and in cloudy weather. In addition, the methods combine to achieve the same and better results on the quantity of instantaneous heat. The effectiveness seems intensive at solar noon and becomes less at sunset.

The curves of figures 9-11 take account efficiency theoretically and through experimentation of the system over time. Meantime, these results show that the useful and received energies captured by the system are sufficient to indicate the heat or cold needs of the occupants all day long. Results saw that the two curve patterns coincided during the experiment. This justifies the best performance of the methods used.

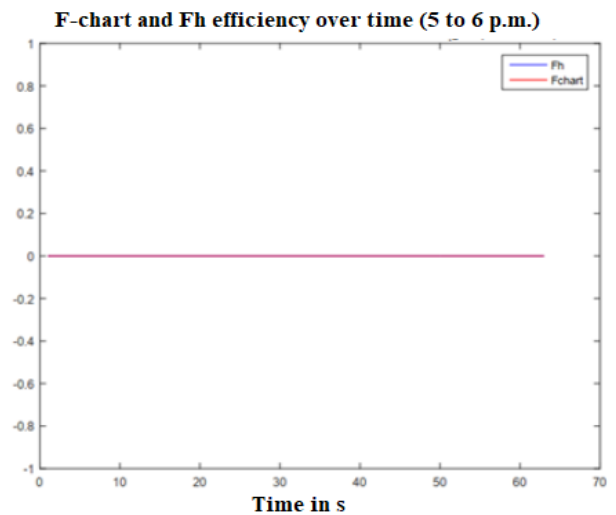


Figure 11: F-chart and Fh efficiency (analytical method and hyperbolic function) over time (5 to 6 p.m.)

The exchanger ensures cooling by lowering the temperature from 1°C to 10°C and provides hot air production with its 4 liter box (40cm×10cm×10cm).

The used technique based on low-temperature heated floors can be transformed into cooling floors.

The low floor helps to reduce indoor temperature. The system conditions: no visible emitter, no movement of air or dust have many advantages. This system is connected in a network of pipes embedded in the floor.

The cooling floor has good emission efficiency because there is little loss in the diffusion of freshness, especially since this remains rather localized at the bottom of the room. It can be recommended for moderate cooling where summer sunshine has been reduced by blinds or any other means of shade.

The distribution of hot and normal air works thanks to fresh air taken from the roof via the solar air collector. To cover F tends to 1, the exchanger consumes around 30% of electrical energy, the remaining 70% being drawn from its environment.

In winter, we always exploit solar energy and we have dimensioned it by developing the heating or cooling work and air conditioning

VI. CONCLUSION

To conclude, the system analysis shows that the thermal evolution of the home requires several factors for air renewal and management of solar gain. The evaluation of exergy flows establishes a linear, elementary model essential to the limitation of thermal discomfort during summer or winter and to the regulation of passive and active air conditioning of a room.

The simplified approach to heating or cooling a building involves practical and methodical conditions to determine the acceptable and ideal temperature in order to control and charge the significant energy needs of the basic equipment of a room.

In short, the complexity of a relationship between energetics and exergetics leads to the following conclusion: The exergy method is more advantageous in the search for a thermal environment in a home because it depends entirely on passive energy, but it remains difficult to dimension given that it is based on the natural climatic conditions of the home which are random variables. This method is more cost-effective in cooling buildings during summer periods. On the other hand, the energy method is easier to manage in the search for perfect air conditioning during winter periods since it combines the active solar energy of a solar collector. Apart from the summer and winter periods, it is also necessary to combine the two methods to have a comfortable and stable daytime and nighttime climate in a home.

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