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Optimizing the Spherical Surface Form Error in Ultra-Precision Turning using Central Composite Design and Genetic Algorithm

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Abstract— *This study investigates the influence of cutting parameters on the form error during spherical surface machining on an ultra-precision lathe. A total of 17 experiments were conducted based on the Central Composite Design (CCD), with parameters selected within the machine's recommended operating range. A predictive model of the form error was developed to simulate and evaluate machining accuracy. The results show that simultaneously increasing spindle speed and depth of cut tends to reduce the form error, while a higher feed rate generally leads to greater deviation. To optimize the process, a Genetic Algorithm (GA) was applied, resulting in an optimal parameter set of spindle speed 1667 rpm, feed rate 5 µm/min, and depth of cut 8 µm, yielding a minimum form error of 0.877 µm. This research not only establishes an effective model for predicting form error but also provides a foundation for optimizing the diamond turning process, contributing to improved surface quality in single-point diamond turning (SPDT).*

Keywords— *SPDT, lens, spherical, form error, genetic algorithm*.

I. INTRODUCTION

Form error refers to the discrepancy between the actual machined surface and its intended theoretical profile, which can significantly impact the functionality of components, particularly in high-precision fields such as optics. In ultra-precision machining (UPM), maintaining tight control over form error, often within the nanometer range, is essential to meet stringent quality requirements. Study [1] investigated the effect of tool misalignment on the geometric precision of convex spherical surfaces and the resulting cutting forces during diamond turning. Meanwhile, study [2] examined the effects of spindle imbalance on form error in single-point diamond turning (SPDT).

UPM is a cutting-edge manufacturing technology capable of achieving exceptionally high accuracy, enabling the production of components with tight dimensional tolerances, ultra-smooth surfaces, and superior finishes. It finds broad application in industries such as optics, aerospace, automotive, telecommunications, and biomedical engineering [3]. UPM techniques are generally divided into four main categories: cutting, grinding, polishing, and non-traditional methods such as electron beam and ion beam figuring. Among these, ultra-precision cutting utilizes ultra-hard tools, most notably diamond tools, to achieve surface roughness at the nanometer scale, encompassing processes such as turning, milling, boring, and compound machining [4]. Of particular importance, SPDT is extensively employed for fabricating optical surfaces, delivering micrometer-level form accuracy and nanometer-level surface roughness.

In study [5], Mao Mukaida conducted a study on the machining of spherical lenses using the Slow Tool Servo method with diamond turning. The material used in this research was single-crystal silicon. The results showed that arrays of spherical microlenses were successfully fabricated with a form error of approximately 300 nm and a surface roughness of about 6 nm. In the same field of precision machining, Min Li explored an alternative approach by employing the weak chemical coordinated thickening polishing method [6]. This technique utilizes abrasive polymer clusters capable of flexibly removing material through a micro-cutting mechanism. Using 9Cr18 steel as the workpiece material and applying the process to spherical surfaces, they successfully fabricated ultra-smooth mirrors with a surface roughness of Ra 25 nm, a roundness error of 4 µm, and a high form accuracy with a PV value of 1.16 µm. Yuetian Huang investigated the application of diamond turning and ion beam figuring for machining spherical aluminum components in [7]. Their results showed that the form error was nearly halved, decreasing from 226 nm to 113 nm, while the surface roughness improved from 3.02 nm to 2.86 nm. Study [8] develops a cutting force prediction model for diamond turning of FSP-treated polycrystalline copper, considering the Hall-Petch effect and residual stress. FSP refines grain size, increases hardness, and improves surface finish (Sz from 23.399 nm to 18.667 nm), but process optimization is needed for varying cutting conditions.

Optimization algorithms have demonstrated greater power and flexibility compared to conventional optimization techniques [9]. In study [10], a comparative analysis of multiple optimization algorithms was conducted, highlighting Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) as the most effective population-based search methods. Owing to its robust global search capability, GA is particularly effective in exploring large and complex search spaces, enabling the identification of globally optimal solutions [11]. Genetic Algorithm (GA) has been successfully applied in numerous studies to optimize surface roughness in both conventional and advanced machining processes, involving materials such as alloy steel [12], ductile iron [13], and Inconel [14]. In addition to single-objective optimization, GA has also been effectively employed for multi-objective optimization in machining. For

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instance, study [15] utilized GA to optimize machining parameters during the turning of composite materials.

Based on the reviewed studies, the objective of this paper is to predict the form error of spherical copper surfaces in diamond turning, identify the key machining parameters influencing form error, and determine the optimal machining conditions to minimize it. The form error optimization is performed within a defined range of machining parameters, including spindle speed, feed rate, and depth of cut. A regression model is developed to predict form error by varying these cutting parameters. Copper is selected due to its excellent machinability, high thermal conductivity, and its ability to achieve ultra-smooth surfaces, making it highly suitable for precision optical components. The Central Composite Design (CCD) is chosen for its effectiveness in providing accurate regression modeling while requiring a reasonable number of experiments. The response surface methodology (RSM) is constructed based on the CCD and is subsequently used as the objective function in GA to efficiently explore the design space and determine the optimal machining parameters.

II. EXPERIMENT

A. *Experimental methodology*

A total of 17 experiments were conducted based on three cutting parameters: spindle speed n (rpm), feed rate F (mm/min), and depth of cut t (µm), all within the machine's recommended range and suitable for the cutting tool.

Fig. 1. Workpieces.

The experiment utilized a copper workpiece (Fig. 1) with a diameter of 30 mm, a height of h = 20 mm, and a spherical radius of R = 19.5 mm. The chemical composition of the workpiece material is presented in Table 1.

TABLE I. Chemical composition of copper alloy.

	Cu	Zn	Pb	Sn	Fe	Ni	Al	Si	Mn
%	54.12	40.94	2.69	0.97	0.75	0.38	0.11	0.02	<0.02

The cutting tool used is a diamond turning tool NN60R0635mWGC-MS0454, with the specifications shown in Table 2.

TABLE II. Cutting tool parameters.

	Radius	Rake Angle	Cutting Height	Conical Clearance
Value	0.684 mm	-25°	7.475 mm	12°

Machining was performed on the Nanoform® X ultra-precision lathe. The workpiece was mounted on the lathe using a specialized fixture. The runout of the chuck was adjusted to ensure that the P-V value was less than 0.005 µm to minimize system error. The machining process was carried out under laboratory environmental conditions.

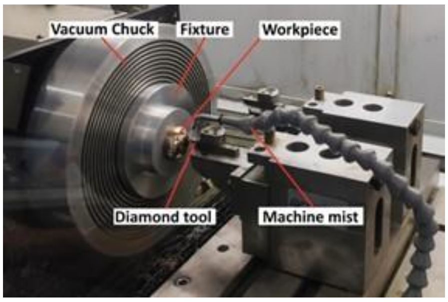

Fig. 2. Machining system.

The machining system (Fig. 2) consists of a vacuum chuck, the copper workpiece, a fixture, the diamond turning tool, and a misting cooling system. The form error of the workpiece was measured along a center crossing curve using the Form Talysurf® i-Series PRO profiler (Fig. 3), which offers high-precision evaluation of optical surfaces with a 20 mm measurement range and 0.2 nm resolution.

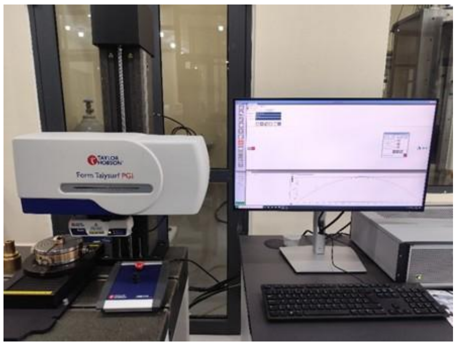

Fig. 3. Form Talysurf® i-Series PRO profiler.

B. *Response surface methodology*

CCD is a widely used experimental design method within RSM, introduced to efficiently develop accurate second-order (quadratic) models for process optimization . CCD combines a factorial or fractional factorial design with center points and a group of axial (or star) points that allow the estimation of curvature in the response surface. Unlike Box-Behnken Design, CCD includes experimental runs at both the center and extreme regions (along the axes) of the design space, enabling a more thorough evaluation of variable interactions and nonlinear effects. The inclusion of multiple center points improves the reliability of the model by providing information on experimental repeatability and inherent variability. In this study, machining parameters are coded into five levels corresponding to the low, high, center, and axial values, as shown in Table 3.

TABLE III. The factors and levels of RSM based on BBD.

Factor	Parameter	Unit	Level				
			-α	-1	0	1	α
A	Spindle speed (n)	rpm	823.44	1000	1500	2000	2176.56
B	Feed rate (F)	mm/min	1.47	5	15	25	28.53
C	Depth of cut (t)	µm	0.94	2	5	8	9.06

The regression equation was developed using Design Expert software with a CCD experimental model in RSM, where the input consists of 3 factors. A total of 17 experiments were conducted, with 3 center-point experiments.

TABLE IV. Central Composite design and results.

No.	n (rpm)	F (mm/min)	t (µm)	Form Error (µm)
1	2000	5	2	0.9421
2	1000	5	2	1.0101
3	1000	25	8	1.1185
4	1500	15	5	1.0488
5	1000	25	2	1.1920
6	1500	15	5	1.0285
7	2000	5	8	0.8853
8	1500	15	5	1.0530
9	2000	25	2	1.0538
10	1000	5	8	0.9588
11	1500	1.47	5	0.8443
12	2000	25	8	0.9836
13	1500	15	9.06	1.0436
14	823.44	15	5	1.1502
15	2176.56	15	5	1.0409
16	1500	28.5313	5	1.0298
17	1500	15	0.94062	1.1010

III. Results and Discussion

A. Results of the ANOVA analysis

Fig. 4 presents the results of the ANOVA analysis and the correlation coefficients. The F-value column reflects the goodness-of-fit of the regression model, indicating the difference in variability between groups of mean values.

Source	Sum of Squares	df	Mean Square	F-value	p-value	
Model	0.1249	9	0.0139	85.63	< 0.0001	significant
A-n	0.0271	1	0.0271	167.39	< 0.0001	
B-F	0.0552	1	0.0552	340.80	< 0.0001	
C-t	0.0093	1	0.0093	57.43	0.0001	
AB	0.0022	1	0.0022	13.36	0.0081	
AC	6.050E-07	1	6.050E-07	0.0037	0.9530	
BC	0.0002	1	0.0002	0.9774	0.3558	
A^2	0.0039	1	0.0039	24.11	0.0017	
B^2	0.0261	1	0.0261	161.20	< 0.0001	
C^2	0.0009	1	0.0009	5.42	0.0528	
Residual	0.0011	7	0.0002			
Lack of Fit	0.0008	5	0.0002	0.9219	0.5938	not significant
Pure Error	0.0003	2	0.0002			
Cor Total	0.1260	16				

Fig. 4. The ANOVA and coded coefficients for the prediction model.

A high F-value suggests that the model is better at explaining the variation in the data compared to a model without influential factors. If the F-value is large and the P-value is less than 0.05, the model is considered statistically significant. With an F-value of 85.63 and a P-value less than 0.05, the model demonstrates high statistical significance and is consistent with the experimental data.

P-V	=
+1.05	
-0.0482	* A
+0.0688	* B
-0.0283	* C
-0.0164	* AB
-0.0003	* AC
-0.0044	* BC
+0.0241	* A²
-0.0624	* B²
+0.0114	* C²

Fig. 5. Final Equation in Terms of Coded Factors.

Fig. 5 shows the coefficients of the regression equation. The coefficients, marked in Fig. 4, have a significant impact on the value of the regression equation due to their high reliability (p-value < 0.05), while other coefficients can be excluded due to their negligible impact. The regression equation for the form error obtained is as follows:

$$PV = 1.046 - 0.048A + 0.068B - 0.028C - 0.016AB + 0.024A^2 - 0.062B^2 \quad (1)$$

B. Optimization using genetic algorithm

GA was proposed by Professor John Holland from the University of Michigan in 1975. It is an optimization algorithm that simulates the mechanisms of genetic inheritance and natural selection, based on Darwin's theory of biological evolution. The Genetic Algorithm is one of the most practical, highly effective, and powerful optimization techniques. It provides a general framework for solving complex problems, such as nonlinear, multimodal, and multi-objective problems. It can find global optimal solutions more effectively than traditional optimization methods. The algorithm flow is illustrated in Fig. 6.

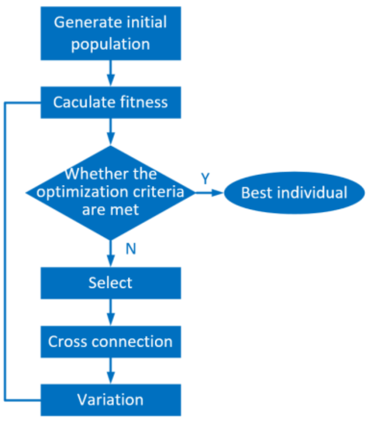

Fig. 6. GA process.

In this study, the Matlab Optimization Toolbox was used to implement GA. Equation (1) was used as the fitness function, with a population size of 100, a crossover fraction of 0.8, and a stopping condition set at 300 generations. The parameters of the algorithm are shown in Table 5.

TABLE V. Genetic Algorithm parameters.

Number	Parameter	Value
1	Population Size	100
2	Max Generations	300
3	Crossover Fraction	0.8

The optimal results are shown in Fig. 7. The optimal form error value is 0.877 µm, with the encoded technological parameters A = 0.667, B = -1, and C = 1, corresponding to the cutting parameters: spindle speed n = 1667 rpm, feed rate F = 5 µm/min, and depth of cut t = 8 µm.

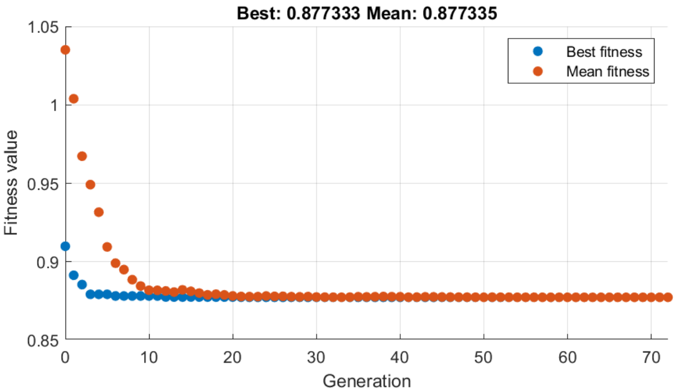

Fig. 7. Variation of best fitness with number of generations.

Optimal results show that, at constant depth of cut, form error increases as the spindle speed decreases. This is because, at lower speeds, the tool remains in contact with the workpiece for a longer duration per revolution, making the cutting process more susceptible to low-frequency vibrations and mechanical oscillations from the machine. In ultra-precision machining, where the tool radius is small and system stability is critical, these vibrations can easily cause tool path deviation, resulting in surface deformation and increased form error.

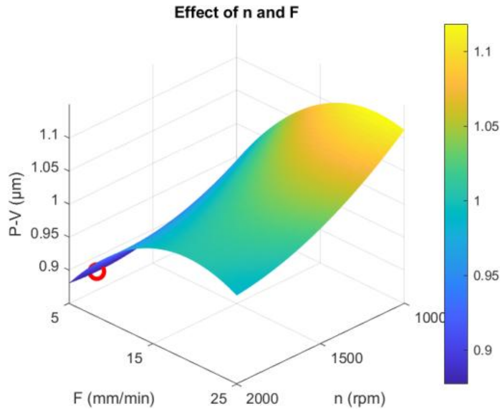

Fig. 8. Effect of feed rate (F) and spindle speed (n) on form error.

Conversely, when spindle speed is kept constant but the depth of cut is reduced, the volume of material removed per pass decreases, resulting in weaker cutting forces that are insufficient to stabilize the tool. This makes the tool more prone to rebound or slight deviation from the ideal cutting path due to elastic deformation of the workpiece and fixture. For copper, which has relatively high ductility and elasticity, insufficient cutting force may prevent the tool from achieving the intended depth, leading to significant form errors. Additionally, a shallow depth of cut increases the tool's sensitivity to vibration and mechanical disturbances from the surface layer of the workpiece.

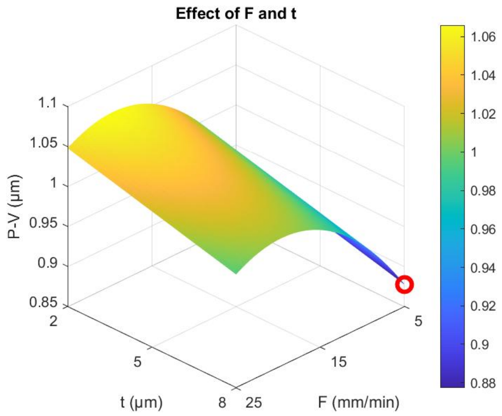

Fig. 9. Effect of feed rate (F) and deep of cut (t) on form error.

Regarding feed rate, form error tends to increase as the feed rate rises, since the spacing between consecutive tool paths becomes larger, resulting in a rougher surface and greater geometric deviation. However, when the feed rate exceeds F > 20 mm/min, a slight decrease in form error is observed. This may be due to a shift in the cutting regime toward a "strong continuous cut," where the increased cutting force helps stabilize the tool and reduces minor vibrations. Moreover, the shorter cutting time at higher feed rates minimizes heat accumulation and localized thermal deformation at the cutting zone, contributing to improved form accuracy.

When the feed rate is kept constant, form error is not only affected by machine stability but also significantly influenced by spindle speed and depth of cut. Fig. 10 show that simultaneously increasing both parameters leads to a noticeable reduction in form error due to improved cutting stability.

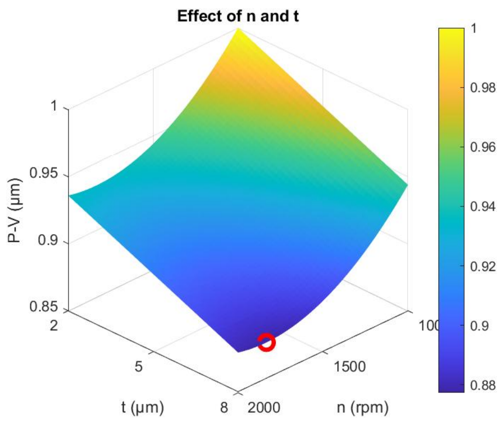

Fig. 10. Effect of deep of cut (t) and spindle speed (n) on form error.

Higher spindle speed shortens the tool-workpiece contact time, reducing the impact of low-frequency vibrations and stabilizing the tool path. It also improves chip evacuation and prevents material buildup at the cutting zone, enhancing surface

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quality. Meanwhile, a larger depth of cut generates sufficient cutting force to stabilize the tool, minimizing rebound or deflection caused by elastic deformation or vibrations, especially important when machining ductile materials like copper.

Conversely, reducing both spindle speed and depth of cut weakens cutting forces, making the tool more prone to instability and vibration, which increases geometric errors such as waviness, roundness deviation, or form inaccuracy.

Based on the above analysis, form error in ultra-precision diamond turning of copper is influenced by spindle speed, depth of cut, and feed rate. Increasing spindle speed and depth of cut enhances process stability, reduces vibration and tool path deviation, thereby lowering form error. Conversely, when these two parameters are reduced, weaker cutting forces make the tool more prone to instability. A high feed rate increases form error due to a rougher surface, but when it exceeds $F > 20$ mm/min, the error may slightly decrease thanks to a more stable cutting regime. Therefore, to achieve high precision, all three cutting parameters must be optimized simultaneously.

IV. CONCLUSION

In this study, the Response Surface Methodology (RSM) was employed to design experiments for evaluating the influence of machining parameters on form error in ultra-precision turning of spherical copper alloy surfaces. Additionally, the Genetic Algorithm was applied to identify optimal machining conditions that minimize form error.

1. The developed predictive model revealed significant effects from linear, quadratic, and interaction terms on form error. ANOVA findings suggested that feed rate had the most significant effect, followed by spindle speed and depth of cut.
2. Optimization using the Genetic Algorithm yielded a minimum form error of 0.877 µm with the optimal parameters: spindle speed of 1667 rpm, feed rate of 5 µm/min, and depth of cut of 8 µm. Experimental verification confirmed the reliability and practical applicability of both the model and the optimization method.
3. The analysis of parameter influences provides a practical foundation for selecting suitable cutting conditions in ultra-precision machining of spherical surfaces, thus enhancing machining quality.

However, this study was limited to a specific range of machining parameters and did not consider other factors such as surface roughness or tool wear. The model could be extended to multi-objective optimization or integrated with advanced artificial intelligence techniques such as neural networks for improved efficiency. The findings have strong potential for application in manufacturing high-precision components such as optical parts, precision molds, and other products requiring exceptional accuracy, while also paving the way for more effective optimization strategies in precision engineering.

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