International Research Journal of Advanced Engineering and Science
ISSN (Online): 2455-9024

Dynamic Analysis and Balance Control of 2D Cubli Model Applying Biomimetic Optimization Algorithm

Nguyen Xuan Thien*, Vu Huy Trung, Nguyen Hong Quan
Academy of Military Science and Technology, Hanoi, Vietnam
*nguyenthienvtl8x@gmail.com

Abstract—*This paper presents the results of dynamic modeling and control of 2D Cubli model using PID control system. Dynamics equations are established to describe the motion of the 2D Cubli and the reaction wheel. Frictional factors that impede movement are also considered. The parameters of PID control system are searched by the Ant Colony Optimization (ACO) algorithm. The control problem is designed to control the equilibrium position and control the trajectory. The numerical simulation results show that the control error is small, the system meets the requirements of equilibrium position and trajectory in the shortest time. Furthermore, this shows the efficiency of the ACO optimization algorithm. Research results are the basis for building realistic 2D Cubli models and developing 3D Cubli models.*

Keywords—*Cubli model, dynamics, PID control, biomimetic algorithm.*

I. INTRODUCTION

The cube robot is a type of the inverted pendulum capable of moving in space and is widely used to study control algorithms [1]. It can balance itself on one of its edges or a corner (Fig. 1) and can jump back to a leveling position from a resting position. This balance is established by the movement of the inertial wheels on the three faces of the cube. They rotate and produce a moment of inertia that meets the equilibrium requirements of the cube [2].

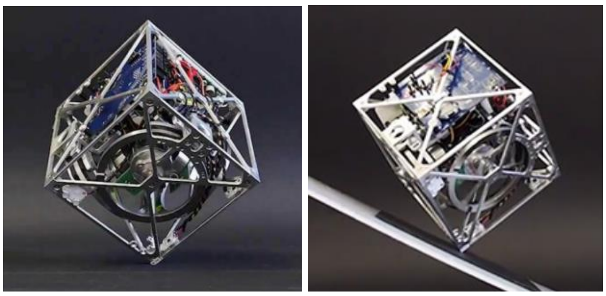

Fig. 1. 3D Cubli model [3]

The self-balancing cube is not only used to develop control algorithms but also has many applications in the aerospace field such as artificial satellites that control the state in zero-gravity space without resorting to the action of any external force to navigate [4]. 3D inverted pendulum dynamics modeling and control were developed very early in [5]. A 15x15x15 cm cube can be moved from a resting position to a balanced position on one of the sides, and then jumping onto one of the corners designed and manufactured in [6]. Pendulum model developed in [7] with momentum control magnetic blocks and brake system. This pendulum can move in many different directions. When considering only one side of the self-balancing cube, it becomes an inverted pendulum system in two dimensions, also known as a 2D Cubli. This block consists of a square pendulum face and a inertial wheel which is a rotating disk fixed at the center of the pendulum. The rotating disk and the 2D cube are a closed system with conserved momentum. The force of inertia generated by the inertial wheel serves to balance the pendulum. The Cubli 2D model is used to simplify cube models nonlinear problem, creating the foundation for building the control algorithm for these 3D cubes. This 2D model is very suitable in studying and verifying nonlinear control algorithms as well as approach method [8]. The stability of the 2D Cubli inverted pendulum considered in [9] is based on the moment exchange technique. The PID and LQR control systems are described in [10] and [11] to control the Cubli 2D model. The control results show that the PID and LQR systems can completely meet the requirements of the model's balance and jumping ability. However, the problem of parameter selection for the controller still needs to be further developed based on the use of optimization algorithms. On the other hand, the development of new control problems for the inverted pendulum cube model is still a matter of interest as the application of new optimization algorithms and artificial intelligence techniques [12]. Currently, although there are not many scientific publications on this model, it is still developed with new control methods and serves many new purposes.

This paper focuses on building dynamic models and controlling Cubli 2D model. The dynamics equations is built on the basis of the Lagrange - Euler energy method with the calculation of the total kinetic and potential energy of the system. The traditional PID control system is used to solve the problem of position control and trajectory control for the rotation angle of 2D Cubli. The parameters of the PID control system are optimally searched on the basis of the ACO algorithm with the control error objective function being the smallest. The research results have important implications in building a 3D cube model control platform.

II. MATERIALS AND METHODS

A. Dynamics modelling the 2D Cubli

Consider the 2D Cubli model as shown in Fig. 2. In which, Fig. 2a describes the equilibrium state of the system. The coordinate system fixed in the vertical plane is $(OXY)_0$ and the coordinate system attached to the Cubli is $(OXY)_C$. The

Nguyen Xuan Thien, Vu Huy Trung, and Nguyen Hong Quan, “Dynamic analysis and balance control of 2D Cubli model applying biomimetic optimization algorithm” *International Research Journal of Advanced Engineering and Science*, Volume 8, Issue 4, pp. 118-122, 2023.

coordinate system $(OXY)_w$ is attached to the inertial wheel. The rotational angle of the Cubli relative to axis $(OY)_0$ is q_c and the rotational angle of the inertial wheel relative to axis $(OY)_w$ is q_w. The L_c and L_w are the distance from the center of gravity G of Cubli to O_0 and the distance from O_w to O_0. Fig. 2b and Fig. 2c show position of 2D Cubli at equilibrium.

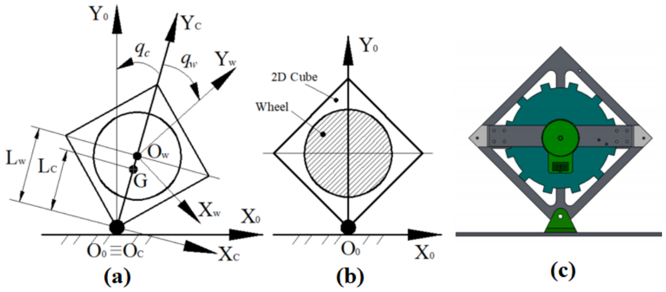

Fig. 2. 2D Cubli model

The general definition of the generalized variable vector of the system is $\mathbf{q}=\begin{bmatrix} q_c & q_w \end{bmatrix}^T$. The angular velocity and angular acceleration vectors of the system are $\dot{\mathbf{q}}=\begin{bmatrix} \dot{q}_c & \dot{q}_w \end{bmatrix}^T$ and $\ddot{\mathbf{q}}=\begin{bmatrix} \ddot{q}_c & \ddot{q}_w \end{bmatrix}^T$. The system has only the rotation of the Cubli and the relative rotation of the inertial wheel with the power from the motors. The moments caused by the friction force acting on the Cubli are $C_{rc}\dot{q}_c$ and $C_{rw}\dot{q}_w$, respectively. According to Fig.1a, the angular velocity of the Cubli is $\dot{q}_c$ and the relative angular velocity of the inertial wheel is $(\dot{q}_c+\dot{q}_w)$. The rotational kinetic energy of the system is determined as follows

$$T=\frac{1}{2}I_{total}\dot{q}_c^2+\frac{1}{2}I_w(\dot{q}_c+\dot{q}_w)^2 \tag{1}$$

Where, I_{total} and I_w are the moment of inertia of the whole system and the moment of inertia of the wheel, respectively. It is approximated that $I_{total}=I_C+I_w$ with I_C is moment of inertia of 2D Cubli.

The potential energy of the system is described as follows

$$V=L_c m_t g\cos q_c \tag{2}$$

where, $m_t=m_c+m_w$ is the total mass of the system and $g=9.81(m/s^2)$ is the gravity acceleration.

The Lagrange function is considered as follows [13]

$$L=T-V \tag{3}$$

The Lagrange differential equations are defined as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)-\frac{\partial L}{\partial \mathbf{q}}=\tau \tag{4}$$

Where, τ is a generalized force vector without potential energy that includes the driving torque of the motors and the torque caused by the friction force.

The motion differential equation of 2D Cubli is established as follows

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{G}=\tau \tag{5}$$

In which, the component matrices are presented in detail as follows

$$\mathbf{M}=\begin{bmatrix} I_{total} & 0 \\ 0 & I_{total}I_w \end{bmatrix};\mathbf{C}=\begin{bmatrix} C_{rc} & -C_{rw} \\ -C_{rc}I_w & C_{rw}(I_{total}+I_w) \end{bmatrix};$$
$$\mathbf{G}=\begin{bmatrix} L_c m_t g\sin q_c \\ -I_w L_c m_t g\sin q_c \end{bmatrix};\tau=\begin{bmatrix} T_{motor} \\ T_{motor}(I_{total}+I_w) \end{bmatrix} \tag{6}$$

And T_{motor} is the driving torque of the motor.

B. PID controller and ACO algorithm

The PID controller [14] has been widely used in the industry but it is hard to determine the optimal or near optimal PID parameters using classical tuning methods as Ziegler Nichols. This paper presents the ACO algorithm to find the suitable parameters of the PID controller for 2D Cubli model. The Ant colony optimization (ACO) algorithm is a probabilistic technique that finds the optimal path in a defined range based on the ant colony's search behavior from their nest to a food source.

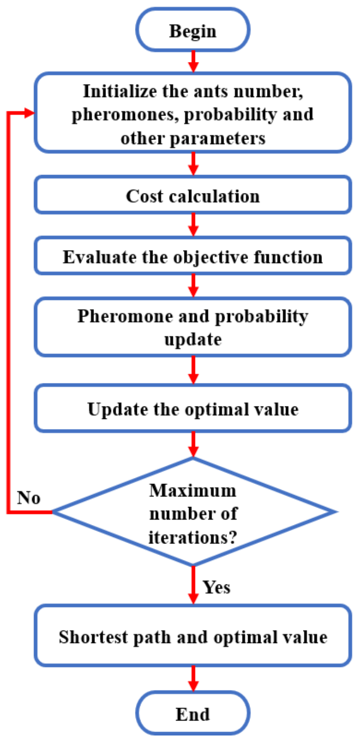

Fig. 3. ACO algorithm diagram

This algorithm was developed by Gambardella and Dorigo in 1991 and is a type of meta-heuristic optimization algorithm [15]. The basic idea of the ACO algorithm is to simulate the communication process between ants using pheromones to learn and improve the path to the food source. Pheromone is a chemical secreted by ants while on the road to mark and attract other ants. The effect of pheromones depends on the distance and quality of the food source. Ants tend to choose paths with more pheromones, but there is also a small probability of discovering new paths. Through many iterations, the ants will find the shortest and most efficient path from the nest to the food source. Fig. 3 depicts the basic steps to implement the ACO algorithm.

Suppose a k ant goes from node i to node j with the probability as

$$p_{i,j}^{k} = \frac{(\tau_{i,j}^{\alpha})(\eta_{i,j}^{\beta})}{\sum(\tau_{i,j}^{\alpha})(\eta_{i,j}^{\beta})} \quad (7)$$

Where, $\tau_{i,j}$ is the amount of pheromone on the i,j edge; α is the parameter to control the influence of $\tau_{i,j}$; $\eta_{i,j}$ is the statistical information on edge i,j that allows finding the best value or visibility (usually reaching the $1/d_{i,j}$ value with d being the distance); β is the parameter that controls the influence of $\eta_{i,j}$. The pheromone value stored in the path of the k ant is $\Delta\tau_{ij}^{k}$ on edge i,j . Pheromone values updated by all ants that have completed the trip are as follows

$$\tau_{ij} \leftarrow (1-\rho).\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad (8)$$

Where, ρ is the volatility of the pheromone; m is the number of ants participating in the search. If the ant walks on edge i,j , then $\Delta\tau_{ij}^{k} = Q / L_k$ and vice versa $\Delta\tau_{ij}^{k} = 0$. Where, L_k is the length of the trip of the k ant and Q is a constant. If the environment is static or the fluctuations are small, the small rate of evaporation of the pheromone is the best choice for optimal performance. If the environment fluctuates greatly, it is necessary to increase the evaporation rate value to high in order to quickly find the shortest path to the food source.

C. Numerical simulation results

The control problem is built in two cases. Case 1 is position control. Case 2 is path control. The control problem in case 2 is to investigate the periodic oscillation of the pendulum. Structural controller of system is designed as in Fig. 4 for 2D Cubli model.

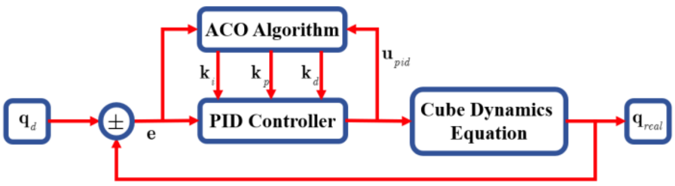

Fig. 4. Control diagram 2D Cubli model

The geometrical and dynamics parameters of the model are described in Tab. 1.

The control objective in the position control problem is to satisfy the equilibrium position with angles $q_c = 0$ and $q_w = 0$. The desired trajectory in the trajectory control problem is $q_c = \cos\frac{\pi t}{2}(rad)$ and $q_w = \cos\frac{\pi t}{4}(rad)$. The parameters set in the optimization algorithm ACO preferences are described in Tab. 2.

TABLE I. THE GEOMETRICAL AND DYNAMICS PARAMETERS

Parameters	Symbols	Values	Units
Distance from G to O_0	L_C	0,077	m
Distance from O_w to O_0 .	L_w	0,101	m
Mass of 2D Cubli	m_c	0,057	kg
Mass of inertial wheel	m_w	0,115	kg
Total mass of system	m_t	0.172	kg
The moment of inertia of the 2D Cubli	I_C	$0,39.10^{-3}$	kgm^2
The moment of inertia of the wheel	I_w	$1.17.10^{-3}$	kgm^2
The moment of inertia of the system	I_{total}	$0.887.10^{-3}$	kgm^2
Effect of friction factor on 2D Cubli	C_{rc}	$1,02.10^{-3}$	kgm^2/s
Effect of friction factor on inertial wheel	C_{rw}	$0,05.10^{-3}$	kgm^2/s

TABLE II. INITIAL PARAMETERS OF ACO ALGORITHM

ACO parameters	Values
Number of iterations	10
Number of Ants	10
Alpha and beta coefficient	0.8 and 0.2
Evaporation rate	0.7
Number of variables	3
Lower boundary	[0.1; 0.1; 0.1]
Upper boundary	[500; 500; 500]
number of nodes for each variable	1000

The objective function J to be achieved in the ACO algorithm is described as follows

$$J = \int_{0}^{+\infty} |\mathbf{e}(t)| dt \quad (9)$$

Where, $\mathbf{e}(t)$ is the error vector of the rotational angles q_c and q_w between the desired and control values. The optimal values in the two cases of position and trajectory control are presented in Tab. 3.

TABLE III. OPTIMAL PID VALUES

Optimal parameters		Kp	Ki	Kd
Position control	Joint 1	425.44	270.82	0.1
	Joint 2	493.49	493.99	417.43
Trajectory control	Joint 1	452.46	22.12	10.61
	Joint 2	158.23	375.4	130.7

The simulation results of the 2D Cubli's equilibrium position control is shown in Fig. 5, Fig. 6 and Fig. 7. Accordingly, the equilibrium position of the Cubli and the wheel is reached after 4.5(s), the overshoot value has the maximum value respectively 0.04 (rad) and 0.02 (rad), the time to move to the equilibrium position is very small (0.5s), the static error value reaches zero after 6(s).

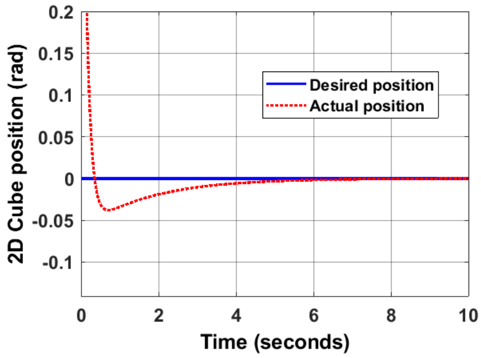

Fig. 5. Control value q_c

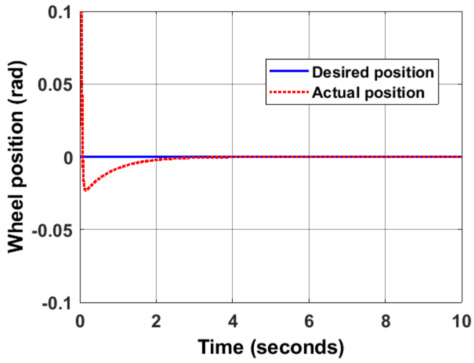

Fig. 6. Control value q_w

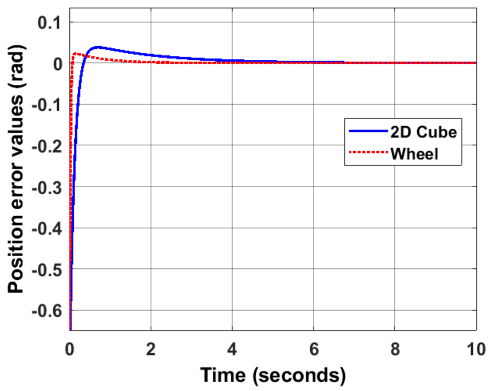

Fig. 7. Position error value q_c và q_w

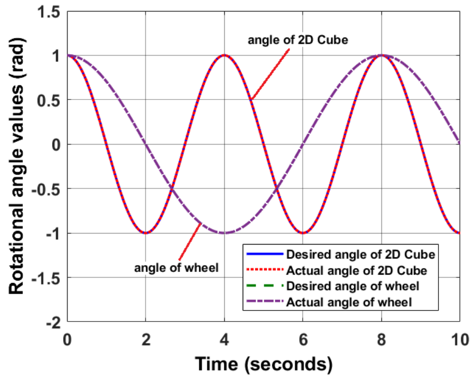

Fig. 8. Cubli and wheel trajectory control results

The results of trajectory control are shown in Fig. 8 and Fig. 9. Accordingly, the control trajectory is performed by a PID control system with parameters optimized by the ACO algorithm to follow the desired trajectory. This shows that the system control is hight quality. Fig. 9 depicts the trajectory control error with small value. The largest error when controlling the 2D Cubli's trajectory is $10^{-4}(rad)$, while the maximum wheel control error is $0.3\times10^{-4}(rad)$.

Fig. 10 and Fig. 11 show the torque values that control the motion trajectory of the 2D Cubli and the wheel.

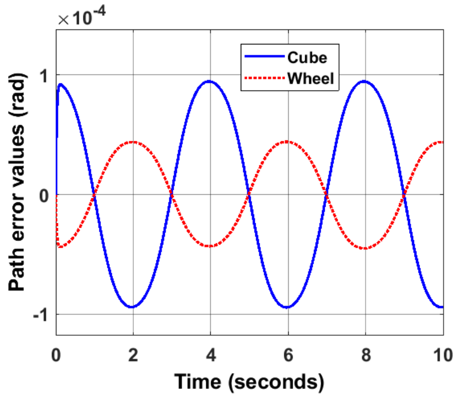

Fig. 9. Control error value

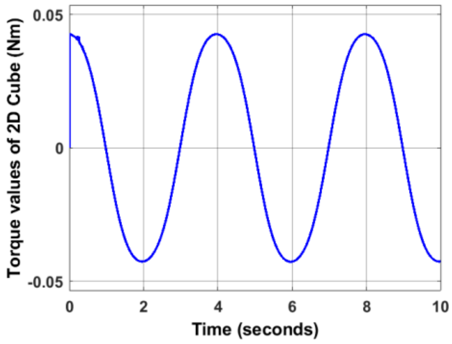

Fig. 10. Control torque value for 2D Cubli

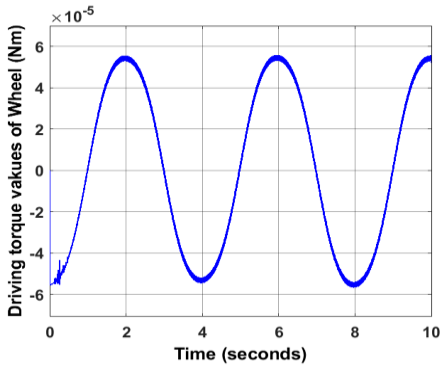

Fig. 11. Control torque value for inertial wheel

This path has a periodic form, so the control torque value also follows a specific rule. The maximum 2D Cubli control torque value reaches 0.04 (Nm), the inertial wheel control torque value reaches small value and is close to zero.

III. CONCLUSIONS

This paper describes the results of dynamic analysis and balance control of the 2D Cubli inverted pendulum robot system in the plane with the basic movements of Cubli and the inertial wheel. The Lagrange energy method is used to build the dynamics equations. The PID control system with parameters optimized by the ACO algorithm shows the efficiency and stability of the system in the cases of balance control and periodic oscillation control. The optimal efficiency according to the given objective function of the ACO algorithm is also recorded. The research results allow to evaluate the nonlinearity of the model and serve for the development of 3D Cubli models along with more modern control systems.

REFERENCES

[1] P. Reist and R. Tedrake, "Simulation-based lqr-trees with input and state constraints," in Robotics and Automation (ICRA), 2010 IEEE International Conference on, may 2010, pp. 5504 –5510.

[2] R. L. Mark, W. Spong, P. Corke. Nonlinear control of the inertia wheel pendulum. Automatica, 1999.

[3] https://idsc.ethz.ch/research-dandrea/research-projects/archive/cubli.html

[4] M. Gajamohan, M. Merz, I. Thommen, R. D'Andrea. The Cubli: A Cube that can Jump Up and Balance. 2012 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2012. Vilamoura, Algarve, Portugal.

[5] A. Nalin, D. D. Bernstein, H. McClamroch, J. Shen, A. K. Sanyal. Dynamics and control of a 3d pendulum. In 43rd IEEE Conference on Decision and Control, pages 323–328, University of Michigan, December 2004.

[6] T. Widmer, M. Gajamohan, M. Muehlebach, R. D'Andrea. The cubli: A reaction wheel-based 3d inverted pendulum. In 2013 European Control Conference, pages 268–274, Switzerland, July 2013.

[7] K. G. John, W. Romanishin, R. Daniela. M-blocks: Momentumdriven, magnetic modular robots. IROS, 2013.

[8] H. K. Khalil, Nonlinear Systems, Pearson India; 3th edition (2014)

[9] J. Meyer, N. Delson, and R. de Callafon, "Design, modeling and stabilization of a moment exchange based inverted pendulum," in 15h IFAC Symposium on System Identification, Saint-Malo, France, 2009, pp. 462 – 467.

[10] S. L. Elena. Cubic Structure Capable of Balancing. Bachelor's Thesis. University POLITEHNICA of Bucharest. 2015.

[11] D. Posada. Design and control of an inertia wheel cube 2D prototype with a reaction wheel. Bachelor of Science in Mechanical Engineering. Universidad De Los Andes. 2017.

[12] B. Andrievsky, "Global stabilization of the unstable reactionwheel pendulum," Automation and Remote Control, vol. 72, pp. 1981–1993, 2011.

[13] D. Morin. Introduction to classical mechanics with problems and solutions. Cambridge University Press, 2008.

[14] K. Astrom and T. Hagglund. PID Controllers: Theory, Design and Tuning. Instrument Society of America, USA, 1995.

[15] M. Dorigo, V. Maniezzo, A. Colorni (1991) The ant system: an autocatalytic optimizing process, Technical Report TR91-016, Politecnico di Milano.

Nguyen Xuan Thien, Vu Huy Trung, and Nguyen Hong Quan, "Dynamic analysis and balance control of 2D Cubli model applying biomimetic optimization algorithm" *International Research Journal of Advanced Engineering and Science*, Volume 8, Issue 4, pp. 118-122, 2023.