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Teaching Research and Application of Real Analysis

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Abstract**—***Real analysis are an important foundational course in mathematics majors in higher education institutions. Due to the abstract and difficult nature of the theory, it makes learning difficult for students. Therefore, it is necessary to change traditional teaching methods to enhance students' enthusiasm for learning real analysis. Combining practical application background and analogizing with the courses such as mathematical analysis and point set topology, heuristic teaching methods are adopted to guide students to learn basic concepts and theories, and improve their understanding level of basic concepts and theories, and stimulate students' initiative in learning.*

Keywords**—***Application, Analogical connection, Heuristic, Real analysis*.

I. INTRODUCTION

In mathematics majors of higher education institutions, real analysis is an important foundational course that has a profound impact on mastering the basic ideas of modern abstract analysis, improving abstract thinking and mathematical expression abilities, deepening understanding of mathematical analysis knowledge, and deepening understanding of relevant content in middle school mathematics.

Through the study of this course, students should have a correct understanding of the purpose of establishing Lebesgue integrals, master the process of establishing Lebesgue measures and integrals on straight lines, and understand the connections and differences between Lebesgue integrals and Riemannian integrals.

Through learning, students will master the concepts of convergence by measure, convergence almost everywhere, and basically uniform convergence, and clarify the differences and connections between several types of convergence.

Through learning, students can proficiently master and correctly use several important theorems in this course, such as the Levy monotone convergence theorem for non negative measurable sequences, the Lebesgue control convergence theorem, Fatou lemma, the Newton-Leibniz formula for Lebesgue integrals, and the Fubini theorem

Due to the abstraction and difficulty of the theory of real analysis, it makes it very difficult for students to learn. Therefore, it is necessary to change traditional teaching methods to enhance students' enthusiasm for learning real analysis.

II. LEARNING PRACTICAL APPLICATION BACKGROUND

Based on practical application background, we explain relevant concepts and theories to enhance students' learning enthusiasm.

The concepts of real analysis are numerous and complex, and make learning dull and uninteresting. If appropriate application examples can be added to teaching to make students feel the profound application background behind complex definitions, it can easily stimulate students' enthusiasm for learning and improve their learning effectiveness.

For example, when discussing finite and non-finite functions, students are prone to confusion about functions with infinite values at a certain point, and believe that such functions cannot exist, and this is an impossible thing in primary and secondary schools. In fact, such functions do exist, such as the infinitely deep square well function in quantum mechanics

$$v(x)=\begin{cases}0, 0<x<a,\\ \infty, x\notin(0,a),\end{cases}$$

which has important physical significance. It represents that the potential field of a particle within [0, a] is 0, and it is infinite outside the interval. As the potential field of an infinite particle outside this interval cannot leave the interval (0, a) and go outside the interval. Furthermore, we can obtain the differential equation in quantum mechanics

$$-y''(x)=\lambda y(x)$$

with the boundary conditions

$$y(0)=y(1)=0.$$

In fact, the subsequent course of real analysis--functional analysis, is a mathematical discipline developed in the context of quantum mechanics Through this explanation, students feel that such a function is meaningful and different from previous functions, which is a research object in real analysis.

In the teaching of this course, some appropriate content can also be added to help students understand the purpose of learning. For example, when teaching Cantor sets, a series of questions such as what the circumference of a snowflake is equal to can be asked. Then the modern fractal geometry pioneered by French mathematician B.B. Mandelbrot is introduced, such as whether dimensions are all integers and how to describe the scale of measurements. When describing measures and integrals, content such as random measures and Ito stochastic integral can be introduced. Stochastic differential equations are a very important tool in modern financial mathematics, which can be used to establish research and development models for financial derivatives such as futures, stocks, and bonds, and predict important economic situations and trends.

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III. Adopt Heuristic Teaching Methods

Adopt heuristic teaching methods, and guide students to learn basic concepts and theories, and stimulate students' initiative in learning.

The main object of research on real analysis is the Lebesgue integral theory, which has undergone a long foundational process, including set theory, measure theory, measurable function theory, etc. But it is only a general explanation that may be sudden for students. We can guide them to learn relevant theories step by introducing a series of questions.

When learning the cardinality or potential of a set, firstly we can ask such question: which set contains more elements for the set of positive integers N or the set of positive even numbers

$$\{2,4,6,\ldots\}?$$

This will arouse the thinking of the students and stimulate students' initiative in learning real anysis. Although the number of the elements of the rational number set Q and that of N are same, Q is dense in the real number set R.

As seen in mathematical analysis, the Dirichlet function

$$D(x)=\begin{cases}1, x\in Q\\ 0, x\notin Q\end{cases}$$

is not continuous. However, we found that the set of points with function value 1 is a rational point set, and the set of points with function value 0 is an irrational point set. These two sets are very irregular. Are these sets measurable? If measurable, how to measure the "length" of these irregular sets? This is the measurable problem of sets.

Furthermore, we use measurable sets to study the properties of functions and obtain a broader class of functions - measurable functions. And we find that the above function is not Riemann integrable. Can we establish a new integration theory to study the integrability of this type of function? Through this series of explanations, students will understand that real analysis is the extension and continuation of Riemann integrals in mathematical analysis, and is the fundamental theory of modern analytical mathematics, with important theoretical value.

Inserting mathematical allusions, celebrity stories, and theorem proofs into classroom teaching can greatly enhance students' interest in learning. For example, when teaching the generation of real analysis, we start with the following mathematical question: Is it correct for continuous functions to be differentiable except for individual points? Weierstrass constructed a function and proved that the function is not differentiable at any point. This conclusion prompted people to study more properties of the function, which functions are continuous, which functions are differentiable, which functions are integrable, whether to modify the definition of integral, and so on, which prompted the birth of real analysis.

IV. Connect With Previous Courses

By analogy with courses such as mathematical analysis, linear algebra, and point set topology, we aim to improve our understanding of basic concepts and theories.

Real analysis is the deepening and extension of mathematical analysis, and is a topic of discussing differential and integral in a broader context. Therefore, when learning similar concept and theories, it is important to pay attention to their connections and differences. For example, When it comes to metric spaces, we can compare metric spaces with linear spaces in linear algebra. A metric space is a non empty set equipped with a metric structure, while a linear space is a non empty set equipped with two operations: addition and multiplication, and remains closed on these two operations. The former discusses its topological structure, while the latter focus on its algebraic structure. For example, the set [0,1] equipped with the usual distance can be a metric space, while it cannot be a linear space according to the usual linear operations.

In mathematical analysis, a type of function we study is a continuous function, but the limit function of a continuous function sequence may not be continuous. That is to say, the equality

$$\lim_{n\to\infty}\int_a^b f_n(x)dx=\int_a^b \lim_{n\to\infty} f_n(x)dx$$

may not hold. If we add conditions for uniform convergence, we can ensure that the limit function is continuous. But for measurable function sequences in real analysis, the limit function of measurable function sequence is measurable. Comparing in this way helps us to have a deeper understanding of the differences between the two types of functions.

When explaining the Lebesgue integral of a non negative measurable function, we obtain that if the function $f(x)$ is non-negative and measurable on the measurable set E, and

$$\int_E f(x)dx\geq 0$$

then it equals to 0 almost everywhere. However, in mathematical analysis, if the function $f(x)$ is non-negative and continuous on a closed interval $[a,b]$, then it equals to 0 everywhere on the closed interval. The conclusions is different and the methods of the proofs are very different. When studying the problem of permutation between two repeated integrals, Riemann integrals in mathematical analysis generally require the continuity of binary functions, while Lebesgue integrals require the integrability of binary functions, etc. When studying commutative order of the limit and integrals of function sequence, Lebesgue integral requires that the function sequence converge almost everywhere and is controlled by a non negative integrable function, while Riemann integral requires uniform convergence and continuity, which is a strong requirement.

When explaining the relationship between everywhere convergence and uniform convergence of function columns, an example from mathematical analysis can be introduced

$$f_n(x)=x^n, x\in[0,1].$$

We obtain that this function sequence converges to

$$f(x)=\begin{cases}0, x\in[0,1)\\ 1, x=1\end{cases}$$

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everywhere, but does not uniformly converge on $[0,1]$. However, upon careful study, it was found that this function sequence converges uniformly after removing a set whose measure can be arbitrarily small. So, is this phenomenon accidental? Our answer is no, this is an inevitable conclusion that leads to the famous EropoB theorem.

V. Conclusions

By the above methods, some definitions and theories become relatively clear and intuitive in the course of real analysis. At the same time, students learn the definition and properties of Lebesgue integral through comparative methods, and finally summarize the similarities and differences with Riemann integral, as well as the relationship between the two classes of integrals. This makes it relatively easy to understand the content of multiple integrals, repeated integrals etc. in real analysis.

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