International Research Journal of Advanced Engineering and Science
ISSN (Online): 2455-9024

Optimization of the Electrical Energy Billing System for Residential in the City of Kinshasa in Prepayment Mode

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Abstract**—** *In this article, we present different mathematical models to optimize the system of electric energy billing in the residential area of the electrical network. The equations developed address the problem of the dynamics of electricity energy consumption in the residential area. This optimization is carried out on the residential users of prepayment meters; whose study area is taken as a case in the commune of Lemba city province of Kinshasa in the Democratic Republic of Congo. The optimization of the billing in the residential area shows the quasi budget deficits and the hidden costs to the residents who use the low voltage electrical network in prepayment mode.*

Keywords**—** *Optimization, Billing system, Electrical energy, City of Kinshasa, Prepayment mode.*

I. INTRODUCTION

Currently, most residential customers have a post-paid billing system. 85% of SNEL's residential subscribers are billed post-payment, compared to 15% for prepayments; progressive billing in three tranches is in effect in Kinshasa as in many cities in the country. The price per kWh is an increasing function of the level of consumption [1-5].

The electricity sector in Kinshasa is affected by crises in the continuity of electrical service. The Kinshasa Regional Distribution Department found that 60% of households were moderately satisfied with the quality of electricity, compared to 30% who said that the quality of electricity was very poor. Only 10% were satisfied with the quality of the energy provided.

These costs often depend on the technology deployed and the performance of the sector [5-9].

The objective of this article is to explore the budgetary viability of the billing of electrical energy consumed in a residential area in prepayment mode. The problem is to identify the individual consumption of each resident and determine its daily and monthly consumption dynamics. The optimization of billing in the residential area shows the quasi-budgetary deficits and hidden costs to the residents who use the low voltage electricity network in prepayment mode.

In Kinshasa, the studies revealed that several subscribers complain about the cost of the energy actually consumed, which is 1.8 times more expensive. This work is therefore based on a problem that aims to mature several thoughts. First of all, it is a question of understanding the principle of residential pricing, which represents the most important part of the energy demand. We propose billing alternatives based on the most widespread models in the world.

II. DATA AND METHODOLOGIES

The residential data collected and available concern the most densely populated communes using prepayments in Kinshasa. All subscribers with a prepaid meter received their electricity continuously over the period. These data cover the period from August 2018 to November 2021. We will focus only on residential customers in this article, as this research only covers household subscribers.

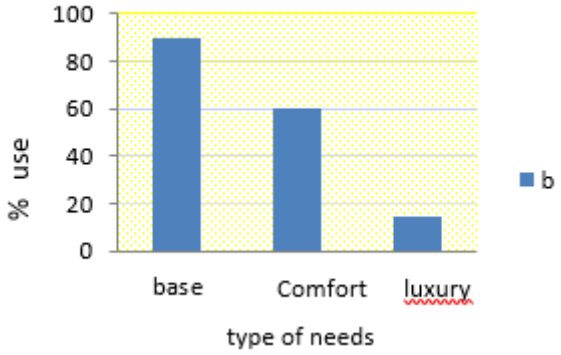

Figure 1: Classification of use by type of need in Kinshasa C.A.E, 2011.

It can therefore be seen that on average, Kinshasa households spend about 28.9% of their income to meet their electrical energy expenses. According to the hypotheses, we can therefore conclude that most households in Kinshasa are in a situation of energy insecurity insofar as they spend more than 10% of their income on energy expenses. From the above, we can deduce that:

$$q(t) = \begin{cases} po * q \\ p1 * q \\ 250 * p1 + (q - 250) * p2 \end{cases} \quad (1)$$

If q≥20
If 20<q≤250
If q>2500
Where p_0 is the price of the first band (78 FC) for all consumption less than or equal to 20 kWh per month, p1 is the

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price of band 1 (109 FC) for all consumption between 21 kWh and 250 kWh per month, and finally p2 is the price of band 2 for all consumption above 250 kWh per month.

Data analysis

Of the 2880 invoices analyzed, there are respectively 838 invoices belonging to the first bracket (20kWh per month), 1866 invoices belonging to the second bracket (21 to 250 kWh) and 176 invoices belonging to the third bracket (Figure 2). This represents a distribution of 28.4%, 64.8% and 6.8% by increasing band. The brackets reflect the income level of the households. Only 5% of individuals have consumption above 250kWh.

These results indicate that needs 1 mainly concern lighting and television (social bracket), needs 2 cover needs 1 in greater proportions as well as the use of household appliances and finally needs 3 concern the use of energy consuming equipment such as air conditioners, washing machines, ...

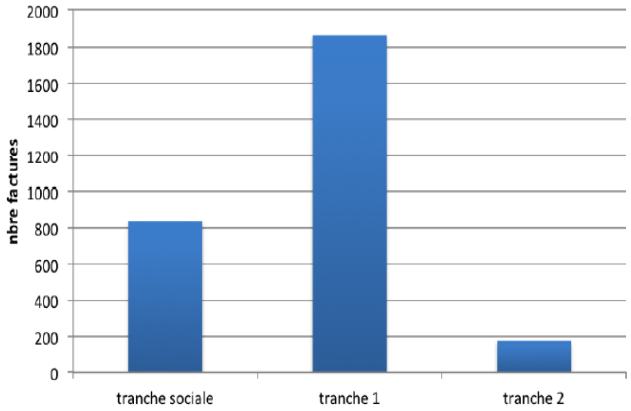

Figure 2: Distribution of invoices by bracket

Ramsey-Boiteux billing

In line with the Ramsey pricing assumptions on second-tier pricing, the question here is to determine the optimal price for which the difference between price and marginal cost is inversely proportional to the elasticity of demand. In our case, to simplify the analysis, we assume that all consumers have the same elasticity, given that the monopoly has no information on household incomes. We therefore assume that the effect of a change in tariffs on the level of consumption is homogeneous, whatever the subscriber. This second-tier pricing allows monopolies to reduce their deficit while maximizing the collective surplus.

The equations allow us to determine the optimal price resulting from a Ramsey Boiteux pricing. This price is a function of marginal cost, elasticity and the Lagrangian constant. This hypothesis respects the principle of the minimum right of access to electricity by respecting that the monopoly charges the energy consumed at its marginal cost. Moreover, the social price in the reference tariff is not subject to value added tax, which justifies that at this price, the marginal revenue equals the marginal cost.

$$\frac{p - C_m}{p} = \frac{\lambda}{1 + \lambda}\left(\frac{1}{\xi}\right) \quad (2)$$

$$\frac{p - C_m}{p} = \frac{\lambda}{\xi * (1 + \lambda)}$$
$$\xi * (p - C_m)(1 + \lambda) = p * \lambda$$
$$p = \frac{\xi * (p - C_m)(1 + \lambda)}{\lambda}$$
$$\xi * p(1 + \lambda) - (C_m)(1 + \lambda) = p * \lambda$$
$$p^* = (\xi(1 + \lambda) - \lambda) = \xi(C_m)(1 + \lambda)$$
$$p^* = \frac{\xi(C_m)(1 + \lambda)}{\xi(1 + \lambda) - \lambda} \quad (3)$$
$$\xi = |\xi| \& \xi^*(1 + \lambda) \neq \lambda \quad (4)$$

The difference of 65 FC between the reference marginal cost and the calculated Ramsey price thus represents the costs and expenses incurred by the monopoly to operate.

Peak billing

Peak billing is therefore aimed at consumers to be efficient because the energy called during peak periods costs more to generate. Given the price distortion, it is therefore a matter of a consumer being less energy-intensive during peak periods because he will be billed for energy at a higher price.

In fact, it shows a window from 06:00 to 08:00 (morning) and a second window from 18:00 to 22:00 (evening) as shown in Figure 3. We therefore deduce that: q2=6 and q1 =18. That is, 25% of the daily price of a peak consumption against 75% off-peak.

We maintain the reference prices for tranches 1 and 2 insofar as they are higher than the marginal costs and therefore supposed to capture a net surplus for the monopoly. Let p1 = 109 and p2 = 115. For the price, we assume that p(2)=115 and p(1)= 109. The idea is to maintain the prices of the network tariffs for the reference blocks 1 and 2. We therefore summarize the peak pricing as the following system of equations:

$$\begin{cases} T_1 = p1 * q1 \\ T_2 = p2 * q2 \end{cases} \quad (5)$$

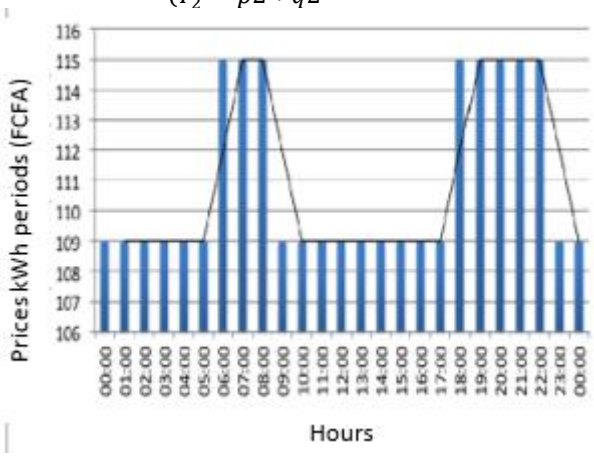

Figure 3: Daily electricity trend

III. Prepayment of a Household in Kinshasa

Prepaid tariffs are not yet regulated in DR Congo in a general way, and in particular to subscribers of the city of Kinshasa. Several price indexes have been charged in the prepayment system in the city of Kinshasa since 2016, with rates in Congolese francs ranging from 50 to 250. Table shows the consumption point of subscribers in a study area.

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ISSN (Online): 2455-9024

Consumption point of the subscribers

Joint consumption of the month of February

Table 1: Joint consumption point of the month of February

N°	PT (W)	C_i (kWh)	Cm_{28}(kWh)
1	1382	6.1499	172.1972
2	2382	10.5999	296.7972
3	1655	7.36475	206 .213
4	5382	23,9499	670.5972
5	2850	12.6825	355.11
6	3755	16.70975	467. 873
7	5250	23.3625	654.15
8	4382	19.4999	545.9972
9	3382	15.0499	421.3972
10	6382	28.3999	795.1972
11	1950	8.6775	242.97
12	1175	5.22875	146.405
13	2075	9.23375	258.545
14	4015	17.86675	500.269
15	3015	13.41675	375.669

Consumption February and December

Table 2: Consumption point February and December

N°	TP (W)	$_{29}$ cm (kWh)	cm $_{30}$ (kwh)	$_{31}$ cm (kWh)
1	1382	178.3	184.4	190.6
2	2382	307.3	317.9	328,5
3	1655	213.5	220.9	228.3
4	5382	694.5	718.4	742.4
5	2850	367,7	380.4	393.1
6	3755	484,5	501.2	518
sept	5250	677.5	700.8	724.2
8	4382	565.4	584.9	604.4
9	3382	436.4	451.4	466.5
dix	6382	823.5	851.9	880.3
11	1950	251.6	260.3	269
12	1175	151.6	156.8	162.09
13	2075	267.7	277.01	286.2
14	4015	518.1	536	553.8
15	3015	389.08	402.5	415.9

IV. Results and Payment Simulation

Daily payment results and analysis

The cost of the prepayment dedicated to the consumption of electrical energy, depends on the tariff set for a kilowatt hour. It is important that each subscriber can know the daily cost of his residential consumption.

For a subscriber whose daily consumption range is between 5.22875kWh and 28.3999 kWh. Figure (4) shows that for a tariff set at fifty Congolese francs, the range of daily cost varies from 261.4375 to 1419.995 Congolese francs.

February 28-day payment results and analysis

It is important that each subscriber be able to know the cost of the February twenty-eight day prepayment for his residential consumption. For a subscriber whose daily consumption range is between 146.405 kWh and 795.1972 kWh, the curve in Figure (5) shows that for a tariff set at one hundred Congolese francs, the daily cost range varies between 14,640.5 and 79,519.72 Congolese francs.

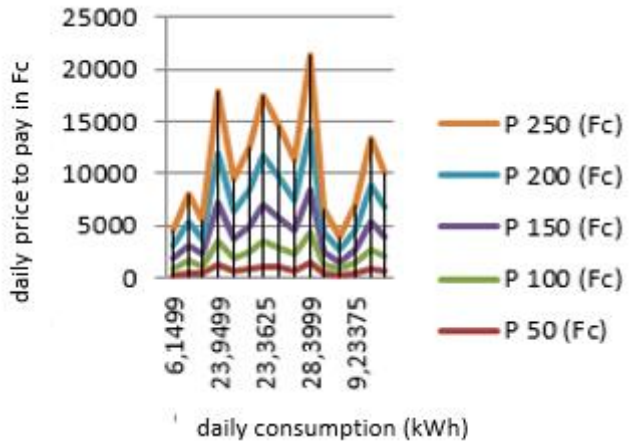

Figure 4: Results and analysis of daily payment

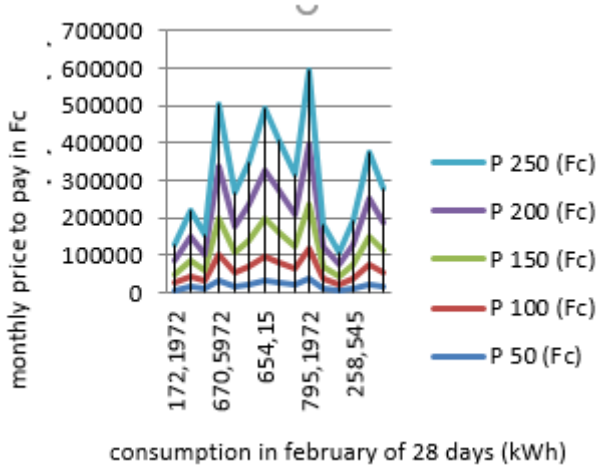

Figure 5: February 28-day payment results and analysis

February 29-day payment results and analysis

It is important for each customer to know the cost of the February twenty-nine day prepayment for his residential consumption.

For a subscriber whose daily consumption range is between 151.63375 kWh and 823.5971 kWh, the curve (6) shows that for a tariff set at one hundred and fifty Congolese francs, the daily cost range varies between 22,745.0625 and 123,539.565 Congolese francs.

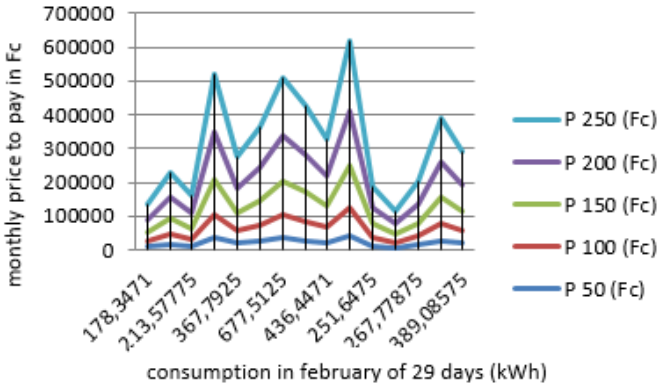

Figure 6 : February 29-day payment results and analysis

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ISSN (Online): 2455-9024

30-day monthly payment results and analysis

It is important for each subscriber to know the cost of the thirty-day monthly prepayment for his residential consumption. For a subscriber whose daily consumption range is between 156.8625 kWh and 851.997 kWh, the curve shows that for a tariff set at two hundred Congolese francs, the daily cost range varies between 31,372.5 and 170,399.4 Congolese francs.

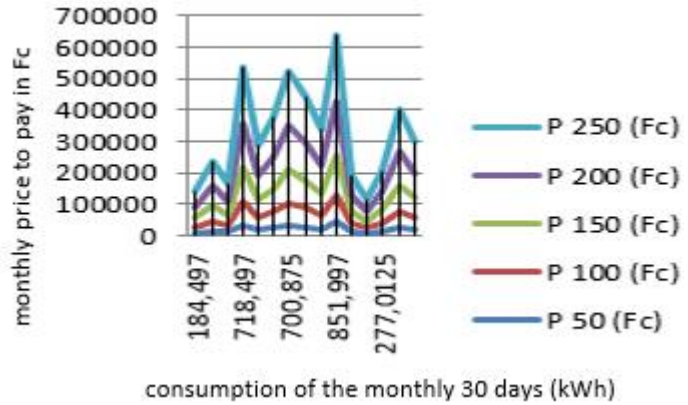

Figure 7: 30-day monthly payment results and analysis

31-Day Monthly Payment Results and Analysis

It is important for each subscriber to know the cost of the monthly prepayment at thirty-one days for his residential consumption. For a subscriber whose daily consumption interval is between 162.09125 kWh and 880.3969 kWh. On figure (8), we have represented the curve of analysis of the monthly payment of 31 days according to the Congolese francs. This curve shows that for a tariff set at two hundred and fifty Congolese francs, the range of daily costs varies between 40,522.8125 and 220,009.225 Congolese francs.

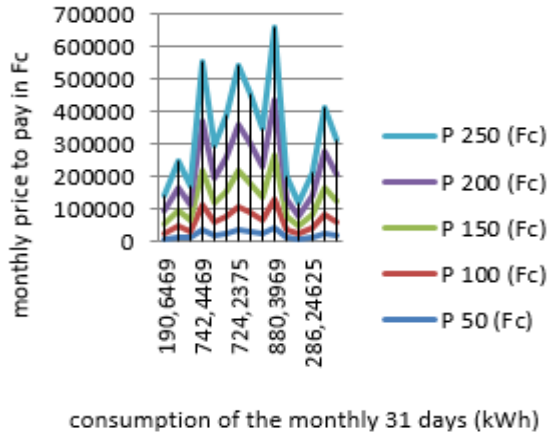

Figure 8 : 31-day monthly payment results and analysis

V. Evolution Of Subscriber Consumption

The subscriber in prepaid mode knows in advance his daily, monthly and annual consumption. This is done in order to adjust the budget forecast on the savings to be rented to the prepaid electric energy.

Figure 9 shows the power of the subscribers in kW according to the consumption of electrical energy. The blue curve shows the evolution of daily consumption, which varies from 5.228 to 28.3999 kWh, the red curve shows the evolution of consumption for the month of February to 28 days, which varies from 146.405 to 795.1972 kWh, the black curve shows the evolution of consumption for the month of February to 29 days, which varies from 151, 6337 to 823.5971 kWh, the yellow curve shows the evolution of monthly consumption for thirty days, which varies between 156.8625 to 851.997 kWh, the green curve shows the evolution of monthly consumption for thirty-one days, which varies between 162.012 to 880.396 kWh.

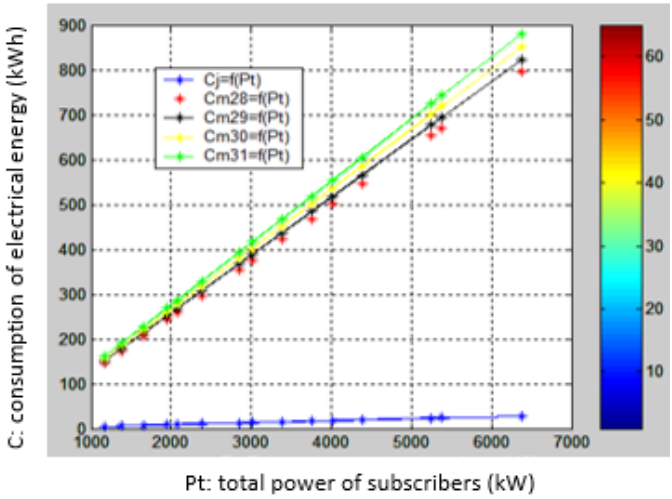

Figure 9: Prepayment simulation by device type

Results and Interpretation of 50 CF Pricing

On Figure 10, we have plotted the energy consumption points in the study area against the price to be paid at the rate of 50 Congolese francs. The blue curve shows the daily payment pattern, which varies from 261.4375 FC to 1,419.995 FC; the red curve shows the payment pattern for the month of February at 28 days, which varies from 7,320.25 FC to 39,759.86 FC; the black curve shows the payment pattern for the month of February at 29 days, which varies from 7. 581.6875 FC to 41.179.855 FC, the yellow curve gives the monthly payment pattern at thirty days which varies between 7.843.125 FC to 42.599.85 the green curve gives the monthly payment pattern at thirty-one days which varies between 8.100.6 FC to 44.019.845 FC.

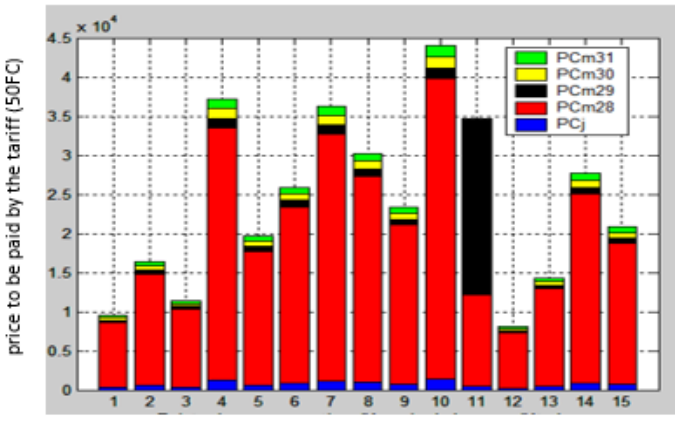

Figure 10: Simulation of prepayment prices at 50 FC

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Results and interpretation of 100 CF pricing

Figure 11 shows the variations in consumption as a function of the price to be paid by the 100 Congolese francs tariff. The green curve shows the daily payment pattern, which varies from 522.875 FC to 2,839.99 FC; the black curve shows the payment pattern for the month of February at 28 days, which varies from 1,640.5 FC to 7,9519.72 FC; the yellow curve shows the payment pattern for the month of February at 29 days, which varies from 15. 163.375 FC to 82.359.71 FC, the red curve gives the monthly payment pattern at thirty days which varies between 15.686.25 FC to 85.199.7 FC, the blue curve gives the monthly payment pattern at thirty-one days which varies between 16.209.125 FC to 88.039.69 FC.

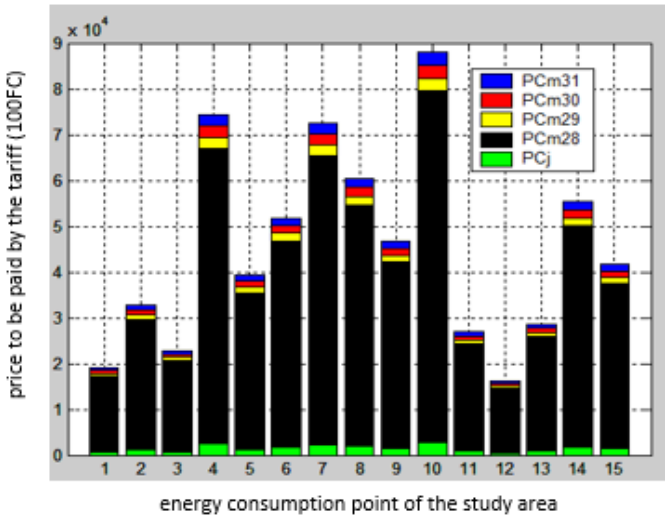

Figure 11 : Simulation of prepayment prices at 100 FC

Results and interpretation of 150 CF pricing

Figure 12 shows the variations in consumption as a function of the price to be paid by the tariff 100 Congolese francs. The red curve shows the daily payment pattern, which varies from 784.3125 FC to 4259.985 FC; the green curve shows the payment pattern for the month of February at 28 days, which varies from 21,960.75 FC to 119,279.58 FC; the yellow curve shows the payment pattern for the month of February at 29 days, which varies from 22. 745.0625 FC to 123.539.565 FC, the curve in blue color gives the monthly payment pattern at thirty days which varies between 23.529.375 FC to 127.799.55 FC, the curve in black color gives the monthly payment pattern at thirty-one days which varies between 24.313.6875 FC to 132.059.535 FC.

Results and interpretation of 200 CF pricing

Figure 13 shows the variations in consumption as a function of the price to be paid by the 200 Congolese franc tariff. The yellow curve shows the daily payment pattern, which varies from 1,045.75 CFA francs to 5,679.98 CFA francs; the blue curve shows the payment pattern for the month of February with 28 days, which varies from 29,281 CFA francs to 159,039.44 CFA francs; the black curve shows the payment pattern for the month of February with 29 days, which varies from 30,326.75 CFA francs to 164,719.42 CFA francs; the green curve shows the payment pattern for the month of February with 29 days, which varies from 30,281 CFA francs to 159,039.44 CFA francs. 326.75 FC to 164,719.42 FC, the green curve gives the monthly payment pattern at thirty days which varies between 31,372.5 FC to 170,399.4 FC, the red curve gives the monthly payment pattern at thirty-one days which varies between 32,418.25 FC to 176,079.38 FC.

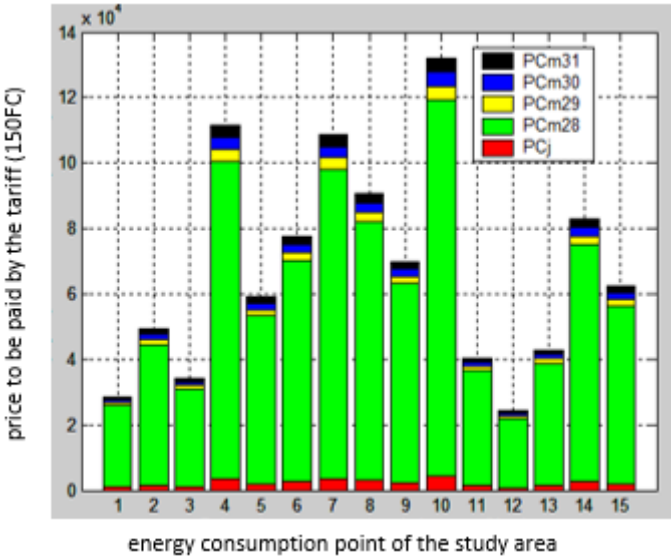

Figure 12 : Simulation of prepayment prices at 150 FC

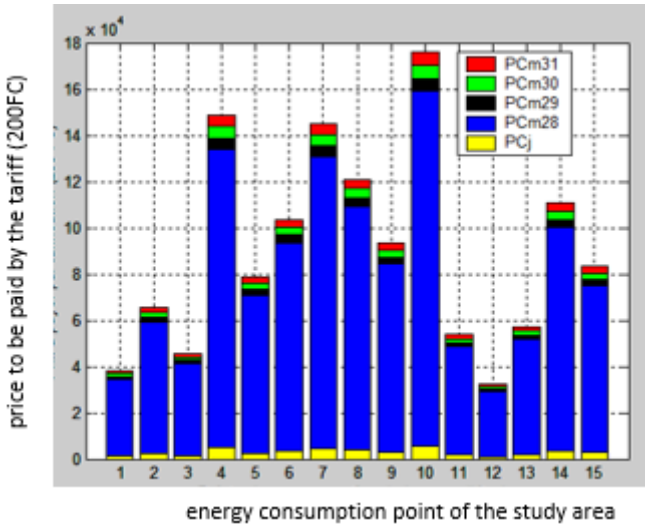

Figure 13 : Simulation of prepayment prices at 200 FC

Results and interpretation of the 250 CF pricing

Figure 14 shows the variations in consumption as a function of the price to be paid by the 250 Congolese franc tariff. The blue curve shows the daily payment pattern, which varies from 1,307.1875 CFA francs to 7,099.975 CFA francs; the red curve shows the payment pattern for the month of February with 28 days, which varies from 36,601.25 CFA francs to 198,799.3 CFA francs; the black curve shows the payment pattern for the month of February with 29 days, which varies from 37. 908.4375 FC to 205.899.275 FC, the curve in green color gives the monthly payment pattern at thirty days which varies

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between 39.522.8125 FC to 212.999.225 FC, the curve in yellow color gives the monthly payment pattern at thirty-one days which varies between 40.503 FC to 220.099 FC.

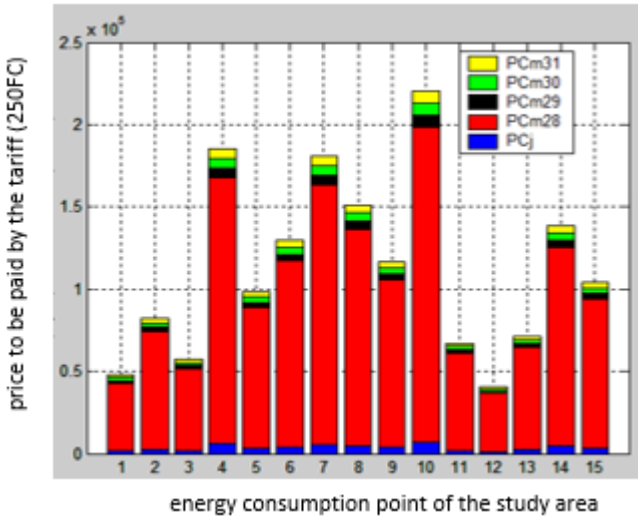

Figure 14: Simulation of prepayment prices at 250 FC

VI. CONCLUSION

This article was about optimizing the system of electric energy billing in residential consumption. The identification method used for each residence reflects the mode of consumption of electrical energy. We applied this method on the daily and monthly consumption, resulting from the simulation with Excel software. The numerical results allow us to budget the payment of the billing of energy consumed in a residential area in prepayment mode. The developed method allowed us to evaluate the residential tariff system in force in DR Congo. Using available electricity bills, we were able to determine the cost of electricity per block. Our analysis revealed a number of findings such as the inadequacy of the billing bands and the prices charged.

In general, our results call into question the effectiveness of the current post-paid tariffs insofar as the consumption volumes billed do not necessarily reflect the real levels of users.

However, the implementation of these new forms of pricing also implies the improvement of metering methods. Prepaid pricing or smart meters are alternatives to be explored to reduce reading bias and payment defaults.

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