International Research Journal of Advanced Engineering and Science
ISSN (Online): 2455-9024

Partial Differential Equations for Shallow Water Systems

Cosmus Emmanuel Mwangangi[1], Benard Kipchumba[2]
[1]Mathematics, University of Nairobi, Nairobi, Nairobi, Kenya
[2]Mathematics, University of Sussex, Brighton, East Sussex, BN1 9QH, United Kingdom

Abstract *— Several studies have been done on the systems of shallow water with specific goals. In this paper, we derive the equations of shallow water systems, find their numerical solutions, simulate the results and study the behavior of shallow water waves over time with the aim of addressing disasters of water reservoirs. We have derived the partial differential equations of shallow water systems in three dimensions, solved them using finite difference method, implemented the solution for the system in Matlab and performed some simulations (for one dimension) to observe the behavior of waves over time. We observe that in the process of constructing water reservoirs, for example water dams, there is need to put into consideration the strength of any possible occurrence of waves.*

Keywords—*Equations of Shallow water systems, PDEs for shallow water systems.*

I. INTRODUCTION

A partial differential equation (popularly known as PDE) is an identity that relates independent variables, the dependent variable, and the partial derivatives of the dependent variable [1]. Generally, we can say that a partial differential equation is a relation containing one or more partial derivatives of unknown function depending on two or more independent variables [2].

Partial differentiation and partial integration occur even in ordinary processes of calculus where partial differential equations do not occur [3]. Partial differential equations have many diverse applications and are frequently encountered when modelling real life problems. The second order linear partial differential equations, in particular, have many engineering and scientific applications. Examples of this class of equations include the Laplace equation, the heat equation and the wave equation [4].

Shallow water equations are commonly used in many diverse areas of science. Of the many applications of shallow water equations, one of their most relevant applications is in geophysical fluid dynamics, where they provide an approximation for rotating stratified fluids. Their important property is that they can be obtained as the amplitude equations for the vertical normal modes of the continuously stratified fluids. For this reason, they become very useful in the analysis of large scale atmospheric circulation. Even though they are an essential tool in theoretical studies, these shallow water equations are also very useful in the process of designing Numerical Weather Prediction models by creating an ideal test model for the evaluation of time integration methods as well as for the investigation of various aspects of space discretization [5].

As a result of these numerous advantages and uses of Shallow water equations, several researchers have done several studies in connection to these equations. Philippe Courtier and Jean-Francois Geleyn, in July 1988, developed a global numerical weather prediction model with variable resolution as an application to the shallow-water equations [6]. In their study, they implement a spectral global shallow-water model with variable resolution.

They prove that the only non-trivial conformal mapping which exists between the two spheres is based on the transformation introduced by Schmidt, but the pole of the collocation grid has no longer to coincide with the pole of dilatation. They then implement the technique in an explicit model, where only minor modifications to a uniform resolution model are needed. From their study, they find out that the semi-implicit scheme and the nonlinear normal mode initialization work satisfactorily and in addition, results obtained from their one day forecasts show that the method is successful in dealing with the shallow-water equations.

In 2010, Enrique D.Fernández, Nietoa, Pascal Nobleb Jean and Paul Vila published a paper (Shallow Water equations for Non-Newtonian fluids). In this paper, they provide a consistent thin layer theory for some Non-Newtonian fluids that are in-compressible and flowing down an inclined plane under the effect of gravity. They also do the derivation of shallow water models in the case of power-law fluids and Bingham fluids [7].

In December 2017, Anna Geyer and Ronald Quirchmayr published a paper (Shallow water equations for equatorial tsunami waves). In this paper, they present derivations of shallow water model equations of Korteweg–de Vries and Boussinesq type for equatorial tsunami waves in the f-plane approximation and discuss their applicability. In their derivations, they consider two-dimensional one-layer oceanic flows in the equatorial region. They then use the methods from asymptotic analysis to derive two shallow water model equations for waves of small amplitude from the f-plane approximation of the Euler equations for divergence-free in-compressible fluids [8]. These are just a few of the many studies that have been carried out on the system of shallow water equations. In our study we present the derivations of the shallow water equations in three dimensions from the Navier Stokes equations where we start by deriving the Navier Stokes equations.

We there after solve these shallow water equations using finite difference method and implement the same in Matlab software, but in one-dimension to study the behavior of the waves in shallow waters over time. Our main goal is to establish whether we can use these partial differential equations for shallow water systems to control disasters associated with the breaking of water reservoirs (dams).

An example is the tragedy that occurred on 9th May 2018 at around 7:15 p.m in Nakuru, Kenya where one of the eight dams located in Solai, Nakuru County, burst its banks draining down close to 190 million liters of water and sweeping away homes, people and social amenities, leading to deaths of more than 47 people [20]. This is the main motivation for this paper; to find out if we can use the partial differential equations for shallow waters during the construction of Dams to control and avoid disasters.

Model Development

Shallow Water Equations

Shallow water equations are hyperbolic or parabolic partial differential equations that govern fluids flow in coastal regions, rivers and channels. In other words, the shallow water equations describe the evolution of a hydro-static homogeneous (constant density), in-compressible flow on the surface of the sphere. They describe a thin layer of fluid with constant density in hydro-static balance, bounded from below by the bottom topography and from above by a free surface. They exhibit a rich variety of features, because they have many conservation laws. The propagation of a tsunami can be described accurately by these equations until the wave approaches the shore [9]. Shallow water system can also be applied when constructing water reservoirs such as dams. They can be used to control the disasters that can be associated with such reservoirs. They are derived from the physical conservation laws for mass and momentum and are valid for problems in which vertical (height) dynamics can be neglected compared to horizontal effects (length).

Derivation of Shallow Water Equations

To derive the systems of equations for shallow waters, we follow four basic steps as outlined in [10]. These are:

I Derive the Navier-Stokes equations from the conservation laws.
II Ensemble average the Navier-Stokes equations to account for the turbulent nature of ocean flow.
III Specify boundary conditions for the Navier-Stokes equations for a water column.
IV Use the boundary conditions to integrate the Navier-Stokes equations over depth.

We follow the derivation technique step by step as in [4].

Derivation of the Navier-Stokes Equations

In this step, we shall derive the Navier-Stokes Equations from the conservation laws. We proceed as follows:

Mass Conservation

For the case of mass conservation, we consider mass balance over a control volume Ω and then proceed to derive the continuity equation. Thus,

$$\frac{d}{dt}\int_{\Omega}\rho dV = -\int_{\partial\Omega}(\rho\vec{v})\cdot\vec{n}dA \tag{3.1}$$

where $\frac{d}{dt}\int_{\Omega}\rho dV$ is the time rate of change of total mass in Ω, $\int_{\partial\Omega}(\rho\vec{v})\cdot\vec{n}dA$ is the net mass flux across boundary of Ω, ρ is the fluid density (Kg/m^3),

$$\vec{v} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

is the fluid velocity (m/s), and $\vec{n}$ is the outward unit normal vector on the boundary ∂Ω.

We then need to re-express equation (3.1). Let us recall the Gauss' Divergence Theorem [7] that we are going to use to re-express equation 3.1 (to ensure that the integration in both left-hand and right-hand sides is over the surface but not boundary). The Theorem states that:

$$\int_{\Omega}\nabla\cdot F dA = \int_{\partial\Omega}F\cdot\vec{n}d\Omega, \tag{3.2}$$

Where Ω is the region of the plane, and ∂Ω is the boundary of the plane.

Applying this theorem to the right hand side of equation 3.1, we obtain

$$\frac{d}{dt}\int_{\Omega}\rho dV = -\int_{\Omega}\nabla\cdot(\rho\vec{v})dV. \tag{3.3}$$

Now, to obtain continuity equation, we apply Reynold's Transport Theorem to equation (3.3) (to get rid of the derivative operator outside the left-hand integral). Recall that the Reynold's Transport Theorem [11], [12] is as shown in equation (3.4).

$$\frac{d}{dt}\int_{\Omega_t}\rho dV = \int_{\Omega_t}\left(\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{v}\right)dV. \tag{3.4}$$

The proof to this theorem can be found in [13].

Applying this theorem to the left-hand side of equation (3.3) we obtain equation (3.5).

$$\frac{d}{dt}\int_{\Omega}\rho dV = \int_{\Omega}\left(\frac{\partial\rho}{\partial t} + \rho\nabla\cdot\vec{v}\right)dV\ . \tag{3.5}$$

It is also important to note that for a fixed domain Ω; that is, not evolving with time, we can interchange $\frac{d}{dt}$ with integral (Also, $\nabla\rho = 0$ for in-compressible fluid).

This implies that equation (3.3) becomes equation (3.6).

$$\int_{\Omega}\frac{\partial\rho}{\partial t}dV + \int_{\Omega}\nabla\cdot(\rho\vec{v})dV = 0. \tag{3.6}$$

Assuming that ρ is smooth, then equation (3.6) becomes equation (3.7);

$$\int_{\Omega}\left(\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v})\right)dV = 0. \tag{3.7}$$

Because Ω is arbitrary, we obtain the continuity equation (3.8).

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \tag{3.8}$$

Momentum Equation

In the next step, we derive the momentum equation, by considering linear momentum balance over a control volume Ω. That is,

$$\frac{d}{dt}\int_{\Omega}\rho\vec{v}dV = -\int_{\partial\Omega}(\rho\vec{v})\vec{v}\cdot\vec{n}dA + \int_{\Omega}\rho\vec{b}dV + \int_{\partial\Omega}\vec{T}\vec{n}dA \tag{3.9}$$

If we apply Gauss's theorem to the first and the third right-hand side of equation (3.9), we obtain equation (3.10);

$$\frac{d}{dt}\int_{\Omega}\rho\vec{v}dV + \int_{\Omega}\nabla\cdot(\rho\vec{v}\vec{v})dV - \int_{\Omega}\rho\vec{b}dV - \int_{\partial\Omega}\nabla\cdot\vec{T}dV = 0. \tag{3.10}$$

Applying Reynold's Transport Theorem to the first term of equation (3.10), we obtain equation (3.11);

$$\int_{\Omega}\frac{\partial}{\partial t}(\rho\vec{v})dV + \int_{\Omega}\nabla\cdot(\rho\vec{v}\vec{v})dV - \int_{\Omega}\rho\vec{b}dV - \int_{\partial\Omega}\nabla\cdot\vec{T}dV = 0\ . \tag{3.11}$$

With the assumption that $\rho\vec{v}$ is smooth, and the fact that Ω is arbitrary, we obtain equation (3.12):

$\frac{\partial}{\partial t}(\rho\vec{v}) + \nabla\cdot(\rho\vec{v}\vec{v}) - \rho\vec{b} - \nabla\cdot\vec{T} = 0$ (3.12)

This is the momentum equation.

Our next step is now to obtain the Navier-Stokes equation from the continuity and momentum equations already obtained above. In order to achieve this, we make some assumptions about the fluid in the areas of the density ρ, the body forces $\vec{b}$ and the stress tensor $\vec{T}$. These assumptions are:

The fluid is incompressible, that is, the volume and density does not change with pressure.

Salinity and temperature of the fluid are constant (sea water).

Sea water is a Newtonian fluid (constant viscosity, zero shear rate and zero shear stress, that is, the shear rate is directly proportional to the shear stress). This affects the form of $\vec{T}$. Recall that the gravity is a single body force and this implies that $\rho\vec{b} = \rho\vec{g} + \rho\overrightarrow{b_{others}}$.

For the case of Newtonian fluid, $\vec{T} = -p\vec{I} + \vec{t}$ where p is the pressure and $\vec{t}$ is the matrix of stress terms. That is,

$$t = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{bmatrix} \quad (3.13)$$

Therefore, we finally obtain our Navier-Stokes equations in 3D as in equations 3.14 and 3.15.

$\nabla\cdot\vec{v} = 0$ (3.14)

$\frac{\partial}{\partial t}\rho\vec{v} + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p + \rho\vec{g} + \nabla\cdot\vec{t}.$ (3.15)

Expansion of Navier-Stokes Equations

To be able to find the numerical solutions to the Navier-Stokes equations obtained, there is need to expand them first. Note that;

$$\nabla\cdot\vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}. \quad (3.16)$$

$$\nabla p = \begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{pmatrix} \quad (3.17)$$

Performing some algebraic operations in equations (3.14) and (3.16), we obtain the following expanded Navier-Stokes equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.18)$$

$$\begin{cases} \frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \\ \frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \\ \frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\rho g + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial(\tau_{zz} - p)}{\partial z} \end{cases}$$

(3.19)

Integration of the Navier-Stokes Equations

To obtain the partial differential equations of the shallow water systems, we integrate the expanded Navier-Stokes equations, subject to the following boundary conditions:

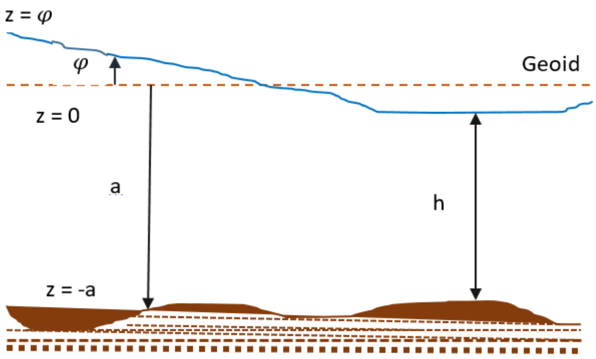

Figure 1 (h<<l) is the pictorial display of the shallow water system, from which we state the boundary conditions.

Note that:

1 $\varphi = \varphi(t,x,y)$is the elevation (in metres) of the free surface relative to the geoid.

2 $a = a(x,y)$is the bathymetry (in metres), measured positive downward from the geoid.

3 $h = h(t,x,y)$is the total depth (in metres) of the column. Note that$h = \varphi + a$.

Boundary Conditions

- At the bottom (z = -a) there is no slip, that is (u = v= 0) and no normal flow:

  $u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} + w = 0.$ (3.20)

  and the bottom shear stress is;

  $$\tau_{bx} = \tau_{xx}\frac{\partial a}{\partial x} + \tau_{xy}\frac{\partial a}{\partial y} + \tau_{xz}. \quad (3.21)$$

  $$\tau_{by} = \tau_{xy}\frac{\partial b}{\partial x} + \tau_{xy}\frac{\partial b}{\partial y} + \tau_{yz}. \quad (3.22)$$

  where τ_{bx} and τ_{by} are the specified bottom frictions.

- At the free surface, $(z = \varphi)$, there is no relative normal flow:

  $$\frac{\partial\varphi}{\partial t} + u\frac{\partial\varphi}{\partial x} + v\frac{\partial\varphi}{\partial y} - w = 0. \quad (3.23)$$

  p = 0, and surface shear stress is:

  $$\tau_{sx} = -\tau_{xx}\frac{\partial\varphi}{\partial x} - \tau_{xy}\frac{\partial\varphi}{\partial y} + \tau_{xz}. \quad (3.24)$$

  $$\tau_{sy} = -\tau_{xy}\frac{\partial\varphi}{\partial x} - \tau_{yy}\frac{\partial\varphi}{\partial y} + \tau_{yz}. \quad (3.24)$$

Z-Momentum Equation

Before we can integrate over the depth, we need to examine the momentum equation for vertical velocity. By a scaling argument, all the terms except the pressure derivative and the gravity term are small [14]. Therefore, the z-momentum equation reduces to equation (3.25).

$$\frac{\partial p}{\partial z} = \rho g. \quad (3.25)$$

Integrating (3.25) over the depth we obtain

$$p = \rho g h. \quad (3.26)$$

Which is the hydrostatic pressure distribution.

Therefore,

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial h}{\partial x}. \qquad (3.27)$$

And

$$\frac{\partial p}{\partial y} = \rho g \frac{\partial h}{\partial y}. \qquad (3.28)$$

The First Equation of Shallow Water Systems

We integrate Navier-Stokes equation (3.18) over the depth of water to obtain the first equation of shallow water systems. That is:

$$\int_{-a}^{\varphi} \nabla \cdot \vec{v} dz = \int_{-a}^{\varphi} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dz = 0 \qquad (3.29)$$

This simplifies to equation (3.30):

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) = 0, \qquad (3.30)$$

where

$$\bar{u} = \frac{1}{h}\int_{-a}^{\varphi} u dz \qquad (3.31)$$

$$\bar{v} = \frac{1}{h}\int_{-a}^{\varphi} v dz \qquad (3.32)$$

Which is the first equation of shallow water systems, the depth-averaged continuity equation.

The Second Equation of Shallow Water Systems (x-momentum equation)

To obtain the second equation of shallow water systems, we integrate the left hand side of the x-momentum equation (the first equation in the system of Navier-Stokes equations 3.19) over the depth. That is, we find the value of

$$\int_{-a}^{\varphi} \left(\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) + \frac{\partial}{\partial z}(\rho uw)\right) dz (3.33)$$

To perform this integration, we need to recall the Leibnitz integral rule which states that:

$$\int_a^b \frac{\partial f}{\partial z} dx = \frac{\partial}{\partial z}\int_a^b f(x,z)dx + f(a,z)\frac{\partial a}{\partial z} - f(b,z)\frac{\partial b}{\partial z} (3.34)$$

The integral (3.33) simplifies to equation (3.35):

$$\rho\frac{\partial}{\partial t}(h\bar{u}) + \rho\frac{\partial}{\partial x}(h\bar{u}^2) + \rho\frac{\partial}{\partial y}(h\bar{u}\bar{v}) + \{DifferentialAdvectionTerms\}(3.35)$$

The differential advection terms account for the fact that the average of the product of two functions is not the product of the averages.

Now, we integrate the right-hand side of the x-momentum equation, the first equation of (3.19), over the depth of the water.

$$\int_{-a}^{\varphi} \left(\frac{\partial \tau_{xx}}{\partial x} - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right) dz. \qquad (3.36)$$

We apply the Leibnitz integral rule to the first, third and the fourth terms of the right-hand side of equation (3.36). Therefore, equation (3.36) becomes (3.37):

$$\frac{-1}{2}\rho g \frac{\partial h^2}{\partial x} + \tau_{sx} - \tau_{bx} + \frac{\partial}{\partial x}(h\bar{\tau}_{xx}) + \frac{\partial}{\partial y}(h\bar{\tau}_{xy}). (3.37)$$

Where;

$$\bar{\tau}_{xx} = \frac{1}{h}\int_{-a}^{\varphi} \tau_{xx} dz \qquad (3.38)$$

and

$$\bar{\tau}_{xy} = \frac{1}{h}\int_{-a}^{\varphi} \tau_{xy} dz \qquad (3.39)$$

Therefore, the first equation in (3.19) becomes (3.40):

$$\rho\frac{\partial}{\partial t}(h\bar{u}) + \rho\frac{\partial}{\partial x}(h\bar{u}^2) + \rho\frac{\partial}{\partial y}(h\bar{u}\bar{v}) = \frac{-1}{2}\rho g \frac{\partial h^2}{\partial x} + \tau_{sx} - \tau_{bx} + \frac{\partial}{\partial x}(h\bar{\tau}_{xx}) + \frac{\partial}{\partial y}(h\bar{\tau}_{xy}). \qquad (3.40)$$

This simplifies to

$$\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}(h\bar{u}^2) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) = -\frac{1}{2}g\frac{\partial h^2}{\partial x} + \frac{1}{\rho}\left[\tau_{sx} - \tau_{bx} + \frac{\partial}{\partial x}(h\bar{\tau}_{xx}) + \frac{\partial}{\partial y}(h\bar{\tau}_{xy})\right]. \qquad (3.41)$$

The Third Equation of Shallow Water Systems (y-momentum equation)

To obtain the third equation of the shallow water systems, we integrate the second equation of the system of equations in (3.19) (the y-momentum equation). Integrating the left hand side of this y-momentum equation over the depth we obtain equation (3.42)

$$\rho\frac{\partial}{\partial t}(h\bar{v}) + \rho\frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \rho\frac{\partial}{\partial y}(h\bar{v}^2) + \{DifferentialAdvectionTerms\}(3.42)$$

Similarly, integrating the right hand side of the y-momentum equations of (3.19), we obtain (3.43)

$$\frac{-1}{2}\rho g \frac{\partial h^2}{\partial y} + \tau_{sy} - \tau_{by} + \frac{\partial}{\partial x}(h\bar{\tau}_{xy}) + \frac{\partial}{\partial y}(h\bar{\tau}_{yy}). (3.43)$$

Combining equation (3.85) and equation (3.86), we obtain the third equation of the shallow water systems as equation (3.44)

$$\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}(h\bar{v}^2) = -\frac{1}{2}g\frac{\partial h^2}{\partial y} + \frac{1}{\rho}\left[\tau_{sy} - \tau_{by} + \frac{\partial}{\partial x}(h\bar{\tau}_{xy}) + \frac{\partial}{\partial y}(h\bar{\tau}_{yy})\right] \qquad (3.44)$$

Therefore, our shallow water equations in conservative are:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) = 0, \qquad (3.45)$$

$$\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}(h\bar{u}^2) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) = -\frac{1}{2}g\frac{\partial h^2}{\partial x} + \frac{1}{\rho}\left[\tau_{sx} - \tau_{bx} + \frac{\partial}{\partial x}(h\bar{\tau}_{xx}) + \frac{\partial}{\partial y}(h\bar{\tau}_{xy})\right]. \quad (3.46)$$

$$\frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}(h\bar{v}^2) = -\frac{1}{2}g\frac{\partial h^2}{\partial y} + \frac{1}{\rho}\left[\tau_{sy} - \tau_{by} + \frac{\partial}{\partial x}(h\bar{\tau}_{xy}) + \frac{\partial}{\partial y}(h\bar{\tau}_{yy})\right] \quad (3.47)$$

It is worthy to note that, the shallow water equations for the in-compressible flow are not easy to solve. Therefore, to be able to numerically solve these shallow water equations, we assume that our flow is inviscid one (which is still okay considering that we are dealing with shallow water equations). This is the flow of inviscid fluid, meaning that the viscosity is zero. This means that all the terms with shear stress are equivalent to zero.

With this assumption, the three equations simplify to the equations showed below (3.48)

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) + \frac{\partial}{\partial y}(h\bar{v}) = 0. \\ \frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}(h\bar{u}^2) + \frac{\partial}{\partial y}(h\bar{u}\bar{v}) = -\frac{1}{2}g\frac{\partial h^2}{\partial x} \\ \frac{\partial}{\partial t}(h\bar{v}) + \frac{\partial}{\partial x}(h\bar{u}\bar{v}) + \frac{\partial}{\partial y}(h\bar{v}^2) = -\frac{1}{2}g\frac{\partial h^2}{\partial y} \end{cases} \quad (3.48)$$

Numerical Schemes for Partial Differential Equations

Numerical methods were first put into use as an effective tool for solving partial differential equations (PDEs) by John von Neumann in the mid-1940s [15]. They have become indispensable tools for the quantitative solution of differential equations that express the behavior of any system in the universe. Numerical methods are applicable to the solution of differential equations that represent mathematical models of underlying real system [16]. There are many numerical methods that have been developed and widely used to study different problems and when one is choosing a particular numerical method to solve a given problem, he or she may consider several factors which include; the ease in applying the method to the problem being solved, the efficiency of the method when compared to other numerical methods and the robustness of the numerical methods [17]. Some of these numerical methods include finite difference method, method of lines, finite element method, gradient discretization method, finite volume method, spectral method, mesh-free methods, domain decomposition methods and multigrid methods. In this work we are going to consider the finite difference method as it is the easiest numerical method to implement for numerical simulation.

We are going to derive the numerical solutions for the one-dimensional Shallow Water Equations (x-direction). Therefore, equations (3.48) become

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\bar{u}) = 0, \\ \frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}(h\bar{u}^2) = -\frac{1}{2}g\frac{\partial h^2}{\partial x}, \end{cases} \quad (3.49)$$

where,

1 $h = h(x,t)$.
2 $u = u(x,t)$.

We now employ Finite Difference Method to solve these two partial differential equations.

Finite Difference Method

The finite difference method is the most direct approach that is used to discretize partial differential equations. One considers a point in space where he or she takes the continuum representation of the equations and replaces it with a set of discrete equations, called finite-difference equations. The finite difference method is defined on a regular grid and for this reason, it can be used for very efficient solution methods.

In other words, the finite-difference method belongs to the so-called grid-point methods.

In the grid-point methods a computational domain is covered by a space-time grid and each function is represented by its values at grid points. The space-time distribution of the grid points may be, in principle, arbitrary, but it significantly affects the accuracy of the approximation. Usually, no assumption is made about the values in-between the grid points. A derivative of a function is approximated by the so-called finite-difference formula which uses values of the function at a specified set of the grid points [18].

We prefer the finite difference method because it easy to increase the "element order" to get higher-order accuracy since it is defined dimension per dimension. Also, its efficient implementations are much easier than for finite-element and finite-volume methods. To find the numerical solutions for our differential equations, we are going to follow the procedure displayed in [19].

Development of the Difference Equations

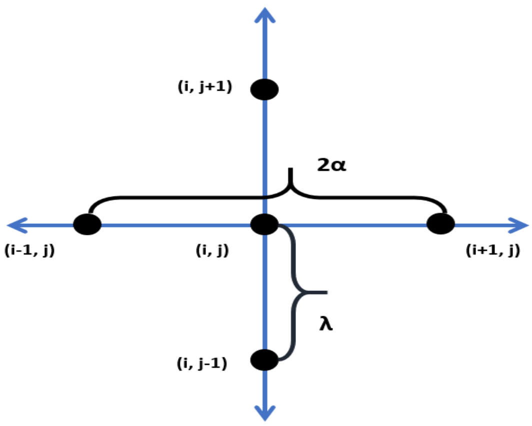

Figure 2: Diagrammatic demonstration of finite difference method

To be able to derive the finite difference equations for equations in 4.1, we assume that we know the values of h(x, t) and u(x, t) at time t_{j-1} (at the point j-1), and find the value at time t_j (at the point j) [19].

Figure 2 gives an illustration of finite difference, where for simplicity we have denoted (X_{i-1},t_j), (X_i,t_j), (X_{i+1},t_j), (X_i,t_{j+1}), and (X_i,t_{j-1}) by $(i-1,j)$, (i,j), $(i+1,j)$, $(i,j+1)$, and $(i,j-1)$ respectively. We start with forward difference for time as follows;

$$\left(\frac{\partial}{\partial t}h(x,t)\right)_{i,j-1} = \frac{h(x_i,t_j) - h(x_i - t_{j-1})}{\lambda}. \quad (4.1)$$

For simplicity, we denote the values with subscripts with the subscript terms. That is,

$$\left(\frac{\partial}{\partial t}h(x,t)\right)_{i,j-1} = \frac{h_{i,j} - h_{i,j-1}}{\lambda}, \quad (4.2)$$

where $\lambda = (j-(j-1))$.

$$\left(\frac{\partial}{\partial t}(\mathrm{h(x,t)\bar{u}(x,t)})\right)_{i,j-1} = \mathrm{h}_{i,j-1}\frac{\bar{u}_{i,j} - \bar{u}_{i,j-1}}{\lambda} + \bar{u}_{i,j-1}\frac{h_{i,j} - h_{i,j-1}}{\lambda} \quad (4.3)$$

In the next step we find the central difference for the space. We proceed as shown below:

$$\left(\frac{\partial}{\partial x}(\mathrm{h(x,t)\bar{u}(x,t)})\right)_{i,j-1} = \mathrm{h}_{i,j-1}\frac{\bar{u}_{i,j} - \bar{u}_{i,j-1}}{2\alpha} + \bar{u}_{i,j-1}\frac{h_{i,j} - h_{i,j-1}}{2\alpha}, \quad (4.4)$$

where 2α = (i+1) − (i−1).

Lastly, we have:

$$\left(\frac{\partial}{\partial x}(\mathrm{h(x,t)\bar{u}^2(x,t)})\right)_{i,j-1} = \mathrm{h}_{i,j-1}\bar{u}_{i,j-1}\frac{\bar{u}_{i+1,j} - \bar{u}_{i-1,j}}{\alpha} + \bar{u}^2_{i,j-1}\frac{h_{i+1,j} - h_{i-1,j}}{2\alpha}, \quad (4.5)$$

and

$$\left(\frac{1}{2}g\frac{\partial}{\partial x}\mathrm{h}^2(x,t)\right)_{i,j-1} = g\mathrm{h}_{i,j-1}\frac{h_{i+1,j} - h_{i-1,j}}{2\alpha}. \quad (4.6)$$

Therefore, the system of equations (3.49) becomes:

$$\frac{h_{i,j}-h_{i,j-1}}{\lambda} + h_{i,j-1}\frac{\bar{u}_{i,j}-\bar{u}_{i,j-1}}{2\alpha} + \bar{u}_{i,j-1}\frac{h_{i,j}-h_{i,j-1}}{2\alpha} = 0, \quad (4.7)$$

and

$$h_{i,j-1}\frac{\bar{u}_{i,j}-\bar{u}_{i,j-1}}{\lambda} + \bar{u}_{i,j-1}\frac{h_{i,j}-h_{i,j-1}}{\lambda} + h_{i,j-1}\bar{u}_{i,j-1}\frac{\bar{u}_{i+1,j}-\bar{u}_{i-1,j}}{\alpha} + \bar{u}^2_{i,j-1}\frac{h_{i+1,j}-h_{i-1,j}}{2\alpha} + gh_{i,j-1}\frac{h_{i+1,j}-h_{i-1,j}}{2\alpha} = 0, \quad (4.8)$$

respectively.

We now perform some algebraic operations on these two equations to simplify them. Multiplying equation (4.7) by $\bar{u}_{i,j-1}$ and subtracting the results from equation (4.8) we obtain

$$\frac{h_{i,j} - h_{i,j-1}}{\lambda} + h_{i,j-1}\frac{\bar{u}_{i+1,j} - \bar{u}_{i-1,j}}{2\alpha} + \bar{u}_{i,j-1}\frac{h_{i+1,j} - h_{i-1,j}}{2\alpha} = 0, \quad (4.9)$$

and

$$h_{i,j-1}\frac{\bar{u}_{i,j} - \bar{u}_{i,j-1}}{\lambda} + \bar{u}_{i,j-1}h_{i,j-1}\frac{\bar{u}_{i+1,j} - \bar{u}_{i-1,j}}{2\alpha} + gh_{i,j-1}\frac{h_{i+1,j} - h_{i-1,j}}{2\alpha} = 0. \quad (4.10)$$

Multiplying through the two equations by 2αλ, we obtain

$$2\alpha h_{i,j} - 2\alpha h_{i,j-1} + \lambda h_{i,j-1}\bar{u}_{i+1,j} - \lambda h_{i,j-1}\bar{u}_{i-1,j} + \lambda\bar{u}_{i,j-1}h_{i+1,j} - \lambda\bar{u}_{i,j-1}h_{i-1,j} = 0, \quad (4.11)$$

and

$$2\alpha h_{i,j-1}\bar{u}_{i,j} - 2\alpha h_{i,j-1}\bar{u}_{i,j-1} + \lambda\bar{u}_{i,j-1}h_{i,j-1}\bar{u}_{i+1,j} - \lambda\bar{u}_{i,j-1}h_{i,j-1}\bar{u}_{i-1,j} + \lambda g h_{i,j-1}h_{i+1,j} - \lambda g h_{i,j-1}h_{i-1,j} = 0. \quad (4.12)$$

Identification of the known and the unknown terms

To be able to solve our linear equations, it is important to know which terms are known and which terms we are searching for (the unknown terms). After that we need to express our system of linear equations in the form $A\vec{X} = \vec{b}$. From our system, the known terms are $\bar{u}_{i,j-1}$ and $h_{i,j-1}$ at $j-1$, whereas the unknown terms are $\bar{u}_{i,j}$ and $h_{i,j}$ at j. Keeping the unknowns to the left and the known terms to the right-hand side we obtain the equations below:

$$2\alpha h_{i,j} + \lambda h_{i,j-1}\bar{u}_{i+1,j} - \lambda h_{i,j-1}\bar{u}_{i-1,j} + \lambda\bar{u}_{i,j-1}h_{i+1,j} - \lambda\bar{u}_{i,j-1}h_{i-1,j} = 2\alpha h_{i,j-1}, \quad (4.13)$$

and

$$2\alpha h_{i,j-1}\bar{u}_{i,j} + \lambda\bar{u}_{i,j-1}h_{i,j-1}\bar{u}_{i+1,j} - \lambda\bar{u}_{i,j-1}h_{i,j-1}\bar{u}_{i-1,j} + \lambda g h_{i,j-1}h_{i+1,j} - \lambda g h_{i,j-1}h_{i-1,j} = 2\alpha h_{i,j-1}\bar{u}_{i,j-1}. \quad (4.14)$$

Derivation of the System

To come up with the system, we choose as an example, N = 4 (number of discretizations). We thus find the initial solution for the discretized linear equations as follows:

For $i = 1$:

$$2\alpha h_{1,j} + \lambda h_{1,j-1}\bar{u}_{2,j} - \lambda h_{1,j-1}\bar{u}_{0,j} + \lambda\bar{u}_{1,j-1}h_{2,j} - \lambda\bar{u}_{1,j-1}h_{0,j} = 2\alpha h_{1,j-1} \quad (4.15)$$

and

$$2\alpha h_{1,j-1}\bar{u}_{1,j} + \lambda\bar{u}_{1,j-1}h_{1,j-1}\bar{u}_{2,j} - \lambda\bar{u}_{1,j-1}h_{1,j-1}\bar{u}_{0,j} + \lambda g h_{1,j-1}h_{2,j} - \lambda g h_{1,j-1}h_{0,j} = 2\alpha h_{1,j-1}\bar{u}_{1,j-1} \quad (4.16)$$

For $i = 2$:

$$2\alpha h_{2,j} + \lambda h_{2,j-1}\bar{u}_{3,j} - \lambda h_{2,j-1}\bar{u}_{1,j} + \lambda\bar{u}_{2,j-1}h_{3,j} - \lambda\bar{u}_{2,j-1}h_{1,j} = 2\alpha h_{2,j-1} \quad (4.17)$$

and

$$2\alpha h_{2,j-1}\bar{u}_{2,j} + \lambda\bar{u}_{2,j-1}h_{2,j-1}\bar{u}_{3,j} - \lambda\bar{u}_{2,j-1}h_{i,j-1}\bar{u}_{1,j} + \lambda g h_{2,j-1}h_{3,j} - \lambda g h_{2,j-1}h_{1,j} = 2\alpha h_{2,j-1}\bar{u}_{2,j-1} \quad (4.18)$$

For $i = 3$:

$$2\alpha h_{3,j} + \lambda h_{3,j-1}\bar{u}_{4,j} - \lambda h_{3,j-1}\bar{u}_{2,j} + \lambda\bar{u}_{3,j-1}h_{4,j} - \lambda\bar{u}_{3,j-1}h_{2,j} = 2\alpha h_{3,j-1} \quad (4.19)$$

and

$$2\alpha h_{3,j-1}\bar{u}_{3,j} + \lambda\bar{u}_{3,j-1}h_{3,j-1}\bar{u}_{4,j} - \lambda\bar{u}_{3,j-1}h_{3,j-1}\bar{u}_{2,j} + \lambda g h_{3,j-1}h_{4,j} - \lambda g h_{3,j-1}h_{2,j} = 2\alpha h_{3,j-1}\bar{u}_{3,j-1} \quad (4.20)$$

Boundary conditions

As boundary conditions, we set the following;

1. Velocity at the boundaries is zero, implying that the average velocity at the boundaries is also zero. That is $\bar{u}(x_a,t)=\bar{u}(x_\varphi,t)=0$.
2. The height at the boundary is set to be one. That is, $h(x_a,t)=h(x_\varphi,t)=1$.

Therefore, in general we have;

$$\begin{cases} u_{0,j}=0 \\ u_{4,j}=0 \end{cases} \qquad (4.21)$$

And

$$\begin{cases} h_{0,j}=1 \\ h_{4,j}=1 \end{cases} \qquad (4.22)$$

The unknowns are thus;
$\bar{u}_{1,j},\bar{u}_{2,j},\bar{u}_{3,j};h_{1,j},h_{2,j},h_{3,j}$.
Applying the boundary conditions, our six equations now become:

For $i=1$:

$$2\alpha h_{1,j}+\lambda h_{1,j-1}\bar{u}_{2,j}+\lambda\bar{u}_{1,j-1}h_{2,j} = 2\alpha h_{1,j-1}+\lambda\bar{u}_{1,j-1} \qquad (4.23)$$

and

$$2\alpha h_{1,j-1}\bar{u}_{1,j}+\lambda\bar{u}_{1,j-1}h_{1,j-1}\bar{u}_{2,j}+\lambda g h_{1,j-1}h_{2,j} = 2\alpha h_{1,j-1}\bar{u}_{1,j-1}+\lambda g h_{1,j-1} \qquad (4.24)$$

For $i=2$:

$$2\alpha h_{2,j}+\lambda h_{2,j-1}\bar{u}_{3,j}-\lambda h_{2,j-1}\bar{u}_{1,j}+\lambda\bar{u}_{2,j-1}h_{3,j} - \lambda\bar{u}_{2,j-1}h_{1,j}=2\alpha h_{2,j-1} \qquad (4.25)$$

and

$$2\alpha h_{2,j-1}\bar{u}_{2,j}+\lambda\bar{u}_{2,j-1}h_{2,j-1}\bar{u}_{3,j}-\lambda\bar{u}_{2,j-1}h_{i,j-1}\bar{u}_{1,j} + \lambda g h_{2,j-1}h_{3,j}-\lambda g h_{2,j-1}h_{1,j} = 2\alpha h_{2,j-1}\bar{u}_{2,j-1} \qquad (4.26)$$

For $i=3$:

$$2\alpha h_{3,j}+\lambda h_{3,j-1}\bar{u}_{4,j}-\lambda h_{3,j-1}\bar{u}_{2,j}-\lambda\bar{u}_{3,j-1}h_{2,j} = 2\alpha h_{3,j-1}-\lambda\bar{u}_{3,j-1} \qquad (4.27)$$

and

$$2\alpha h_{3,j-1}\bar{u}_{3,j}-\lambda\bar{u}_{3,j-1}h_{3,j-1}\bar{u}_{2,j}-\lambda g h_{3,j-1}h_{2,j} = 2\alpha h_{3,j-1}\bar{u}_{3,j-1}-\lambda g h_{3,j-1} \qquad (4.28)$$

Therefore, our system in matrix form becomes:

$$\begin{bmatrix} 0 & \lambda h_{1,j-1} & 0 & 2\alpha & \lambda\bar{u}_{1,j-1} & 0 \\ 2\alpha h_{1,j-1} & \lambda\bar{u}_{1,j-1}h_{1,j-1} & 0 & 0 & \lambda g h_{1,j-1} & 0 \\ -\lambda h_{2,j-1} & 0 & \lambda h_{2,j-1} & -\lambda\bar{u}_{2,j-1} & 2\alpha & \lambda\bar{u}_{2,j-1} \\ -\lambda\bar{u}_{2,j-1}h_{2,j-1} & 2\alpha h_{2,j-1} & \lambda\bar{u}_{2,j-1}h_{2,j-1} & -\lambda g h_{2,j-1} & 0 & \lambda g h_{2,j-1} \\ 0 & -\lambda h_{3,j-1} & 0 & 0 & -\lambda\bar{u}_{3,j-1} & 2\alpha \\ 0 & -\lambda\bar{u}_{3,j-1}h_{3,j-1} & 2\alpha h_{3,j-1} & 0 & -\lambda g h_{3,j-1} & 0 \end{bmatrix} \times \begin{bmatrix} \bar{u}_{1,j} \\ \bar{u}_{2,j} \\ \bar{u}_{3,j} \\ h_{1,j} \\ h_{2,j} \\ h_{3,j} \end{bmatrix} = \begin{bmatrix} 2\alpha h_{1,j-1}+\lambda\bar{u}_{1,j-1} \\ 2\alpha h_{1,j-1}\bar{u}_{1,j-1}+\lambda g h_{1,j-1} \\ 2\alpha h_{2,j-1} \\ 2\alpha h_{2,j-1}\bar{u}_{2,j-1} \\ 2\alpha h_{3,j-1}-\lambda\bar{u}_{3,j-1} \\ 2\alpha h_{3,j-1}\bar{u}_{3,j-1}-\lambda g h_{3,j-1} \end{bmatrix} \qquad (4.29)$$

II. RESULTS AND DISCUSSIONS

In this chapter, we are going to implement our system in Matlab software, discuss the simulations results, and draw conclusion of our study. We intend to solve for the unknown vector $\vec{X}$ in $A\vec{X}=\vec{b}$ where

$$A = \begin{bmatrix} 0 & \lambda h_{1,j-1} & 0 & 2\alpha & \lambda\bar{u}_{1,j-1} & 0 \\ 2\alpha h_{1,j-1} & \lambda\bar{u}_{1,j-1}h_{1,j-1} & 0 & 0 & \lambda g h_{1,j-1} & 0 \\ -\lambda h_{2,j-1} & 0 & \lambda h_{2,j-1} & -\lambda\bar{u}_{2,j-1} & 2\alpha & \lambda\bar{u}_{2,j-1} \\ -\lambda\bar{u}_{2,j-1}h_{2,j-1} & 2\alpha h_{2,j-1} & \lambda\bar{u}_{2,j-1}h_{2,j-1} & -\lambda g h_{2,j-1} & 0 & \lambda g h_{2,j-1} \\ 0 & -\lambda h_{3,j-1} & 0 & 0 & -\lambda\bar{u}_{3,j-1} & 2\alpha \\ 0 & -\lambda\bar{u}_{3,j-1}h_{3,j-1} & 2\alpha h_{3,j-1} & 0 & -\lambda g h_{3,j-1} & 0 \end{bmatrix}$$

$$\vec{X} = \begin{bmatrix} \bar{u}_{1,j} \\ \bar{u}_{2,j} \\ \bar{u}_{3,j} \\ h_{1,j} \\ h_{2,j} \\ h_{3,j} \end{bmatrix}$$

and

$$\vec{b} = \begin{bmatrix} 2\alpha h_{1,j-1}+\lambda\bar{u}_{1,j-1} \\ 2\alpha h_{1,j-1}\bar{u}_{1,j-1}+\lambda g h_{1,j-1} \\ 2\alpha h_{2,j-1} \\ 2\alpha h_{2,j-1}\bar{u}_{2,j-1} \\ 2\alpha h_{3,j-1}-\lambda\bar{u}_{3,j-1} \\ 2\alpha h_{3,j-1}\bar{u}_{3,j-1}-\lambda g h_{3,j-1} \end{bmatrix}$$

Numerical Results

To obtain the results of the simulation, we have to state some boundary conditions which are very key for our implementation.

Initial and Boundary Conditions:

To implement our system, we are going to use the boundary conditions stated below:

1. $h(x,0)=1+\frac{2}{5}e^{-5x^2}$.
2. $-5<x<5$.
3. $g=1$.
4. $h(x_a,t)=h(x_\varphi,t)=1$.
5. $\bar{u}(x_a,t)=\bar{u}(x_\varphi,t)=0$.
6. $X\in[-5,5]$.

For implementation in Matlab, we let the average velocity $\bar{u}=u$ (treating u as a dummy variable). We do plots at different times to study the variation in height and velocity. We do plots of h, u and hu at t = 0, t = 1, t = 2, t = 3, t = 4, t = 8, t = 10, and t = 40. This will help us to study the behavior of waves (any disturbance in the water) as time increases.
We obtain the following results:

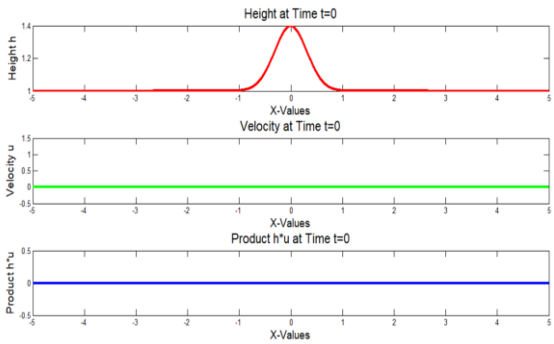

Figure 3: Graphs of h, u and h*u at time t=0

International Research Journal of Advanced Engineering and Science
ISSN (Online): 2455-9024

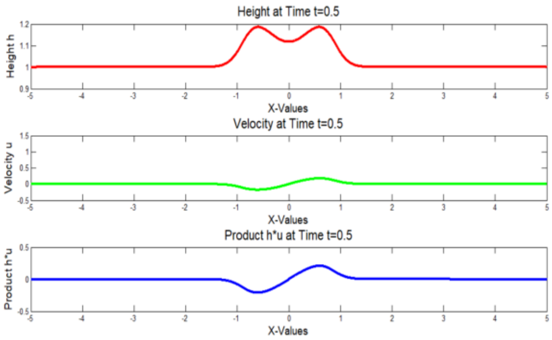

Figure 4: Graphs of h, u and h*u at time t=0.5

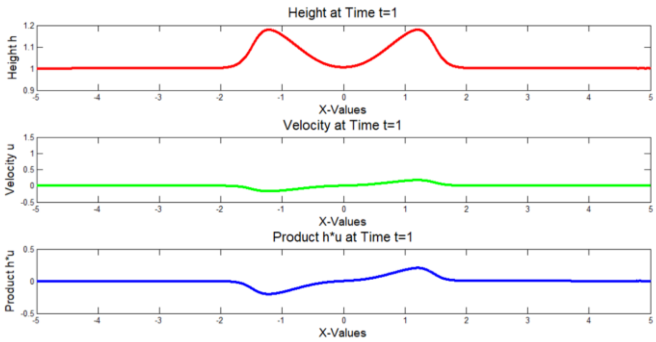

Figure 4: Graphs of h, u and h*u at time t=1

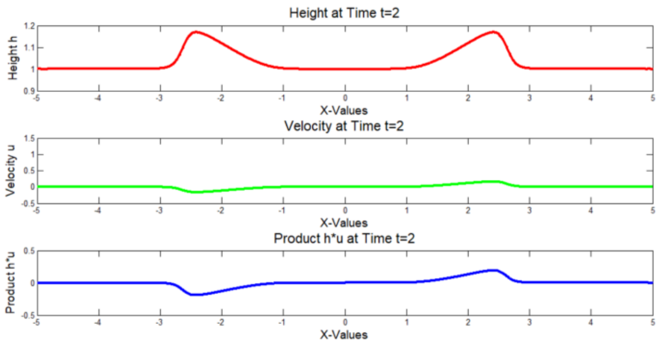

Figure 5: Graphs of h, u and h*u at time t=2

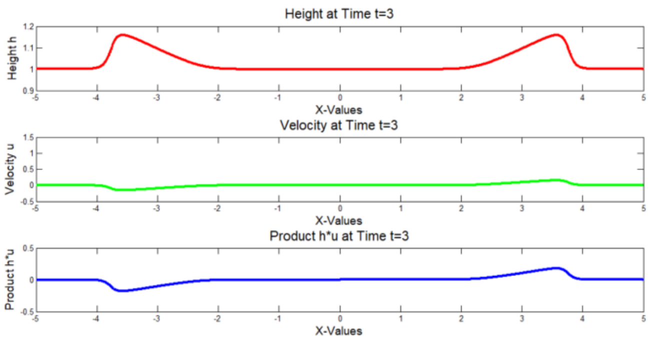

Figure 6: Graphs of h, u and h*u at time t=3

Figure 3 throughout Figure 10 give the diagrammatic simulations of the waves or disturbances happening in a water reservoir. The disturbance in the water causes the waves to occur. The waves migrate or move from the point of disturbance towards the boundaries of the water as can be observed from the waves' simulations at t=0.4, t = 1 up to t =40. Upon reaching the boundaries, the waves retreat back towards the point of initial disturbance depending on the force of the wave and the strength of the walls of the water reservoir.

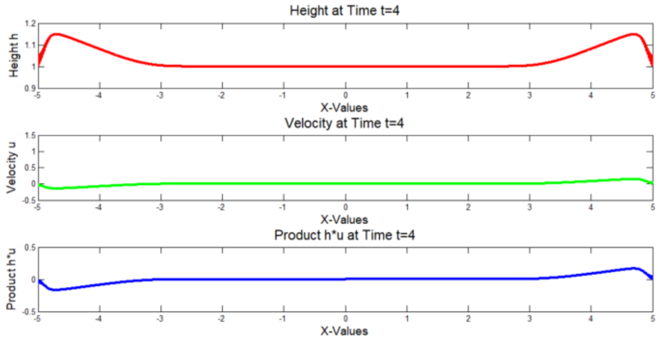

Figure 7: Graphs of h, u and h*u at time t=4

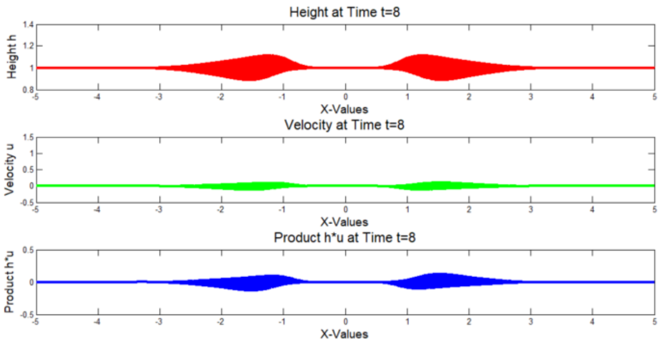

Figure 8: Graphs of h, u and h*u at time t=8

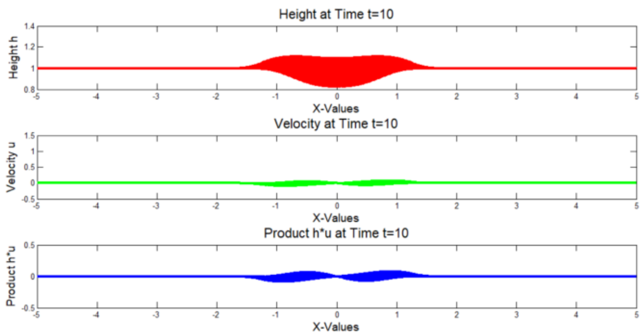

Figure 9: Graphs of h, u and h*u at time t=10

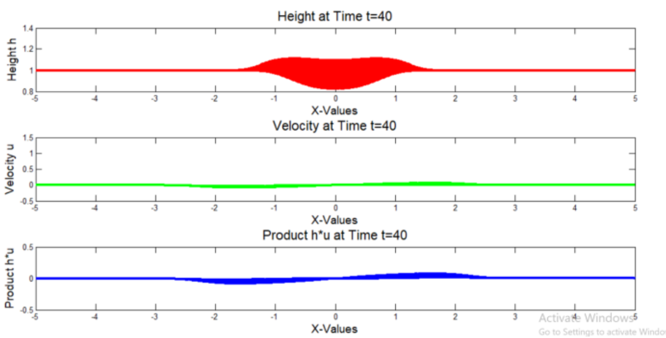

Figure 10: Graphs of h, u and h*u at time t=40

Cosmus Emmanuel Mwangangi and Benard Kipchumba, "Partial Differential Equations for Shallow Water Systems," *International Research Journal of Advanced Engineering and Science*, Volume 8, Issue 1, pp. 151-160, 2023.

The profiles display the motion of waves at different times. It is also evident that the height of the water reservoir contributes much on what is observed at the surface of the water reservoir in case of a disturbance occurring below the surface of the reservoir.

If a disturbance occurs deep at the bottom of the water reservoir, the height of the wave observed at the surface of the water would much depend on the strength of the force causing the disturbance. The stronger the force, the higher the wave observed at the surface of the water and thus the higher the effects of the wave. The profiles displayed by Figure 3 to Figure 10 show the results due to a single disturbance occurring on the water reservoir.

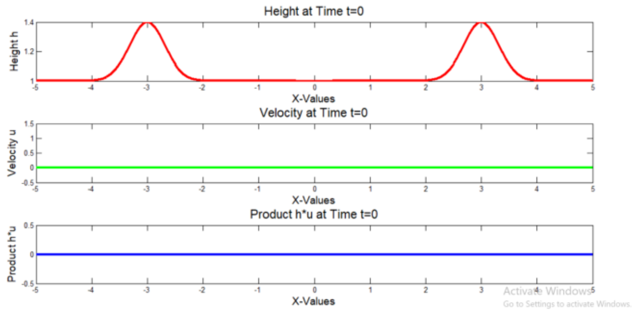

Figure 11. Profiles for double disturbance in the water body when time is t=0

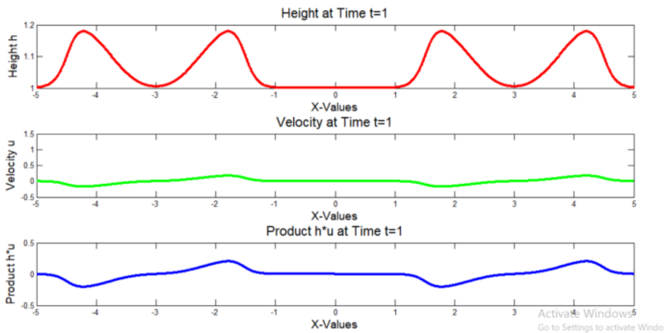

Figure 12. Profiles for double disturbance in the water body when time is t=1

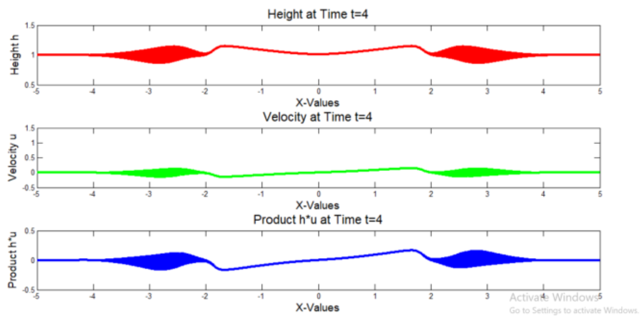

Figure 13. Profiles for double disturbance in the water body when time is t=4

Figure 11 to Figure 15 give the profile display for water waves due to double disturbance occurring in the water reservoir.

Again, the waves start off from the two different points of disturbance occurrence, moving outwards. The waves travel until they hit each other or hit the walls and bounce back. The process repeats itself depending on the strength of the waves until they die off. Thus, the waves occur as a result of disturbance occurring in the water reservoir. The movement of such waves is always outwards from the point of disturbance causing such wave.

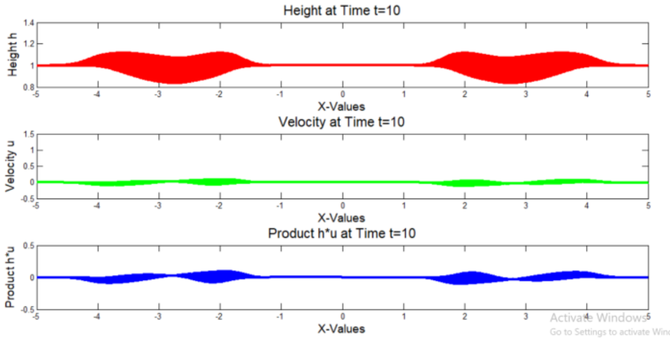

Figure 14. Profiles for double disturbance in the water body when time is t=10

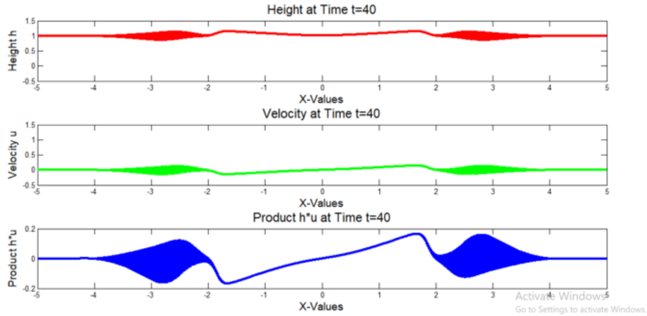

Figure 15. Profiles for double disturbance in the water body when time is t=40

III. Discussion and Conclusion

Finite difference method is an effective numerical method that produces reliable results. The shallow water waves always tend to move from the point of disturbance outwards. Depending on the strength of the forces triggering the waves, the waves may maintain a continuous motion to and from the center of the water body for a longer time. The waves eventually die off if the force triggering the waves dies off. Based on our results, when constructing water reservoirs, it is very important to create strong and high walls to prevent breaking of the reservoirs walls when hit by the waves, hence controlling the disasters that can occur if such breaking occur. The strength of the walls can be established by studying the strength of the waves that are likely to occur at any given time within the waters and also studying the possible highest amount of rains that are likely to occur in the area of construction of the water reservoir.

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