

# Introduction to Analogue Computer Calculations and Applications

Roshen Tariq Ahmedhamdi, Salam Waley Shneen

Energy and Renewable Energies Technology Center, University of Technology, Baghdad, State, Iraq-10066

**Abstract**— In these days, it is rare that we cannot find a house without a computer, and it is also difficult to find a factory or a device and the computer did not interfere in its work or design. There are many types of computers in our world, the analog computer is one of them. Unfortunately few people familiar with this type, therefore, we decided to write this introduction research, which is about analogue computers in terms of their basic components and how to connect and do its calculations and some of its application in order to enable the reader to get an idea of this type of computers the advantage of analogue computers is very high speed, working on real time, deal with continuous signal and more accurate, these specifications of analogue computers will help the designers and manufacturer to make their design and products more efficient with less coast and loss.

**Keywords**— Computer, analogue, analogue setup.

## I. INTRODUCTION

Our basic subject here is electric analog computer, more properly known as electric differential analyzer, but most often referred to simply as the analog computer. When we want to find out what a word means, a good place to start is the dictionary, so let's consider a formal definition of the word. So the definition of Analog is noun as agreement, likeness, or correspondence between the relations of things to one another. For example, the analogy between the heart and pump. A heart and pump are certainly not the same things, a heart is a mass of flesh and a pump is a collection of mechanical parts. So we can define the analog computer as analog computer: is a device in which the problem variables are represented as continuous, varying physical quantities. An analog computer implements a model of the system being studied. The physical form of the analog may be functionally similar to that of the system, figure (1) shows the analog computer.



Fig. 1. Analog computer

## II. COMPARISON BETWEEN ANALOG AND DIGITAL COMPUTERS

It is true that both kinds of computers can generally be used to study the same problems, but there are very significant

differences in the way they solve the problems and the kind of answer they produce.

- One very important advantage of the analog computer is that we can set it up in such a manner that there is a simulation correspondence between the original system and it's analog.
- Another advantage of the analog computer is speed.
- In an analog we have a separate unit for each individual mathematical operation involved in the solution, and these all operate simultaneously, in parallel. In a digital computer, there is only a single computing element, and this one unit must carry out all the operation involved in the solution, in sequence.
- The biggest single advantage of the digital computer over the analog computer is accuracy
- Operates on continuous values while digital computers operate on discrete values.
- Has a quite limited precision.
- Is a perfect match for solving scientific and engineering problems which can be described by differential equation [1].

## III. ANALOG COMPUTER APPLICATIONS

The analog computer can be used as any other computer for calculating and make the basic arithmetic operation, the other application is we have something a device, system or structure that we wish to study or test, but we find it is inconvenient, or dangerous, or expensive to study this thing.

So we try to find an analog, another device, system, or structure, which is sufficiently like the original that by studying it (the analog) we can learn about the original, but which is unlike the original to the extent of being convenient, safe, and inexpensive to study [2].

We can summarize the analog computer applications in

- Calculating.
- Monitoring.
- Simulating.

## IV. MAIN COMPONENTS OF THE ANALOG COMPUTERS

Analogue computers often have a complicated framework, but they have, at their core, a set of components which perform the calculations, which the operator manipulates through the computer's framework. Electrical components might include, Potentiometers, operational amplifier (inverter, summer and integrator) in addition to reference voltage [3].

Now we will discuss each component briefly:

### A. Coefficient potentiometers

Nearly every calculation needs some way to multiply a value by a fixed amount. In an electronic analog computer this is normally done with a potentiometer (variable resistance) connected as a voltage divider. This allows multiplication of an input value with a factor of 1.

The circuit of such a coefficient potentiometer and its graphic representation shown in figure (2) below [4,5]:

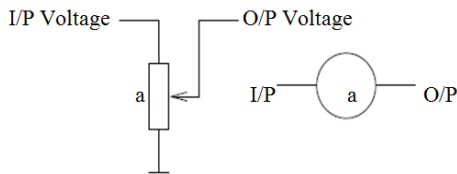


Fig. 2. Potentiometer symbol

**B. Summer**

Operational amplifiers used as summer in analog computers as shown in figure (3) in order to sum (Algebraic summing) any two or more values, the sum of the currents at the summing junction has to be 0 due to the assumed gain  $G = \infty$ , note that a summer always implies a change of sign of the result, the reason for this is electronic not mathematical but this changing does not cause any particular different on the problem result. Note that if the number of inputs is one then the summer will act as inverter [4, 5].

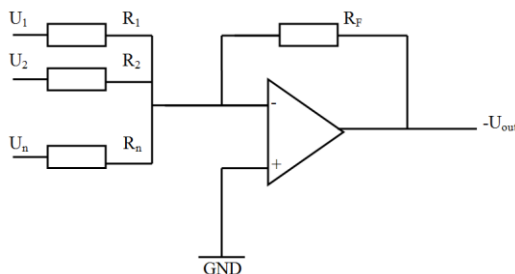


Fig. 3. Operational amplifier as summer

$$\sum_{i=1}^n \frac{U_i}{R_i} = - \frac{U_{out}}{R_F}$$

If  $a_i = \frac{R_F}{R_i}$  then  $-U_{out} = \sum_{i=1}^n a_i U_i$

**B.1 Summing example**

To compute the sum  $-(7 + 2)$  one would setup the analog computer as shown in figure (4), the triangle symbol denotes a summer while the two circles denote two coefficient potentiometers [5].

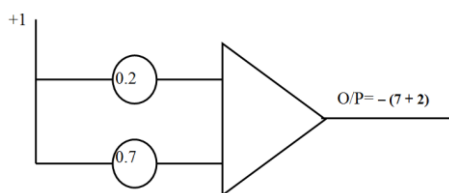


Fig. 4. Example of summing two values

**C. Integrator**

Integration is just as easy as summing, the main difference between a summer and integrator is that the latter uses a

capacitor as its feedback impedance as shown in figure (5) the output voltage of the integrator is.

$$-U_{out}(t) = U_o + \int_0^t \sum_{i=1}^n \frac{1}{R_i C} U_i dt$$

Where  $U_o$  is the so called initial condition, just like a simple summer an integrator changes the sign of its output [4, 5].

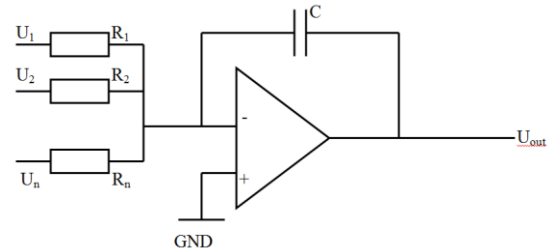


Fig. 5. Operational amplifier as integrator

**D. Reference voltage**

Which is either +1 or -1 machine unite, one machine unite is equal to the 10, 100 volt depend on the computer manufacturer. Finally the following symbols are used in analog computing to denote the various computer elements used in solving as shown in figure (6).

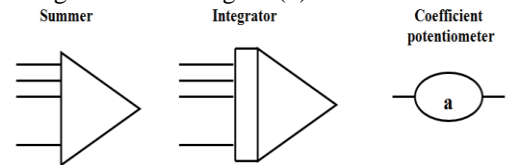


Fig. 6. Symbols used in analog computer setup

**V. TYPES OF DIFFERENTIAL EQUATIONS**

Now that we are generally familiar with the components of the analog computer, we can get down to actually solving some differential equations. In practical situations, one of the biggest jobs is determine what the equations of the system are (transfer function) and getting them into a form suitable for the analog computer. We have indicated that the basic purpose of analog computers is the solution of differential equations, but there are many types of differential equations. Just as the method of mathematical solution of differential equations depends on the particular type of equation. Figure (7) shows the types of differential equations [1].

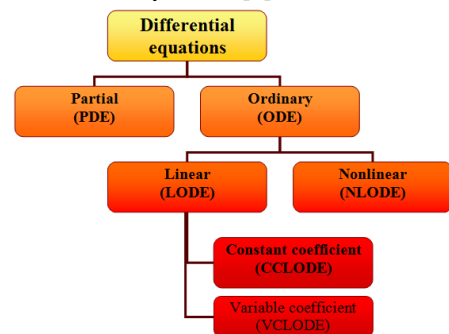


Fig. 7. Classification of Differential equations

Ordinary differential equation (ODE) has only one

independent variable. They may have any number of dependent variable, but all derivatives are with respect to the same independent variable. Partial differential equations (PDE) have more than one independent variable. ODE may be divided into linear (LODE) and nonlinear (NLODE). In LODE, the dependent variables and their derivatives never appear to powers higher than 1, or multiplied by one another. Finally the LODE may be divided into two classes, depending on whether the coefficient of the dependent variable and derivatives are constants (CCLODE) or functions of the independent variable (VCLODE). There are many other possible classifications of differential equations, but that given here is most significant in the sense that the various classifications require different methods of approach on the analog computer [6, 7].

VI. SOLVEING DIFFERENTIAL EQUATION AND DRAWING THE ANALOG SETUP

First of all we must know the differential equation which represents the system under study, after that one must differentiate till we get one of the following cases.

- Differentiate till one get a constant, for example  $f(x) = 2t^2$ .
- Differentiate till one get the higher derivative in term to lower derivative, for example  $f(x) = A \sin(wt)$ .

In the first case:

$$f(x) = 2t^2$$

$$\frac{d(x)}{d(t)} = 4t$$

Make another differentiation because 4t is not a constant so one get:

$$\frac{d^2x}{dt^2} = 4$$

Since number 4 is a constant so stop differentiation, now analog set-up must draw. Since there is differentiation so integrators must be used in other word since the equation contains time parameter so integrators must be used.

Number of integrations depends on number of differentiation done, in our case two differentiations done so two integrators required. Figure (8) shows the simple unscaled analog set-up. Unscaled means that no amplitude and time scaling has been done; these two calculations discussed later [8].

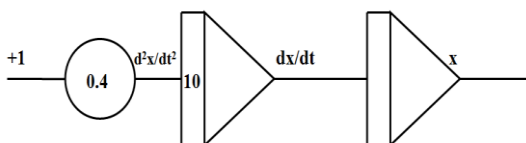


Fig. 8. Complete unscaled analog set-up for the first case

In the same way the unscaled analog set-up for the second case [f(x) = A sin(wt)] obtained as follows:

$$X = A \sin(wt) \tag{1}$$

$$\frac{dX}{dt} = Aw \cos(wt) \tag{2}$$

Since this is not constant or no relation between higher and lower differentiation, continue differentiate.

$$\frac{d^2X}{dt^2} = -Aw^2 \sin(wt) \tag{3}$$

So there is a relation between equation 1 and 3 as shown in equation 4.

$$\frac{d^2X}{dt^2} = -w^2 X \tag{4}$$

Therefore stop calculation because the second case is achieved. And now we draw the analog set-up. The complete unscaled analog set-up is shown in figure 9, the input sign to the initial condition is depend on the equation itself and one must note that the integrator is invert the sign of the inputs just like the summer.

Initial condition can be calculated by substitute time by zero so for X variable is

$$X(t) = A \sin(wt)$$

$$X(0) = A \sin(0) = \text{Zero}$$

There for integrator number 1 initial condition is zero

While for integrator 2 the initial condition can be calculated as follow:

$$dX/dt = AW \cos(wt)$$

$$dX(0)/dt = AW \cos(0) = AW$$

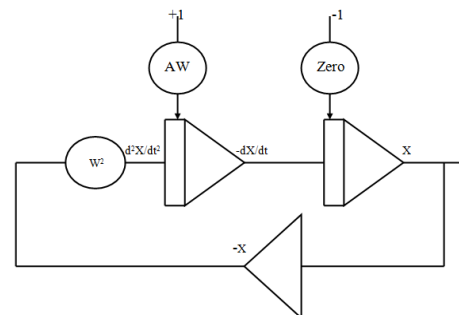


Fig. 9. Complete unscaled analog set-up for the second case

VII. ANALOG COMPUTER CALCULATIONS PROCEDURE

A. Drawing analog setup

To solve the equation below which is a second order homogeneous differential equation which is represent mass, spring damper equation:

$$m\ddot{x} + D\dot{x} + Kx = 0 \tag{5}$$

- Where m=Mass
- D=Damping coefficient
- K=Spring constant
- X=Displacement

First of all we put the higher derivative at left hand side and the other on the right side we get:

$$m\ddot{x} = -D\dot{x} - Kx$$

Divide by (m) :

$$\ddot{x} = \frac{D}{m} \dot{x} + \frac{k}{m} x \quad (6)$$

Figure 10 shows the unscaled analog setup

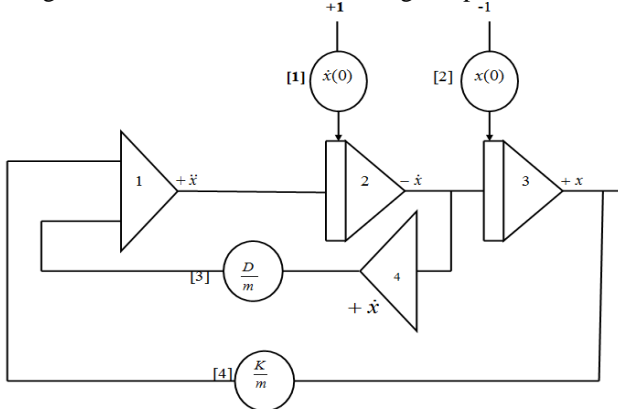


Fig. 10. Unscaled analog setup for equation (5)

### B. Amplitude scaling

Amplitude scaling relates the values of the voltage in the computer to the corresponding values of the problem. Since the analog computer is limited in its operation range depend on the manufacturer, some of them is  $\pm 10$  or  $\pm 100$  volt.

The variables value may be out of range of the computer so amplitude scaling calculations is needed. In order to make this calculation, one must know the maximum value of the variables of the differential equation, the maximum values of the variables of the equation can be estimated in one of the following ways [8]:

- Suggested by the designer [Design of water tank, pendulum movement].
- Given in the problem.
- Known for example [ Cos, Sin ]

After knowing the maximum value we divide and multiply each variable by its maximum variable so equation (6) become.

$$\frac{\ddot{x}_{max}}{\dot{x}_{max}} \dot{x} = \frac{\dot{x}_{max} D}{\dot{x}_{max} m} \dot{x} + \frac{x_{max} K}{x_{max} m} x \quad (7)$$

Divide equation 7 by  $\frac{\ddot{x}_{max}}{\dot{x}_{max}}$

$$\left( \frac{\ddot{x}}{\ddot{x}_{max}} \right) = \frac{\dot{x}_{max} D}{\dot{x}_{max} m} \left( \frac{\dot{x}}{\dot{x}_{max}} \right) + \frac{x_{max} K}{\dot{x}_{max} m} \left( \frac{x}{x_{max}} \right) \quad (8)$$

The variables  $D$ ,  $m$ ,  $K$ ,  $\ddot{x}_{max}$ ,  $\dot{x}_{max}$  and  $x_{max}$  are all constants so they can be represented as potentiometers coefficient. The final step in amplitude scaling is, consider the integrators only, the input quantity must equal to the output as explained in the following steps.

For example assume that

$$\ddot{x}_{max} = 5, \dot{x}_{max} = 2, x_{max} = 10$$

So for integrator number 2 (of figure 10):

$$\text{The input is } \left( \frac{\ddot{x}}{\ddot{x}_{max}} \right) = \left( \frac{\ddot{x}}{5} \right) = 0.2 \ddot{x}$$

$$\text{The output is } \left( \frac{\dot{x}}{\dot{x}_{max}} \right) = \left( \frac{\dot{x}}{2} \right) = 0.5 \dot{x}$$

The input of the integrator is 0.2 while the output is 0.5, so (0.2) must multiply by 2.5 in order to get equal values between input and output.

And for integrator number 3 (of figure 10), by the same way the input of the integrator 3 is 0.5 while the output is 0.1, so (0.5) must multiply by 0.2 in order to get equal values between input and output (balance in magnitude) [8].

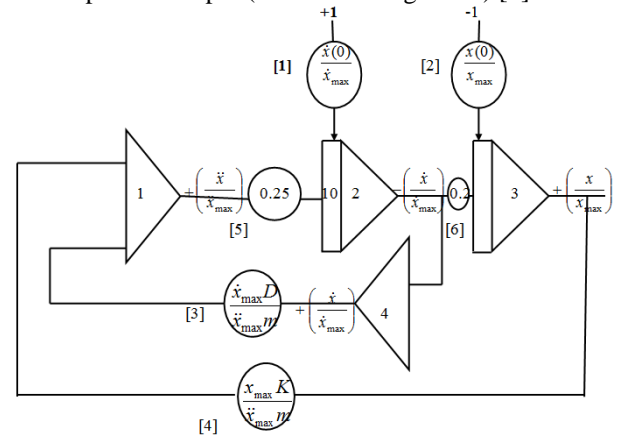


Fig. 11. The complete amplitude scaled analog setup for equation 5.

Note that the integrator number 2 input gain is 10 because the input value must multiplied by 2.5 and since the potentiometer constant is less than 1 so we multiplied by 0.25 and the gain is 10 so we get 2.5.

In the same way for integrator number 3 the input multiplied by 0.2, figure (11) shows the analog setup after finishing the amplitude scaling calculations [8].

### C. Time scaling

The time scaling relates time in the computer solution to real time of the system under study, time scaling calculations needed cause some systems is very slow like rotation of earth around the sun which it will take 1 year on other hand some system is very fast so time scale calculations is very important to change the operation time of the system to a suitable time so one can study it more accurately.

The equation relates this is:

$$\tau = \beta t$$

Where  $\tau$  = computer time

$\beta$  = time scale factor

$t$  = problem time (original or real time)

So if we choose  $\beta > 1$  the equation time will be slower and if  $\beta < 1$  the equation time will be faster. First of all, consider the integrators only, because the operation of integrators depends on time, it integrate the input with respect to time. In order to change the equation of the problem from  $t$  to  $\tau$  we must divide

the equation by  $\beta$ . The value of  $\beta$  is chosen as discussed earlier. So figure (11) will be as shown in figure (12)

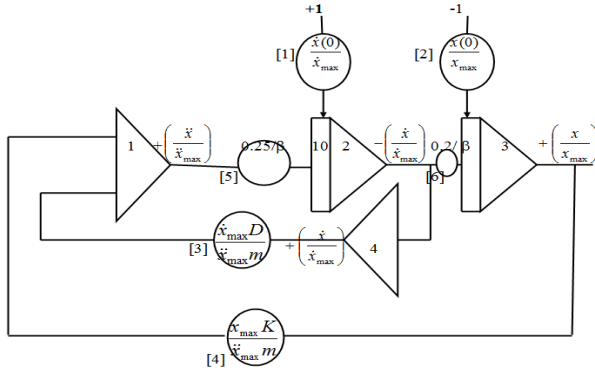


Fig. 12. The complete scaled analog setup for equation 5.

VIII. STATIC CHECK

Finally, static check calculations done in order to get the following:

- Check if the circuit diagram connection is correct.
- Check if the components work properly.

Static check is done by assuming the following conditions:

- Assume time (t) = 0 (time equal to zero).
- All inputs to the integrators are open theoretically.
- Any potentiometer coefficient equal to zero must be changed to any value other than zero static check calculation only, and after we complete the static check calculations we must return the potentiometer coefficient to its original value (because one cannot differ between the potentiometers that work or not, because if the coefficient of the potentiometer is zero the output is zero and if the potentiometer is damage the output is zero too so one cannot differ between them).

This calculation is done by filling two one for potentiometers and the second for amplifier and after filling these sheets we compare the theoretical to the practical results, and if it is same or same with  $\pm 5\%$  so the circuit is correct otherwise the circuit is wrong or some component is not work properly. These two assignment sheets (Tables) of the static check calculations are shown below and it is filled according to example of figure 12 [9, 11].

TABLE 1. Potentiometers assignment sheet

Pot. number	Variable	Pot. Set	Pot. O/P voltage	
			Theoretical	Practical
1	$\frac{\ddot{x}(0)}{x_{max}}$			
2	$\frac{\dot{x}(0)}{x_{max}}$			
3	$\frac{\dot{x}_{max} D}{\dot{x}_{max} m}$			
4	$\frac{x_{max} K}{\dot{x}_{max} m}$			
5	$\frac{\dot{x}_{max}}{10\beta x_{max}}$			
6	$\frac{\dot{x}_{max}}{\beta x_{max}}$			

TABLE 2. Amplifiers assignment sheet

Amplifier number	Function	O/P variable	Amp. O/P voltage	
			Theoretical	Practical
1	$\Sigma$	$\left(\frac{\ddot{x}}{x_{max}}\right)$		
2	$\int$	$\left(-\frac{\dot{x}}{x_{max}}\right)$		
3	$\int$	$\left(\frac{x}{x_{max}}\right)$		
4	INV.	$\left(\frac{\dot{x}}{x_{max}}\right)$		

IX. EXPERIMENTAL RESULTS

For the second order homogeneous equation represented by equation (5), the step response is shown in figure (13,14,15,16). The response is obtained by using storage oscilloscope; the response was taken for different values of spring (K) and damper (D) coefficient.

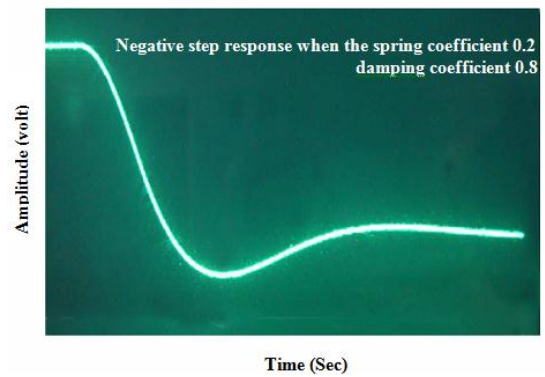


Fig. 13. Relation between time and magnitude for test 1

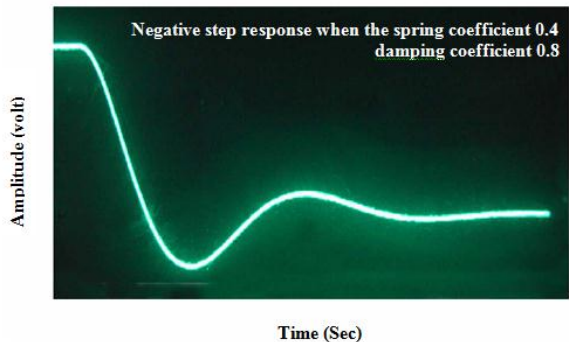


Fig. 14. Relation between time and magnitude for test 2

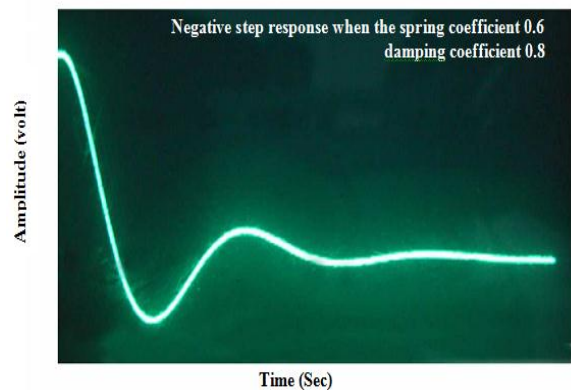


Fig. 15. Relation between time and magnitude for test 3

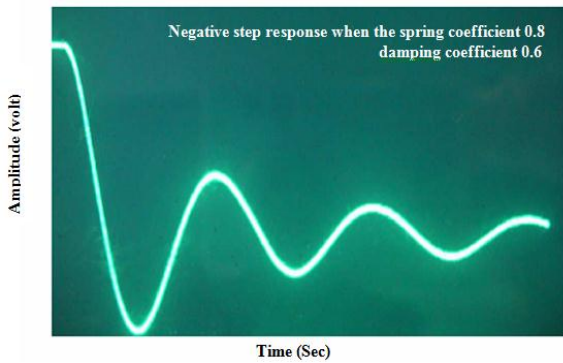


Fig. 16. Relation between time and magnitude for test 4

These results are the outputs of an analogue computer after make a simulation of car damping system (equation 5), and by changing the damping and spring coefficients which can be made by changing the potentiometer constants of the analogue computer, the results show the dynamic characteristics of the negative step response such the steady state error value, response time, oscillation done by spring and the damping force of the damper, increasing the spring coefficient will increase the oscillation and made the response faster but with high overshoot while increase the damping coefficient will cause the response be more damped and slower, these simulations help the manufacturer for design and make it easier to produce with less cost and less loss.

#### X. CONCLUSION

Analog computers are one of the most simple and famous computer that has been used, variable types of differential equations can be solved by using analogue computers.

In order to solve any equation there are few steps that must be done in order to get the answer, these steps can be summarized into the following points:

- Get the mathematical relation of the problem (transfer function).
- Solve the differential equation.
- Draw the analog set up.
- Amplitude scaling calculations.
- Time scaling calculations.
- Static check calculations.

#### REFERENCES

- [1] Charles P. Care, "From analogy-making to modeling: the history of analog computing as a modeling technology", University of Warwick, September 2008.
- [2] Eterman, I. I., "Analogue Computers" New York: Pergamon press, 2005.
- [3] Robert M. Howe, "Fundamentals of the Analog Computers, circuits, technology and simulation", IEEE Control Systems Magazine, 0272-1708/05/IEEE, June 2005.
- [4] Korn, Granino A. Korn, Theresa M., "Electronic Analog Computers : ( D-C Analog Computer", 2nd edition, New York: McGraw-Hill Book Company, Inc., 2005.
- [5] www.play-hookey.com, all about operational amplifiers and experiments 2009.
- [6] Gabriel Nagy, "Ordinary Differential Equations", Michigan State University, 2021.
- [7] Roshen Tariq Ahmedhamdi, Mahdi A. Abdul hussein, "Using Matlab-Simulink for Solving Differential Equations", Journal of Scientific and Engineering Research, Vol. 5, ISSUE 5, ISSN: 2394-2630, PP: 307-314, CODEN (USA): JSERBR, June 2018.
- [8] Dr. Bernd Ulmann, "Analog Computer Applications", Rössler-attractor, 2016
- [9] Weyrick, Robert C. "Fundamentals of Analog Computers". New Jersey: Prentice-Hall, Inc., 2006.
- [10] Fouad Sh. Tahir, Ayad M. Al Jubori, Waleed Ibrahim, Roshen Tariq Ahmed Hamdi, "Design and implement 8085 microprocessor bag system for engineering application"/ Al-Mustansiryia University, journal of basic education Baghdad 2011.
- [11] Abdul Kareem K. A. Raheem, Salam Waley Shneen, Manal Hadi Jaber, Ahmed H. Reja, Design and Simulation of a Second-Order Universal Switched- Capacitor Filter as a 10-Pin Dual-In-Line Package Integrated Circuit, Engineering and Technology Journal, Vol. 30, Issue 18, PP:3175-3191, Publishe University of Technology, 2012.