

# Application of Queue Model in Health Care Sector

Adegoke. O. Folake<sup>1</sup>, Dr. Monica N. Agu<sup>2</sup>, Uchenna Franklin Okebanama<sup>3</sup>

<sup>1</sup>Mathematical Sciences Department, Kogi State University, Anyigba, Kogi State, Nigeria-23401

<sup>2,3</sup>University of Nigeria, Nsukka, Enugu State, Nigeria-23401

Email address: bmuriana685 @ gmail.com

**Abstract**— This paper analyzes the use of queuing model in healthcare with an emphasis on Accident and Emergency Department (AED) of a city hospital. It is widely recognized that, due to high demand and limited resources in these hospitals, a hospital should treat its patients, especially those in need of critical care, in a timely manner but surprisingly this is not achieved in practice, particularly in government-owned health institutions, because of high demand and limited resources in these hospitals. To enhance the level of admittance to care in AED, optimal beds usage is needed and this can be achieved by adequate knowledge of patients flow. This paper establishes the use of queuing models in the healthcare for the flow of in-patient in the hospital, determines the optimal bed count and its performance measure.

**Keywords**— Queuing model, healthcare, optimal beds, in-patient.

## I. INTRODUCTION

Healthcare system is generally hampered with delays from patient having to wait for hours or days before seeing a doctor to patients waiting for bed in hallways. Delays are the outcome of variation between a service's demand and the accessible resources to meet the demand.

If the waiting time and service time are high customers will exit the queue early and this in effect results in frustration and customers' dissatisfaction.

Queues are form when individuals requesting service, typically called customers, reach a service facility and cannot be served promptly. In healthcare delivery systems, patients are typically the customers and either outpatient clinics or hospitals are the service facilities. A common feature of the vast majority of queuing models is that customers are discrete, and the number of customers waiting in the service facilities is integer valued.

Institutions that care for sick and ill people vary enormously in scope and size, ranging from small outpatient clinics to large, city hospitals. In spite of these differences, the healthcare procedures delivered by these institutions can be viewed as queuing systems during which patients arrive, await service, obtain service and then depart.

The healthcare procedures also vary in complexity and scope, but all of them consist of a set of activities and procedures (both medical and para-medical) that the patient must undergo so as to receive the necessary treatment.

## II. RELATED WORKS

Waiting time can be described as a proper assessment of the standard of service provided against the expectations of the customer. Many organizations routinely use queuing models to help determine capacity levels needed to respond to

experienced demands in a timely fashion. Long waiting time in any hospital is considered as an indicator of poor quality (Biju, M. K.; Naeema, K. and Faisal, U. 2001). With rapid change of healthcare system, the use of queuing model has become a prevalent analytical tool (Singh, V. 2007). The queue discipline refers to the order in which members of the queue are selected for service (Hillier F., Lieberman G. 2001). First come, first served (FCFS or FIFO) is the standard queue practice, where customers are served in the order arrival. Although, sometimes there are other service disciplines: last come, first served (which happens sometime in case of emergencies), or service-in-random order and priority rule. (Davies, R. and Davies, H. T. 2005) asserts that reservations first, emergencies first, highest profit customer first, largest orders first, best customers first, longest waiting time in line, and soonest promised date are other examples of queue discipline. (Adeleke. R. A, Ogunwale O. D, Halid O. Y. 2009) considered application of queuing theory to the waiting time of out-patients in a hospital. The average number of patients and the time each patient awaits for service in the hospital were determined. (Weiss E.N, J.O Mcclain. 1985) used the M/G/∞ system to model the queue of patients needing alternative levels of care in an acute care facility whose treatment is completed and are waiting to be transferred to an extended care facility.

## III. THE M/M/S MODEL

The most commonly used queuing model is the M/M/s or Erlang delay model. This model uses a single queue with unlimited waiting room feeding into s identical servers. The customers arrive at a steady rate according to a Poisson method and the service length is exponentially distributed. (These two assumptions are often called Markovian, hence the use of the two in the notation used for the model). One advantage of using the M/M/s model is that it requires only three parameters and so it can be used to obtain performance estimates with very little data. Given an average arrival rate λ, average service duration μ, and the number of servers s, an easy-to-compute formulae are available to obtain performance measures such as the probability that an arrival will experience a positive delay, P<sub>D</sub> or average waiting time of customers in the queue W<sub>q</sub>.

$$P_D = 1 - \sum_{n=0}^{s-1} P_n \tag{1}$$

$$W_q = P_D / (1 - \rho s \mu) \tag{2}$$

$$L_q = \frac{\rho^2}{1 - \rho} \tag{3}$$

$$W_s = W_q + \frac{1}{\mu} \tag{4}$$

$$L_s = \lambda W_s \tag{5}$$

$$\rho = \lambda / su \tag{6}$$

$$p_n = \lambda^n / n! \mu^n P_0 \quad 1 \leq n \leq S \tag{7}$$

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{(\rho s)^n}{n!} + \frac{\rho^s s^{s+1}}{s!(s-\rho s)} \right]^{-1} \quad \rho < 1 \tag{8}$$

$\lambda$  = arrival rate

$s$  = number of servers

$\mu$  = service rate per server

$\rho$  = utilization

Wq = Average wait time in queue

Ws = Average time spent in the system

Lq = Average number of patients in queue

Ls = Average number of patients in the system

$P_0$  = Probability of all servers being idle

$P_n$  = Probability of n patients in the system at any time.

#### IV. QUEUING PROBLEM DESCRIPTION

A queuing system can be described as patients arriving for service, waiting for service if it is not immediate, utilizing the service, and leaving the system after being served. It is characterized by the;

1. Patient Arrival Pattern: In queuing system, it is necessary to calculate the probability pattern of inter-arrival time and what is patients' reaction when they arrive.
2. Patient Service Distribution: Pattern of service time is also important to understand. The pattern will depend on how many patients are on server. Patient arrival patterns and service patterns are considered to be independent
3. Number of Servers: This is the number of servers available for concurrent use by patients. Servers are supplied from a single line or queue
4. System Capacity: The capacity of the system refers to the system's physical limitation, such as a waiting room.
5. Queue Discipline: The discipline explains how patients are treated after they form a queue.

#### V. NUMERICAL SOLUTION AND PROCESS FOR DATA SHEET

We consider a city hospital in the North-Central Nigeria and studied the admissions through its Accident and Emergency department. The selected queuing model suggests that daily admission rates follow a Poisson distribution and that the service times are distributed exponentially, and since AED is generally run independently of other services, its capacity needs can be assessed regardless of other parts of the hospital. We can take the current operating characteristics of a given AED ward as: arrival rate, service rate, and number of servers (number of beds). For the best results, estimates of the arrival rate, service rate and current number of servers are obtained in order to determine the probability that an arriving patient will not find a bed available. A city hospital AED has an average arrival rate  $\lambda$  of 4.5 patients per day at an average length of stay of 2.5 days and the server  $s$ , is 15. Then the M/M/s formula for probability of delay produces an estimate of approximately 23%. To use the M/M/s prescriptively to find the minimum number of servers (beds) needed to attain a target probability of delay, we can use equation 2 and produce a table of results for a broad range of beds' capacities to find the one that best meets the desired target.

Table 1: Probability of Delay and utilization for AED unit.

S	$P_D$ (%)	$\rho$
13	50	0.87
14	36	0.80
15	23	0.75
16	13	0.7
17	7	0.66
18	5	0.63
19	2	0.59
20	1	0.56

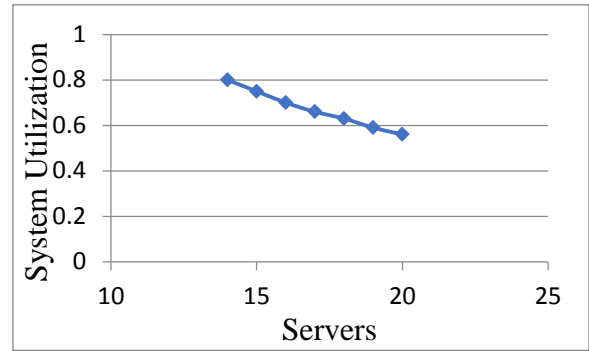


Figure 1: Utilization factor ( $\rho$ ) against server (s)

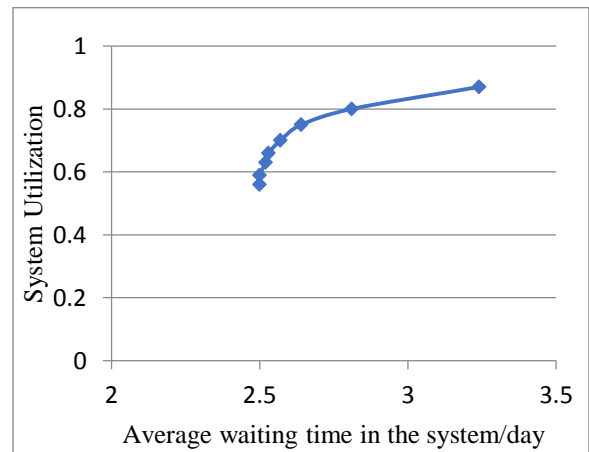


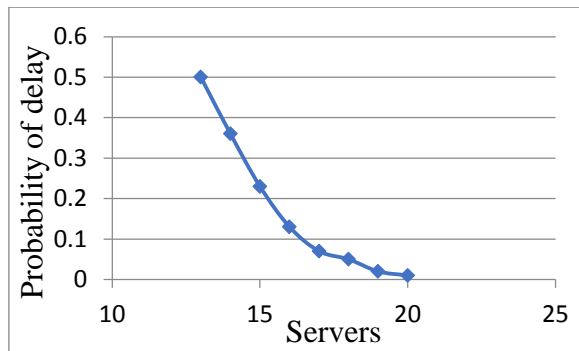
Figure 2: Utilisation Factor ( $\rho$ ) against Average number of patients waiting time in the System ( $W_s$ )

#### VI. PERFORMANCE MEASURE

Though there is no standard delay target, Schneider (1981) suggested that given their emergent status, the probability of delay for an AED bed should not exceed 1%. Applying this criterion, Table 2 indicates that this department should have at least 20 beds. Table 2 also shows the utilization level for each choice of servers (beds) and that at 20 beds, this level is 56%. This is what hospitals call the average occupancy level and it is well below the 85% level that many hospitals and healthcare policy officials consider the minimum target level (Springer International Series, 2006). It is also below the maximum level of 75% recommended by the integrated healthcare association to assure timely access to a bed (Freeman and Poland 1992). So does this example show that as long as an AED unit operates below occupancy level of 75%, the fraction of patients that will be delayed in getting a bed will be very low.

Table 2: Performance Measure of M/M/s

S	$\rho$	$P_D$	$W_s$	$W_q$	$L_q$	$L_s$
13	0.87	0.5	3.24	0.74	3.35	14.6
14	0.80	0.36	2.81	0.32	1.40	12.7
15	0.75	0.23	2.64	0.15	0.65	11.9
16	0.7	0.13	2.57	0.07	0.32	11.6
17	0.66	0.07	2.53	0.03	0.15	11.4
18	0.63	0.05	2.52	0.02	0.07	11.3
19	0.59	0.02	2.50	0.007	0.04	11.3
20	0.56	0.01	2.50	0.003	0.02	11.3



Probability of delay ( $P_D$ ) against Servers (s)

### VII. SUMMARY

This research has attempted to minimize waiting times and offered ample information to hospital interested in using queuing theory to enhance the quality of healthcare services. In the proposed model, we concentrate on queuing system management, waiting time for patients, and social cognitive

approach in AED of a city hospital healthcare system. We determined the probability of delay, the optimal bed count and the performance measure.

### REFERENCES

- [1] Adeleke. R. A, Ogunwale O. D, Halid O. Y (2009) Application of Queuing Theory to Waiting Time of Out-patients in Hospitals. Pacific Journal of Science and Technology Vol. 10(2) 270-274.
- [2] Biju, M. K.; Naeema, K. and Faisal, U. "Application of queuing theory in human resource management in healthcare" pg 1019-1027 June, 2011.
- [3] Brigandi, A.J., Dargon, D.R., Sheehan, M.J. and Spencer III, T., 1994, AT&T's call processing simulator (CAPS) operational design for inbound call centers, *Interfaces* 24: 6-28
- [4] Brusco, M.J., Jacobs, L.W., Bongiorno, R.J., Lyons, D.V. and Tang, B., 1995, Improving personnel scheduling at airline stations. *Operations Research*, 43: 741-751.
- [5] Chelst, K. and Barlach, Z., 1981, Multiple unit dispatches in emergency services, *management Science*, 27: 1390-1409.
- [6] Davies, R., & Davies, H. T. (2005). *Modelling Patient Flows and Resource Provisions in Health Systems*. Omega - Oxford Pergamon Press, 22, 123.
- [7] Hall, R.W., 1990, *Queueing Methods for Service and Manufacturing*. New Jersey: Prentice Hall.
- [8] Hillier F., Lieberman G. *Introduction to Operations Research*. 7th ed. New York: McGraw Hill; 2001.
- [9] Singh, V. "Use of Queuing Models in Health Care", Department of Health Policy and Management, University of Arkansas for medical science, 2007.
- [10] Springer International series Reducing delay in healthcare delivery. University of Southern California. Los Angeles, CA, USA, 2006.
- [11] Weiss E.N, J.O McClain (1986) Administrative days in acute care facilities: a queuing analytic approach *operations Research* Vol. 35 No 1 pp 35-44.