International Research Journal of Advanced Engineering and Science
ISSN (Online): 2455-9024

Analysis of Nonlinear Third Order Phase – Linked Loop

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Abstract— *In the article analysis of nonlinear third order phase linked loop with second order butter worth lowpass filter has been presented .The analysis concerned singular points of local stability of the loop. The investigation of stability has been done using first Lyapunov method and presented results were illustrated by an example.*

Keywords— *nonliner third orderphase, the Lyapunov method, Phase-locked loop model.*

I. INTRODUCTION

In this paper the stability analysis of nonlinear third order analog phase-locked loop is presented. The stability of the singular points of considered system has been examined. The stability analysis of the loop has been done using the Lyapunov method. Similar analysis of global stability of the loop, but with other filter type, was discussed in the [1]. The paper presented continuation of [6] and [7].

II. MODEL AND ANALYSIS

Phase-locked loop model discussed in the article has been shown in fig. 1. The loop consists of the following blocks: the phase detector (PD), the lowpass filter (LF) and the voltage control oscillator (VCO) in the feedback loop. The loop is a nonlinear circuit; however in case when a multiplier is used as a phase detector, the system nonlinear is sine wave type [4]. The nonlinear model of the loop is presented in Fig. 1**.**

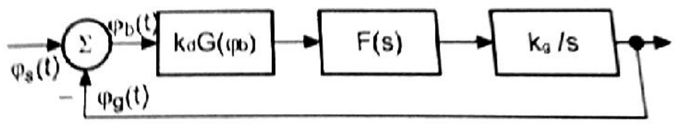

Fig. 1. the PLL Loop

On the assumption, that the input and output signals of the PLL loop is defined as (1):

$$u_s(t) = U_g \sin(w_t t), \qquad (1)$$

$$u_g(t) = U_g \cos(\varphi_g(t)),$$

Where:

$U_{gs} - U_g$ – Input and output signal amplitudes,

w_g – Input signal frequency,

$\varphi_g(t)$ – Instantaneous phase of output signal,

And, that the loop filter (Fig. 1) with cutoff frequency w_g is described by the following Laplace transmittance:

$$F(s) = \frac{U_f(s)}{U_d(s)} = \frac{K}{\left(\frac{s}{w_g}\right)^2 + 1.4142\frac{s}{w_g} + 1} \qquad (2)$$

Where:

K – Gain of the lowpass filter,

It is possible to obtain state equations of the PLL loop. After introducing auxiliary variables:

$$u_f = kU_x \Rightarrow \frac{du_f}{dt} = kU\frac{dx}{dt}, \quad \frac{dx}{dt} = y \Rightarrow \frac{d^2x}{dt^2} = \frac{dy}{dt}, \qquad (3)$$

And

$$\varphi = \varphi_g - w_s t, \quad \tau = w_s t, \quad \vartheta = kw_s T, \quad U = \frac{k_1 U_s U^g}{2}, \qquad (4)$$

$$\rho = \frac{k_g}{w_o}, \quad \sigma = k^2 U w_o T, \quad k = \frac{w_o - w_g}{kUw_o},$$

Where:

k_g – The VCO generator gain,

k_1 – The multiplier gain,

w_o – Middle frequency of the VCO generator.

Applying the averaging method, it is possible to get autonomous averaged equation:

$$\frac{d}{d\tau} = \begin{bmatrix} y \\ \varphi \\ x \end{bmatrix} = \frac{1}{w_s}\begin{bmatrix} -w_g^2(1.4142 w_g^{-1} y + \sin\varphi + x) \\ \frac{\sigma w_s}{\vartheta}(\rho x + k) \\ y \end{bmatrix} \qquad (5)$$

III. STABILITY ANALYSIS

Indirect method of a nonlinear system stability analysis relies on an assignment of a signal or multiple singular points of the system and verification of the loop stability. Assignment of singular points of the system is achieved by comparing the left side of the system state equations (5) to zero. Because of the periodicity of the right side of the system Of equations (5), variable φ can be limited to interval $\varphi \in (-\pi, \pi)$ giving two signaler points for $0 \le k/\rho \angle 1$,

Mohamed A. Shaban and Adel Yahya, "Analysis of Nonlinear Third Order Phase – Linked Loop," *International Research Journal of Advanced Engineering and Science*, Volume 5, Issue 4, pp. 231-233, 2020.

$$(y_0, \varphi_{01}, x_0) = \left(0, are\sin\frac{k}{\rho}, -\frac{k}{\rho}\right), \ldots\ldots\ldots\ldots$$

$$(y_0, \varphi_{02}, x_0) = \left(0, \pi - \arcsin\frac{k}{\rho}, -\frac{k}{\rho}\right) \quad (6)$$

The location of singular points on phase space (y, φ, x) is presented in Fig. 2.

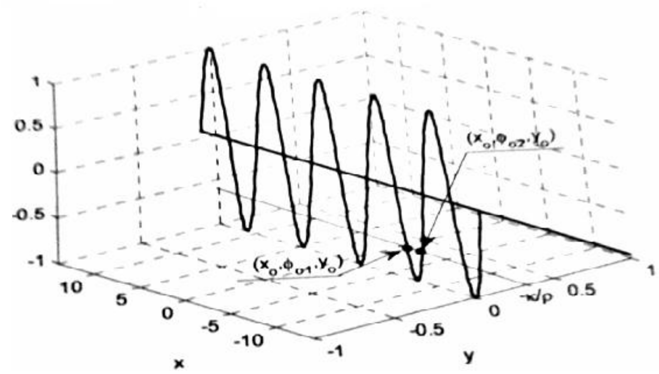

Fig. 2. the location of singular point of system of equation (6)

The determine the stability of the characteristic values of the Jacobi matrix [2], [3] assigned to the system of equation (5) should be calculated characteristitical equation this matrix defined equation:

$$\det\{J_0(y_0, \varphi_{01,2}, x_0) - \lambda E\} = 0, \quad (7)$$

And

$$\lambda^3 + \frac{1.4142 w_g}{w_s}\lambda^2 + \frac{w_g^2}{w_s^2}\lambda \pm \frac{\sigma\rho w_g^2}{\vartheta w_s^2}\sqrt{1-\left(\frac{k}{\rho}\right)^2} = 0 \quad (8)$$

Upper sign in equation (8) refers to point φ_{01} and lower sign refers to point φ_{02}. Further consideration is based upon the following assumption

$$\frac{\sigma\rho w_g^2}{\vartheta w_s^2}\sqrt{1-\left(\frac{k}{\rho}\right)^2} \rangle 0 \Rightarrow \left|\frac{k}{\rho}\right| \langle 1. \quad (9)$$

The discriminate Δ in equation (8) has the following form:

$$\Delta = \left(-\frac{0.1309 w_g^3}{w_s^3} \pm \frac{\sigma\rho w_g^2\sqrt{1-\left(\frac{k}{\rho}\right)^2}}{2\vartheta w_s^2}\right)^2 + \frac{w_g^6}{729 w_s^6} \quad (10$$

This discriminate is positive for:

$$w_g, \vartheta, \rho, \sigma \rangle 0 \quad oraz \left|\frac{k}{\rho}\right| \langle 1. \quad (11)$$

Using the cardano formulas, three roots in equation (8) can be determined (one real and two complexes), which have the following form:

$$\lambda_0 = \frac{1}{15000}\frac{\hat{3}^{\frac{1}{3}} A^{\frac{1}{3}}}{w_g \vartheta} - \frac{8333653}{\hat{3}.5000}\frac{\hat{3}^{\frac{2}{3}}\vartheta w_g^2}{A^{\frac{1}{3}} w_s} - 0.4714\frac{w_g}{w_s},$$

$$\lambda_{1,2} = \frac{1}{30000}\frac{\hat{3}^{\frac{1}{3}} A^{\frac{1}{3}}}{w_s \vartheta} + \frac{8333653}{\hat{3}.5000}\frac{\hat{3}^{\frac{2}{3}}\vartheta w_g^2}{A^{\frac{1}{3}} w_s} - 0.4714\frac{w_g}{w_s} \pm$$

$$\pm i\frac{\sqrt{3}}{2}\left(\frac{1}{15000}\frac{\hat{3}^{\frac{1}{3}} A^{\frac{1}{3}}}{w_s \vartheta} + \frac{8333653}{\hat{3}.5000}\frac{\hat{3}^{\frac{2}{3}}\vartheta w_g^2}{A^{\frac{1}{3}} w_s}\right), \quad (12)$$

Where:

$$A = \hat{3}^{-1}.3 w_g^2 (1473147603\vartheta w_g \mp 5625000000000\sigma\rho w_s\sqrt{B} + 250000\sqrt{6}.(62501198750\vartheta^2 w_g^2 \mp$$
$$441944281089\vartheta\sigma\rho w_g w_s\sqrt{B} + 8437500000000.\sigma^2\rho^2 w_g^2 - 843750000000\sigma^2 w_s^2 k^2)^{\frac{1}{2}})\vartheta^2,$$

$$B = 1 - \left(\frac{k}{\rho}\right)^2.$$

And the constant denoted as, Λ "are assigned to the point φ_{02}.

For the determined roots of equation (8) ($\Delta \rangle 0$), two types of singular points (6) exist: focus and saddle-focus. If one of the determined roots is real and two remaining are conjugate complex, than their sum has the same sign as the real root and it means that the evaluated point is a focus types. However, if their sum has the opposite sign to the real root, then received point is a saddle-focus type [3]. Below, the analysis of received singular points is presented.

A) For different values $\sigma, \rho, \upsilon, w_s, w_g$ and for $0 \le k/\rho \langle 1$ singular points φ_{01} and φ_{02} are the saddle-focus type.

B) For $\sigma = \rho - \upsilon = w_s = w_g = 1$ and $0 \le k \langle 1$ singular point φ_{01} is the focus type, however point φ_{02} is the saddle-focus type.

IV. COMPUTATION EXAMLE

For third order PLL loop (Fig. 1) taking into consideration condition (10) and (12) the following date were assumed:

$$\sigma = 1.25, \vartheta = 1.5, \rho = 0.8, k = 0, U = 2, w_g = 1 \quad (13)$$

The singular point given by equation (5) for interval $\varphi \in (-\pi, \pi)$ (the formula (6)) are:

$$(y_0, \varphi_{01}, x_0) = (0,0,0), (y_0, \varphi_{02}, x) = (0, \pi, 0) \quad (14)$$

The characteristic equation of the state equation of the system (5):

$$\lambda^3 + \frac{1.4142}{1500}\lambda^2 + \frac{1}{2250000}\lambda \pm \frac{1}{3375000} = 0 \quad (15)$$

The roots of this characteristic equation have the following form:

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$$\lambda_0 = -0.007, \lambda_{1,2} = 0.03 \pm i0.0058,$$
$$\lambda_0 = 0.0063, \lambda_{1,2} = -0.036 \pm io.0058. \quad (16)$$

Therefore both singular points are the saddle-focus type (sum of complex roots has opposite sign to the real roots).

V. CONCLUSION

The article present analysis of nonlinear third order phase-locked loop with Butterworth lowpass filter. For this type of the loop singular points of the system were determined and then specifity for different values of the parameter $\sigma, \rho, \upsilon, w_s, w_g$ and for $0 \le k \langle 1$ were analysis as well it has been done using the first Lyapunov method. Presented results were illustrated by an example.

REFERENCES

[1] Abramovitch D.Y."*Analysis and design of a third order phase-lock loop military comm.* Conf, MLICOM 88, New York, vol.2, 23-26 Oct, 1998, pp.55-459.

[2] G. O. Young, "Synthetic structure of industrial plastics," in *Plastics*, 2nd ed., vol. 3, J. Peters, Ed. New York: McGraw-Hill, pp. 15–64, 1964.

[3] W.-K. Chen, *Linear Networks and Systems*. Belmont, CA: Wadsworth, pp. 123–135, 1993.