# Discount Pricing Strategy for Companies Using Game Theory Approach 

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#### Abstract

This research focuses on the discount pricing strategies for the companies specifically cosmetics enterprises considering the price sensitivity of the consumers under Stackelberg game. The numerical results show that the production cost has negative effect on the discount percentage. On the contrary, the price elasticity has positive effect on the discount policy. From the numerical results, some detailed recommendations have been mentioned to increase the business's profit of the companies in different scenarios.


Keywords-Game theory, discount pricing strategy, price sensitivity, optimization.

## I. INTRODUCTION

There are many different factors that determine the economy transformation and significantly affect the development of the industry nowadays. These factors take crucial rolls in the growth or the reduction of a specific industry, some elements which can be mentioned are: import - export policy, taxes, investment strategies, pricing strategy, weather conditions, epidemic proportions, and etc.

Among these factors, pricing policy takes a vital important position in many companies' operational development. To be more specific, pricing strategy is defined as principal tasks of marketing and finance managers in terms of setting prices for products or services, which often plays a significant role in that product's or service's success. Generally, pricing policy performs the way of setting price of products based on expenditures, value of products, market demand and competition with other firms. On the other hand, pricing policy also refers how companies reach their development goals. The products' price can be increased or decreased depending on the market situation and the companies' purpose which is to increase demand or decrease backlog, respectively. Possible pricing strategies include a full price strategy, competitive pricing, discount pricing or a mix of these. In this paper, several questions are answered: (1) What optimal discount pricing level the company should offer to different group of consumers? (2) How does price discount rate affect the quantity breakpoint? (3) What is the relationship between the price discount rate and the production cost?

## II. LITERATURE REVIEW

Lal, R., \& Staelin, R. (1984) investigated why and how a seller develops a discount pricing structure so that they could encourage the consumers to increase their order quantity. They
assumed that the demand is certain and suggested the pricing policies which can be used as a baseline to measure the extent to which buyers pressure the seller to offer discount in return for the buyer place a larger order.

Yang and Zhou (2006) applied Stackelberg game to analyze a two-echelon system, in which a monopoly supplier sold to several groups of homogeneous buyers. The aim was to improve profits, both of channel and of each player.

Lau et al. (2007) analyzed the discount pricing issue by studying a dominant - retailer scenario which means that the retailer has relatively higher market power than the manufacturer/supplier. The paper also recommended a "reverse quantity discount" scheme which is provided by the retailer in order to improve the retailer's profit and coordinate the channel.

Ke, G. Y., \& Bookbinder, J. H. (2012) analyzed the discount policy from the supplier's perspective. The paper concentrated on finding an optimal discount rate corresponded by an optimal quantity breakpoint. The final optimal policies and respective payoffs of both two parties were reflected after the comparison of the optimal quantity breakpoint and discount rate value.

Sadjadi et al. (2018) analyzed the optimal equilibrium solutions when applying Stackelberg game in two - stage supply chain including two manufacturers and one retailer where they compete simultaneously under three factors: price, service and simple price discount contract. The paper illustrated that the performance of supply chain could be improved via the service and price discount contract in terms of the profit, the demand and coordination of supply chain's members.

Zhang et al. (2019) introduced discount pricing policy in word - of - mouth marketing considering the associated word - of - mouth spreading process as a higher dimensional nonlinear differential dynamical system.

Li, C., Chu, M., Zhou, C., \& Zhao, L. (2020) suggested two - period model to investigate different discount pricing strategies of online vouchers for strategic customer behavior.

Nagare, M., Dutta, P., \& Suryawanshi, P. (2020) considered the problem of discount pricing, promotion and ordering of non-instantaneous deteriorating single period product subject to promotional efforts and deterioration by modifying price-and time-dependent demand function.

Based on the previous researches related to pricing strategies conducted, our paper extends to apply Stackelberg game to investigate the discount pricing policies considering the price sensitivity as well as the production cost. The study
also analyzes deeper with the extension into a price break and a discount rate applying for companies specifically cosmetics companies with the set of data from Innisfree Vietnam company for numerical analysis.

## III. Mathematical Model

In reality, after the supplier determines his quantity discount policy, the buyer reacts to that policy and chooses her order quantities and schedules. That is why the Stackelberg equilibrium is used to analyze this non-cooperative gametheoretic model.

The Stackelberg game is a dynamic model of duopoly in which a dominant player (leader) moves first and a subordinate (follower) moves sequentially (Leng, M., \& Parlar, M., 2005). having been informed of the dominant player's move. Such a framework contains the concept of a 'hierarchical equilibrium solution', which analyses and specifies the behaviors of the players when one of them has the ability to enforce his/her strategy on the other. In our model, the Stackelberg equilibrium allows the supplier, considered as the leader, to construct the quantity discount policy. The supplier thus maximizes his own payoff, taking account that the buyer, considered as the follower, is attempting to maximize her payoff.

## A. Notations

## Parameters:

$D$ : Buyer's annual demand.
$P$ : unit acquisition cost without quantity discount for buyer.
$R$ : unit selling price for buyer.
$v$ : unit production cost for manufacturer.
$A_{i}$ : Order processing cost, $i=B$ or $M$ represent for the buyer or manufacturer, respectively.
$H_{i}$ : unit holding cost per year.
$n$ : The absolute value of the product's price elasticity of demand.
$Q$ : Buyer's optimal order quantity (EOQ) before discount.
$Q=\sqrt{\frac{2 D A_{B}}{H_{B}}}$
$q$ : Buyer's actual order quantity with discount price.
$q$ ': Buyer's best order quantity at discount price.
$S_{t}$ : Production quantity of manufacturer in period t without discount.
$s_{t}$ : Production quantity of manufacturer in period t with discount.
$T C_{j}$ : Buyer's total annual costs. $j=0$ or 1 represents the value without or with a quantity discount, respectively.
$\pi_{i j}$ : Profit gained from the product by player $i$ for the case $j$.
$\Pi_{i}$ : Payoff gained by the buyer taking the advantage of the quantity discount.
Nj : Manufacturer's annual production quantity in case j .
K: Manufacture's capacity.

## Indices:

i: Represent for the buyer or manufacturer, $\mathrm{i}=\mathrm{B}$ or M .
j : Represents the value without or with a quantity discount, $j=0$ or 1 , respectively.
t : Production period, $\mathrm{t}=0, \ldots, \mathrm{t}-1, \mathrm{t}$

## Indices:

$\rho$ : Price discount $(0<\rho<1)$. The price will be reduced ( $\rho * 100 \%$ ) when the buyer can satisfy the requirement of quantity order equal or larger than the quantity breakpoint $\varphi$. Otherwise, no discount policy is applied.
$\varphi$ : Quantity breakpoint.

## B. Model

## Objective function:

The objective function of this paper is to maximize the manufacturer's profit, with the formula including the revenue of ( $1+n \rho$ ) $D$ products sold, the elimination of manufacturer's order processing cost, holding cost, and production cost.

$$
\begin{align*}
& \operatorname{Max} \Pi_{M}(\varphi)=-D P \rho(1-n+n \rho)-D v n \rho+A_{M} D \\
& \left(\frac{1}{Q}-\frac{1+n \rho}{\varphi}\right)+\frac{H_{M}}{2}(Q-\varphi) \tag{1}
\end{align*}
$$

## Subject to:

- The rate $\rho$ has to be found in the range from 0 to 1 , in order to meet the basic requirement of percentage.

$$
\begin{equation*}
\rho \in(0,1) \tag{2}
\end{equation*}
$$

- The selling price of buyer after applying quantity discount strategy must be lower than the regular price selling for end-users before receive the discount policy:

$$
\begin{equation*}
R>R * \rho \tag{3}
\end{equation*}
$$

- After discount, the manufacturer must reduce the acquisition for buyer which ensure the cost does not exceed the original cost:

$$
\begin{equation*}
P>P * \rho \tag{4}
\end{equation*}
$$

- Inventory balance constraints has to be always satisfied, which means the inventory in period $t$ equal to the inventory in previous period adding the number of products produced and subtracting the market demand:

$$
\begin{align*}
& \left.S_{t}=S_{t-1}+N_{0}-D \text { (Without discount }\right)  \tag{5}\\
& \left.s_{t}=s_{t-1}+N_{l}-(1+n \rho) D \text { (With discount }\right) \tag{6}
\end{align*}
$$

- The production quantity has not to be exceed the manufacturer's capacity:

$$
\begin{align*}
& N_{0} \leq K(\text { Without discount })  \tag{7}\\
& N_{I} \leq K(\text { With discount }) \tag{8}
\end{align*}
$$

- Also, the production quantity has to cover all demand from market, it means:

$$
\begin{align*}
& N_{0} \geq D \text { (Without discount) }  \tag{9}\\
& N_{I} \geq(1+n \rho) D \text { (With discount) } \tag{10}
\end{align*}
$$

- The quantity breakpoint has to larger than the best order quantity of buyer, since instead of buy the $\varphi$ product quantity which maximize manufacturer 's profit, the
buyer can buy the larger number of product quantity $q^{\prime}$ which maximize the buyer 's profit:

$$
\begin{equation*}
\varphi>q^{\prime} \tag{11}
\end{equation*}
$$

## IV. Results Analysis

Using Mathematica 12.0 software with the data collected from Innisfree Vietnam company, the author derives the results as follow:

TABLE I. Optimal discount policy result when $n=2$

| $n$ | $\rho$ | $\varphi$ | Buyer’s payoff <br> (VND) | Manufacture’s <br> payoff (VND) |
| :---: | :---: | :---: | :---: | :---: |
| 2 | 0.0485209 | 601.05 | $5.14771 \times 10^{\wedge 7}$ | $4.07115 \times 10^{\wedge 7}$ |

When the absolute value of the product's price elasticity of demand is 2 , the optimal discount policy for an order of equaling or exceeding 601 products, the buyer will be discounted about $4.85 \%$ on the total invoice value. Also, the manufacturer's payoff in this case will be maximized to around 40.7 million VND and for the buyer's payoff, gaining around 51.5 million VND.


Fig. 1. Payoffs of manufacturer and buyer
Fig. 1 illustrates the payoffs of the company and the buyer when the order quantity changes from 0 to 1,000 units. It matches with the result in Table I. As can be seen in the graph, at the point order quantity is 601 units, the manufacturer gains the highest profit for its discount strategy.

The result above also indicates that for various scenarios, the company can conduct an equitable quantity discount pricing policy, as long as it satisfies the constraint of ensuring the buyer's payoff be greater than 0 , it means, the discount strategy can help the buyer get a better selection for ordering.

The author also investigates the changing of optimal discount policy and the quantity breakdown when the price sensitivity changes from 2 to 10 and the production cost is fixed at the value of $139,000 \mathrm{VND}$, and $289,000 \mathrm{VND}$, respectively. The result is shown in the graphs below.


Fig. 2. Discount levels with different price sensitivity


Fig. 3. Price sensitivity and quantity breakpoint
Fig. 2 and Fig. 3 show us that the price sensitivity has positive effect on the discount level and the quantity breakpoint. It matches with reality. With higher price sensitivity, it means that the price has more effect on the customers' buying behaviors. Therefore, for this group of customers, the company should offer them more discount to encourage them to buy the products. With concerning more about the price, the customers are more willing to buy more products to get cheaper price, hence, the quantity breakpoint is higher.

For the group of customers who care less about the price which means that their price sensitivity is low, the company can offer lower discount percentage with ensuring that the customers still buy their products as well as increasing their profit by giving higher price for the same product.

It is shown in Fig. 2 and Fig. 3 that the production cost has negative effect on the discount level and the quantity breakpoint. In the real economic circumstance, a business has lots of selection for carrying out an optimal discount policy, and reducing cost of manufacturer is one of the functional methods. The decrease in production cost will help business saving more money for their other plans or strategies in the future, however, the problem is how the manufacturer still remains the good quality of products. This is a hard question because the company need to observes other aspects of the problem, the manufacturer is required to answer some crucial questions such as will customers care more about product
quality or they accept this situation and keep buying these products? What is the company's main market segmentation? And what level of sensitivity of price the buyers is?

There is a note that if the production cost is substantial high, with other parameter as the same, there will be no practical discount policy offered by company, because the discount percentage becomes so low or it can be fall out of the originally possible range of price discount. In these situation, other inputs need to be adjusted in order to balance the parameters and conduct a reasonable and profitable discount pricing strategy.

## V. Conclusions and Recommendations

Discount pricing strategy plays an important role in business development. With a successful strategy, business can take advantage of gaining more revenue, more customers, and increasing more identification level of the product in market. The policy with obvious process, high applicability will contribute for the great exploration of a brand's computation, maybe there will have some difficulties cannot be avoided, however, with the enormous effects of this strong method, benefits bringing to company is worth for more attention.

This paper concentrates on conducting an appropriate discount policy for companies specifically cosmetics companies. A numerical quantity breakpoint and price discount is proposed from differentiating the buyer's payoff and manufacturer's payoff in terms of maximizing manufacturer's profit and the possibility of employing discount policy from buyer.

The final result was conducted which stretched the best discount percentage associated with an optimal quantity breakpoint. By the output of this model, the companies will have a better business development strategy. Moreover, there are lots of analysis that concentrates on how to set the best discount strategy for different circumstance and how other notations such as production cost and price elasticity of demand influence on discount policy decision.

From results of the paper, the author summarizes some crucial contributions. (1) For the group of consumers who have higher price sensitivity, the company should offer them higher level of discount to encourage them to buy more products. For the group of consumers whose price elasticity is lower, the company can consider offering them lower level of discount so that the company can get more marginal profit for each product. (2) In any cases, to get more discount, the
customers have to make order of a larger amount of products. (3) Production cost and the price discount level have negative relationship.

For the modern economic nowadays, discount pricing strategy cannot be denied its advantages. A policy that almost businesses have employed at least once in order to achieve the enormous benefits for their company. Discount policy maybe not strange for many businesses, however, it is not all companies applying this all-unit discount strategy efficiently and gain the success. The effectively strategy is designed by observing many aspect and prospect of market, as well as deeply understanding the company's market segmentation. It is believed that, this study will give a functional model for acquiring the successful pricing strategy for the business. It turns out to be a strong method for developing the business growing orientation.

The future research can consider more cost such as transportation cost, taxes, insurance cost. Another direction for the next research is that they can integrate the purchase quantity and the transportation discount.

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