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Multiresponse Robust Design: Desirability Function Based on Mean Square Error Criterion

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***Abstract*— *Industrial problems frequently involve meeting specifications for several quality characteristics. This implies simultaneously choosing optimum conditions for those quality characteristics. We propose the method of desirability function based on mean square error criterion to solve this multiple criterion problem when the data are collected from a combined array design. We present an illustrative example on the elastic element of a force transducer.*

***Keywords*— *Combined array design, desirability function, mean square error, multiresponse robust design.*

I. Introduction

For most of the designed experiments based on response surface methodology, the quality characteristic is multidimensional, so it is common to observe multiple responses in experimental situation. In such experiments, the determination of the conditions on the sets of controllable variables that optimize a multiresponse function, minimize the variability and make the set of responses insensitive or less sensitive to the effect of the noise variables is of particular interest. In an effort to solve such multiresponse optimization problems, researchers published several articles addressing different approaches to analyzing multiresponse robust experiments [1].

The objective of the current paper is to propose the method of desirability function (DF) based on mean square error criterion (MSE) when the data are collected from a combined array design. We adopt the MSE criterion, which is originally proposed and applied to the dual response problem by Lin and Tu [2] and we extend the idea for robust design in the case of multiple responses. We define the individual desirability functions based on individual mean square errors of the quality characteristics, commonly referred to as response variables, and incorporate the individual desirability functions into a single function which gives the overall assessment on the desirability of the combined response variables. This function is known as overall desirability function, and it is expressed as the geometric mean of the individual desirability functions [3]. The optimum operating condition is the combination of the controllable variable levels which maximizes the overall desirability function.

The rest of the paper is organized as follows. A brief review of the structure of the multiresponse robust design in combined array design is presented in Section II. In Section III we present a brief review of the mean square error approach. We summarize the use of desirability function approach in Section IV. The method we propose, this is the desirability function based on mean square error criterion, is presented in Section V. An example that illustrates the proposed approach is given in Section VI.

II. Review of Combined Array Design

In this section we present a framework for the multiresponse robust problem when the data are collected from a combined array.

Suppose that k quality characteristics $\left(y^{(1)}, y^{(2)}, \ldots, y^{(k)}\right)$ of a product or process depend on p design (control) factors $\left(x_1, x_2, \ldots, x_p\right)$ and q noise factors $\left(z_1, z_2, \ldots, z_q\right)$. The experimental structure of the combined array design is presented by TABLE I.

TABLE I. Experimental Structure of Combined Array Design.

x_1	$\cdots$	x_p	z_1	$\cdots$	z_q	$y^{(1)}$	$\cdots$	$y^{(k)}$
x_{11}	$\cdots$	x_{1p}	z_{11}	$\cdots$	z_{1q}	$y_{11}^{(1)}$	$\cdots$	$y_{11}^{(k)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{n1}	$\cdots$	x_{np}	z_{r1}	$\cdots$	z_{rq}	$y_{nr}^{(1)}$	$\cdots$	$y_{nr}^{(k)}$

The steps of the proposed framework are as follows:

Step 1: Identify

- The potential control factors $\left(x_1, x_2, \ldots, x_p\right)$.
- The noise factors $\left(z_1, z_2, \ldots, z_q\right)$.
- The response variables $\left(y^{(1)}, y^{(2)}, \ldots, y^{(k)}\right)$.
- The target values for individual response variables $\left(T^{(1)}, T^{(2)}, \ldots, T^{(k)}\right)$.

Step 2: Obtain an appropriate design for involved factors.
Step 3: Execute the experiment. The experimental design must allow for estimation of the selected response model.
Step 4: For each response variable, obtain the dual response surfaces, one for the estimated mean and another for the estimated variance.
Step 5: Calculate the MSE function of each response variable (i.e., $MSE^{(i)}; i = 1, 2, \cdots, k$).

III. Review of Mean Square Error Approach

For k response variables, let $\boldsymbol{y} = \left(y^{(1)}, y^{(2)}, \ldots, y^{(k)}\right)$ denote the vector of multiple responses. We consider the quadratic model described by Myers et al. [4]. For the i^{th} response variable, the matrix notation of the model can be written as follows:

$$y^{(i)}(\boldsymbol{x},\boldsymbol{z}) = \beta_0^{(i)} + \beta^{(i)T}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{B}^{(i)}\boldsymbol{x} + \gamma^{(i)T}\boldsymbol{z} + x^T\Delta^{(i)}\boldsymbol{z} + \varepsilon^{(i)};\quad i = 1,2,\cdots,k, \qquad (1)$$

where

- $y^{(i)}(\boldsymbol{x},\boldsymbol{z})$ denotes the i^{th} response variable,
- $\boldsymbol{x}$ denotes the vector of control factors,
- $\boldsymbol{z}$ denotes the vector of noise factors,
- $\beta_0^{(i)}$ is an intercept,
- $\beta^{(i)}$ is a vector of coefficients for the linear effects in control factors,
- $\boldsymbol{B}^{(i)}$ is a matrix for which the main diagonal entries are the regression coefficients associated with the pure quadratic effects of the control factors and the off-diagonal entries are one-half of the mixed quadratic (interaction) effects of the control factors,
- $\Delta^{(i)}$ is a matrix of the control-by-noise interaction coefficients,
- $\varepsilon^{(i)}$ is a random error.

It is assumed that the $\varepsilon^{(i)}$'s are independently and identically distributed $N\left(0,\sigma^2_{\varepsilon^{(i)}}\right)$ and that all the noise factors are continuous. It is also assumed that, in accordance with the design level centering and scaling, $E(\boldsymbol{z}) = \boldsymbol{0}$ and $Var(\boldsymbol{z}) = \Sigma = \sigma_z^2\boldsymbol{I}$.

After the model in Eq. (1) is fitted to the data from the designed experiment, the corresponding adjusted response model is given by the expression [5]

$$\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})} = \hat{\beta}_0^{(i)} + \hat{\beta}^{(i)T}\boldsymbol{x} + \boldsymbol{x}^T\widehat{\boldsymbol{B}}^{(i)}\boldsymbol{x} + \hat{\gamma}^{(i)T}\boldsymbol{z} + x^T\hat{\Delta}^{(i)}\boldsymbol{z};\quad i = 1,2,\cdots,k. \qquad (2)$$

From Eq. (2) we derive the mean response and variance response surfaces given by the following expressions:

$$E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) = \hat{\beta}_0^{(i)} + \hat{\beta}^{(i)T}\boldsymbol{x} + \boldsymbol{x}^T\widehat{\boldsymbol{B}}^{(i)}\boldsymbol{x};\ i = 1,2,\cdots,k. \qquad (3)$$

$$Var_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) = \left(\hat{\gamma}^{(i)T} + \boldsymbol{x}^T\hat{\Delta}^{(i)}\right)\Sigma\left(\hat{\gamma}^{(i)} + \hat{\Delta}^{(i)T}\boldsymbol{x}\right);\ i = 1,2,\cdots,k.$$

The MSE is an effective method to combine the mean response and variance response into one function.

Three types of MSE functions can be defined as follows (O. Köksoy and Tankut Yalcinoz [6]):

The MSE function for the nominal the best (NTB) case:

$$MSE^{(i)} = \left[E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - T\right]^2 + Var_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right);\ i = 1,2,\cdots,k, \qquad (4)$$

where T is the target value.

The MSE function for the smaller the better (STB) case:

$$MSE^{(i)} = \left[E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right)\right]^2 + Var_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right);\ i = 1,2,\cdots,k. \qquad (5)$$

The MSE function for the larger the better (LTB) case:

$$MSE^{(i)} = \left[E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - H\right]^2 + Var_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right);\ i = 1,2,\cdots,k, \qquad (6)$$

where H is the highest plausible value of $E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right)$.

IV. Review of Desirability Function

The desirability function to simultaneously optimizing multiple equations was originally proposed by Harrington [7]. The common approach is to transform each predicted response variable, $E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right); i = 1,2,...,k$ into a desirability function,

$$d^{(i)} = h\left(E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right)\right),\ 0 \le d^{(i)} \le 1; i = 1,2,...,k \qquad (7)$$

The function $d^{(i)}$ increases as the desirability of the corresponding response variable increases. The individual desirability functions are incorporated into a single function D, called overall desirability. The overall desirability gives the overall assessment of the desirability of the combined response variables. The optimal setting is found by maximizing the overall desirability.

Individual desirability functions

Depending on whether a particular response variable, say $y^{(i)};\ i = 1,2,\cdots,k$ is to be maximized, minimized or assigned to a target value, different desirability functions $d^{(i)};\ i = 1,2,\cdots,k$ are defined as follows [3]:

The nominal the best (NTB) type:

$$d^{(i)} = \begin{cases} 0 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \le L^{(i)} \text{ or } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \ge U^{(i)} \\ \left(\dfrac{E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - L^{(i)}}{T^{(i)} - L^{(i)}}\right)^r & \text{if } L^{(i)} < E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) < T^{(i)} \\ \left(\dfrac{E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - U^{(i)}}{T^{(i)} - U^{(i)}}\right)^r & \text{if } T^{(i)} < E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) < U^{(i)} \\ 1 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) = T^{(i)}. \end{cases} \qquad (8)$$

The smaller the better (STB) type:

$$d^{(i)} = \begin{cases} 1 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \le L^{(i)} \\ \left(\dfrac{E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - U^{(i)}}{L^{(i)} - U^{(i)}}\right)^r & \text{if } L^{(i)} < E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) < U^{(i)} \\ 0 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \ge U^{(i)}. \end{cases} \qquad (9)$$

The larger the better (LTB) type:

$$d^{(i)} = \begin{cases} 0 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \le L^{(i)} \\ \left(\dfrac{E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) - L^{(i)}}{U^{(i)} - L^{(i)}}\right)^r & \text{if } L^{(i)} < E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) < U^{(i)} \\ 1 & \text{if } E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right) \ge U^{(i)}. \end{cases} \qquad (10)$$

The values of $L^{(i)}$ and $U^{(i)}$ are the lowest acceptable value and the highest acceptable value of $E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right)$, respectively. The value r is the weight of $E_z\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right)$ in the process.

Overall desirability function

For k response variables, the overall performance of the process is determined by the overall desirability D, which can be expressed as the geometric mean of the individual desirability functions:

$$D=\left(\prod_{i=1}^{k} d^{(i)}\right)^{\frac{1}{k}}. \tag{11}$$

The optimization problem to solve is then

$$\begin{cases}\text{Maximize } D \\ \text{Subject to } \boldsymbol{x}\in R,\end{cases} \tag{12}$$

where R is the experimental region.

V. Desirability Function Based on Mean Square Error Criterion

We determine the individual MSE function for each response variable, i.e., $MSE^{(i)};\, i=1,2,\cdots,k$ according to the purposes of the experiment.

We calculate the corresponding individual desirability function:

$$d^{(i)}=h\left(MSE^{(i)}(\boldsymbol{x})\right);\ i=1,2,\cdots,k. \tag{13}$$

Individual desirability functions

For the i^{th} response variable, $i=1,2,\cdots,k;$ the corresponding desirability function based on MSE is determined as follows.

Individual desirability function for the smaller the better (STB) type:

$$d^{(i)}=\begin{cases}1 & \text{if } \ MSE^{(i)}(\boldsymbol{x})\le L^{(i)} \\ \left(\dfrac{MSE^{(i)}(\boldsymbol{x})-U^{(i)}}{L^{(i)}-U^{(i)}}\right)^{r} & \text{if } \ L^{(i)}<MSE^{(i)}(\boldsymbol{x})<U^{(i)} \\ 0 & \text{if } \ MSE^{(i)}(\boldsymbol{x})\ge U^{(i)}\end{cases} \tag{14}$$

where $L^{(i)}=MSE^{(i)}(\boldsymbol{x}_{\min})$ and $U^{(i)}=MSE^{(i)}(\boldsymbol{x}_{\max})$, where $\boldsymbol{x}_{\min}$ and $\boldsymbol{x}_{\max}$ are respectively solutions of the following optimization problems:

$$\begin{cases}\text{Minimize } MSE^{(i)}(\boldsymbol{x}) \\ \text{Subject to } \boldsymbol{x}\in R\end{cases} \quad \text{and}$$

$$\begin{cases}\text{Maximize } MSE^{(i)}(\boldsymbol{x}) \\ \text{Subject to } \boldsymbol{x}\in R,\end{cases} \tag{15}$$

where R is the experimental region.

For k response variables, the overall desirability based on MSE is given by the following expression:

$$D=\left(\prod_{i=1}^{k} d^{(i)}\right)^{\frac{1}{k}}. \tag{16}$$

We solve the following optimization problem:

$$\begin{cases}\text{Maximize } D \\ \text{Subject to } \boldsymbol{x}\in R,\end{cases} \tag{17}$$

where R is the experimental region.

VI. Illustrative Example

The illustrative example is a robust design conducted on the elastic element of a force transducer. This is a case study presented by Romano et al. [8]. A transducer is a device that provides an output quantity having a determined relationship to the parameter being measured, i.e.., the force in this case.

When a compressive load is applied to the elastic element, a peculiar strain pattern is created over the central section of the elastic element, where strain peaks due to design factors. The deformation of the element is then measured by a second device which converts it into a measurable output.

The design of the element is intended to minimize the transducer inaccuracy, which originates from two major sources, namely non-linearity and hysteresis. These two indicators define the response variables, i.e., $y^{(1)}$ and $y^{(2)}$, respectively. The non-linearity effect is the ratio between longitudinal strain and transversal strain. The hysteresis indicator is the ratio between maximum strain on the measuring area and longitudinal strain.

This example involves a combined array design with three control factors ($\boldsymbol{x}$) and two noise factors ($\boldsymbol{z}$). Control factors are the three parameters defining the element configuration, namely lozenge angle (x_1), bore diameter (x_2), and half-length of the vertical segment (x_3). Noise factors are the deviation of the lozenge angle from its nominal value (z_1) and the deviation of the bore diameter from its nominal value (z_2). These internal noise factors are admittedly independent. They are also assumed to be normally distributed with zero mean and variances $\pm_1^2$ and $\pm_2^2$, respectively. TABLE II displays the coded and real levels of the factors.

TABLE II. Coded and real levels of the factors.

	Levels		
Design factors	**-1**	**0**	**1**
x_1	15	30	45
x_2	8	11	14
x_3	7	9	11
Noise factors	**-1**	**0**	**1**
z_1	-1.5	0	1.5
z_2	-0.25	0	0.25

The aim of the experiment is to find the settings for the lozenge angle (x_1), bore diameter (x_2), and half-length of the vertical segment (x_3) which achieve a target value of 1 for the non-linearity indicator ($y^{(1)}$) and minimize the hysteresis ($y^{(2)}$).

The experimental design used is a central composite design (CCD) made up of a half-fraction design of a five factor-two level factorial design, star points for control factors only, and a threefold center point. The APPENDIX displays the data for this experiment.

The following prediction equations are obtained for non-linearity and hysteresis indicator, respectively:

$$\widehat{y^{(1)}(\boldsymbol{x},\boldsymbol{z})} = 1.377 - 0.361x_1 - 0.155x_2 + 0.077x_3 + 0.042x_1^2 + \\ +0.007x_2^2 + 0.002x_3^2 - 0.148x_1x_2 + 0.022x_1x_3 + 0.013x_2x_3 + \\ -0.059z_1 - 0.012z_2 + 0.010x_1z_1 - 0.008x_1z_2 - 0.006x_2z_1 + \\ +0.001x_2z_2 + 0.005x_3z_1 + 0.003x_3z_2.$$

$$\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})} = 1.660 + 0.592x_1 + 0.438x_2 - 0.095x_3 + 0.247x_1^2 + \\ -0.123x_2^2 + 0.047x_3^2 + 0.301x_1x_2 - 0.143x_1x_3 - 0.033x_2x_3 + \\ +0.066z_1 - 0.042z_2 + 0.079x_1z_1 + 0.017x_1z_2 - 0.031x_2z_1 + \\ -0.061x_2z_2 - 0.004x_3z_1 - 0.014x_3z_2.$$

The corresponding estimated mean models are:

$$E_{\boldsymbol{z}}\left(\widehat{y^{(1)}(\boldsymbol{x},\boldsymbol{z})}\right) = 1.377 - 0.361x_1 - 0.155x_2 + 0.077x_3 + 0.042x_1^2 + \\ +0.007x_2^2 + 0.002x_3^2 - 0.148x_1x_2 + 0.022x_1x_3 + 0.013x_2x_3.$$

$$E_{\boldsymbol{z}}\left(\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})}\right) = 1.660 + 0.592x_1 + 0.438x_2 - 0.095x_3 + 0.247x_1^2 + \\ -0.123x_2^2 + 0.047x_3^2 + 0.301x_1x_2 - 0.143x_1x_3 - 0.033x_2x_3.$$

The models for the variances are given by:

$$Var_{\boldsymbol{z}}\left(\widehat{y^{(1)}(\boldsymbol{x},\boldsymbol{z})}\right) = 3.925\times10^{-3} - 9.88\times10^{-4}x_1 + 6.84\times10^{-4}x_2 + \\ -6.62\times10^{-4}x_3 + 1.64\times10^{-4}x_1^2 + 3.7\times10^{-5}x_2^2 + 3.4\times10^{-5}x_3^2 + \\ -1.36\times10^{-4}x_1x_2 + 5.2\times10^{-5}x_1x_3 - 5.4\times10^{-5}x_2x_3.$$

$$Var_{\boldsymbol{z}}\left(\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})}\right) = 0.04312 + 0.009x_1 + 1.032\times10^{-3}x_2 + \\ +6.48\times10^{-4}x_3 + 0.00653x_1^2 + 4.682\times10^{-3}x_2^2 + 2.12\times10^{-4}x_3^2 + \\ -6.972\times10^{-3}x_1x_2 - 1.108\times10^{-4}x_1x_3 + 1.956\times10^{-3}x_2x_3.$$

It is assumed that z_1 and z_2 are uncorrelated and $\pm_{z_1}^2 = \pm_{z_1}^2 = 1$.

Next, the individual MSE functions, i.e., $MSE_{NTB}^{(1)}$ for the non-linearity indicator and $MSE_{STB}^{(2)}$ for the hysteresis indicator, are computed from the two response variables. These functions are given by the following expressions:

$$MSE_{NTB}^{(1)} = \left[E_{\boldsymbol{z}}\left(\widehat{y^{(1)}(\boldsymbol{x},\boldsymbol{z})}\right) - 1\right]^2 + Var_{\boldsymbol{z}}\left(\widehat{y^{(1)}(\boldsymbol{x},\boldsymbol{z})}\right).$$

$$MSE_{STB}^{(2)} = \left[E_{\boldsymbol{z}}\left(\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})}\right)\right]^2 + Var_{\boldsymbol{z}}\left(\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})}\right).$$

As the MSE error criterion implies the minimization of the MSE, the corresponding desirability function is of the smaller the better type. The individual desirability function corresponding to each of the individual MSE, i.e., $MSE_{NTB}^{(1)}$ and $MSE_{STB}^{(2)}$, has the following form:

$$d^{(i)} = h\left(MSE^{(i)}(\boldsymbol{x})\right)$$
$$= \begin{cases} 1 & \text{if } MSE^{(i)}(\boldsymbol{x}) \le L^{(i)} \\ \left(\dfrac{MSE^{(i)}(\boldsymbol{x}) - U^{(i)}}{L^{(i)} - U^{(i)}}\right)^r & \text{if } L^{(i)} < MSE^{(i)}(\boldsymbol{x}) < U^{(i)} \\ 0 & \text{if } MSE^{(i)}(\boldsymbol{x}) \ge U^{(i)};\ i = 1,2;\ r = 1 \end{cases}$$

where $L^{(i)} = MSE^{(i)}(\boldsymbol{x}_{\min})$, $U^{(i)} = MSE^{(i)}(\boldsymbol{x}_{\max})$, $\boldsymbol{x}_{\min}$ and $\boldsymbol{x}_{\max}$ are respectively solutions of the following optimization problems:

$$\begin{cases} \text{Minimize } MSE^{(i)}(\boldsymbol{x});\ i = 1,2 \\ \text{Subject to } \boldsymbol{x} \in R \end{cases} \quad \text{and}$$

$$\begin{cases} \text{Maximize } MSE^{(i)}(\boldsymbol{x});\ i = 1,2 \\ \text{Subject to } \boldsymbol{x} \in R. \end{cases}$$

TABLE III displays the values for $L^{(i)}$'s and $U^{(i)}$'s .

TABLE III. Ranges for individual mean square errors.

$L^{(1)}: 0.00281$	$L^{(2)}: 1.0562$
$U^{(1)}: 0.706$	$U^{(2)}: 11.8$

The overall desirability function is determined as follows:

$$D = \left(\prod_{i=1}^{2} d^{(i)}\right)^{\frac{1}{2}} = \left[\prod_{i=1}^{2} h\left(\widehat{MSE^{(i)}(\boldsymbol{x})}\right)\right]^{\frac{1}{2}}.$$

The optimum operating conditions are obtained by solving the following optimization problem:

$$\begin{cases} \text{Maximize } D \\ \text{Subject to } -1 \le x_j \le 1,\ j = 1,2,3. \end{cases}$$

TABLE IV presents the results obtained.

TABLE IV. Optimal settings and related estimates.

	$y^{(1)}$	$y^{(2)}$
$\boldsymbol{x}_{\text{optimal}}$	$(0.47202, -0.84483, -1)$	
$E_{\boldsymbol{z}}\left(\widehat{y^{(i)}(\boldsymbol{x},\boldsymbol{z})}\right);\ i = 1,2$	1.337	1.598
$Var_{\boldsymbol{z}}\left(\widehat{y^{(2)}(\boldsymbol{x},\boldsymbol{z})}\right);\ i = 1,2$	0.00363	0.0558
$\widehat{MSE^{(i)}(\boldsymbol{x})};\ i = 1,2$	0.117	2.61

VII. Conclusions

The main objective of this paper is to present the desirability function based on mean square error criterion as a method for analyzing the data obtained from a multiresponse robust design considering a combined array design. To attain the objective, a general framework for a multiresponse robust problem when the data are collected from a combined array design is presented and brief reviews on mean square error approach and desirability function are given. An illustrative example from the literature is presented.

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APPENDIX

Experimental results for the force transducer experiment.

Treatment	x_1	x_2	x_3	z_1	z_2	y_1	y_2
1	–1	–1	–1	–1	1	1.810	1.10
2	–1	–1	–1	1	–1	1.690	1.11
3	–1	–1	1	–1	–1	1.900	1.07
4	–1	–1	1	1	1	1.780	1.07
5	–1	1	–1	–1	–1	1.800	1.47
6	–1	1	–1	1	1	1.630	1.18
7	–1	1	1	–1	1	1.920	1.41
8	–1	1	1	1	–1	1.780	1.58
9	1	–1	–1	–1	–1	1.360	1.57
10	1	–1	–1	1	1	1.220	2.03
11	1	–1	–1	1	1	1.480	1.38
12	1	–1	1	1	–1	1.440	1.68
13	1	1	–1	–1	1	0.693	3.37
14	1	1	–1	1	–1	0.616	3.75
15	1	1	1	–1	–1	0.950	2.81
16	1	1	1	1	1	0.817	2.83
17	–1	0	0	0	0	1.790	1.24
18	1	0	0	0	0	1.030	2.46
19	0	–1	0	0	0	1.530	1.23
20	0	1	0	0	0	1.220	1.73
21	0	0	–1	0	0	1.300	1.63
22	0	0	1	0	0	1.440	1.67
23	0	0	0	0	0	1.380	1.73
24	0	0	0	0	0	1.390	1.74
25	0	0	0	0	0	1.400	1.74