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Linear Slip Control for Improved Antilock Braking System

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Abstract— *Traction, braking and stability control of the automotive systems are mostly represented by the wheel slip control. This paper has presented linear slip control for improved antilock braking system. The aim of this work is to design and simulate a slip control system with improved tracking performance in Matlab/Simulink environment, and thereby maintaining an optimal wheel slip ratio. A two degree of freedom proportional plus integral plus derivative (2DOFPID) controller is developed and integrated into a dynamic model of a quarter-car. The developed slip control system was simulated in Matlab/Simulink environment. To show the effectiveness or robustness of the controller, simulations were performed on a straight-line braking operation on two different road surface conditions. In each case the controller was able to bring the system back to the desired trajectory of 10% optimal slip, which signifies a robust control system. It can be seen that the designed controller was able to minimize slip on the two road conditions considered in this paper.*

Keywords— *Controller, Quarter-car, Slip control, Two-DOFPID, Wheel slip.*

I. Introduction

Traction, with respect to automobiles, is the grip of a tyre on a road surface. Traction Control System (TCS) is indeed an "add-on" feature to Anti-lock Braking System (ABS) that enhances traction when a vehicle is moving on a wet or slippery surface, or is moving too quickly for tyres to maintain their grip. Vehicle traction control consists of antiskid braking and anti-spin acceleration; and can enhance performance and controlling. This control focuses on maximizing tyre traction by hindering the wheels from locking during braking and from spinning during acceleration. The difference between ABS and TCS is as a result of the fact that ABS only acts during braking or deceleration while TCS only acts during acceleration. Traction control system is only effective when in the engine is in acceleration mode, and will stop operating if the brake is applied by the operator.

Traction, braking and stability control of automobiles are mostly represented by the wheel slip control. Wheel slip dynamics depends on the following: system parameters, the nature of the road, normal reaction and vehicle speed. The conditions of the road have an influence on the coefficient of friction between the road surface and the tyre, which, in turn, influences the traction force, as well as the slip and reaction force acting on a tyre [11]. The dynamics of the vehicle are influenced by the road slope and the control problem becomes more difficult when the vehicle starts on a slope [17]. The traction force in the longitudinal direction depends on the adhesion coefficient between the tyre and the road surface in contact. The road friction coefficient is also known to depend on the wheel slip as well as the surface condition of the tyre and the road surface. The wheel velocity and the vehicle velocity are related by a nonlinear function referred to as the wheel slip. Vehicle traction control can be realized by wheel slip because of the dependency of the longitudinal traction force on the wheel slip.

Poor road conditions such as sandy, mud, icy, snow, water etc. can adversely affect vehicle acceleration and braking (or deceleration). For example, a vehicle could spin during acceleration or skid when wheel-lock up during braking. The difference between the vehicle speed and wheel speed is referred to as slip. When it occurs, it causes stopping distance to be longer and in sometimes the vehicle loses steering stability which can lead to vehicle crash. Previous researches on slip control for antilock braking system have been presented, yet it is desired to design a control system that is required to maintain an optimum wheel slip ratio with improved tracking performance. This paper aims to design a slip tracking control system based on feedback linearization (FBL) approach combined with two degrees of freedom proportional integral and derivative (2DOFPID) controller that will maintain an optimum wheel slip ratio with improved tracking performance.

Vehicle traction control is the control of tyre forces both in the longitudinal and lateral directions to obtain desired vehicle motion. Vehicle traction control consists of antiskid braking and anti-spin acceleration; and can enhance performance. The antiskid braking is considered in this context. The tyre traction forces come from the tyre/road interaction and they are resolved in two components: one in the longitudinal direction and the other in the lateral direction. In the lateral direction, the tyre traction force depends on the angle of the wheel slip and the control is achieved by the steering angle. The traction force in the longitudinal direction, which is considered in this work, depends on the friction coefficient between the tyre and the road surface. Controlling the forward traction force can be realized in different ways based on the control objective. This paper is concerned with controlling the wheel slip at any referenced value in order to generate a desired amount of longitudinal traction force (slip control) in antilock braking System. A quarter-car model or single tyre model is employed for the modelling.

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II. Related Works on Slip Control

Solyom [16] proposed a slip tracking approach in which the design objective is for each wheel to track a reference trajectory for the longitudinal wheel slip. A quarter-car model is used for the analysis. A gain scheduled Proportion Integral and Derivative (PID) controller was implemented for the design. Braking commenced from an initial speed of 30m/s, and the vehicle achieved stopping distances of between 36m and 41m. One of the advantages in [16] approach is exploring the accuracy of the PID controller and ease of tuning; nevertheless, the model did not consider a number of system dynamics, such as the suspension dynamics, braking actuator, and the pitching effect. Jiang and Gao [6] proposed a nonlinear PID (NPID) controller. The major modification to the linear PID is the fact that a nonlinear function is incorporated to the linear PID. The NPID control algorithm explored the robust control advantages. A comparison between the linear PID and NPID control methods showed that the NPID controller has shorter stopping distance and better velocity performance than the linear PID controller and a loop-shaping controller.

Austin and Morrey [2] in their study reported that some researchers have tried to solve the chattering problem by introducing a saturation function in place of the switching sign function for different road conditions. The introduction of the saturation function eliminates the chattering; however, it introduces a steady state error [2], [4]. Jing et al [7] proposed a moving sliding surface for the slip control based on global sliding mode control technique. The sliding surface moves from an initial condition to the desired sliding surface and as such a fast tracking of the desired slip is achieved, which is unlike the conventional SMC. The aim of technique was to eliminate the reaching phase that brings about chattering in the traditional SMC method. In addition, the work used the radial basis function of the neural network for the sliding mode controller. Simulation results obtained from the method proposed on a quarter-car model was compared with the conventional method which indicated that the proposed method reduced the chattering, but did not eliminate it.

John et al [9] proposed a hybrid system that combines feedback linearization (FBL) and PID controllers to realize the hybrid FBLPID controller for slip control in antilock braking system. It claims that the system is an active safety device in road vehicles, which during hard braking maximizes the braking force between the tyre and the road regardless of the conditions of the road. This is accomplished by regulating the wheel slip around its optimum value. Due to the high nonlinearity of the tyre and road interaction, and uncertainties from vehicle dynamics, a standard PID controller will not suffice. He therefore proposed a nonlinear control design using input-output feedback linearization approach. To enhance the robustness of the non-linear controller, an integral feedback method was employed. The stability of the controller is analyzed in the Lyapunov sense. To demonstrate the robustness of the proposed controller, simulations were conducted on two different road conditions. The results from the proposed method exhibited a more superior performance and reduced the chattering effect on the braking torque compared to the performance of the standard feedback linearization method. However, the hybrid system recorded lower performance index values than the FBL with respect to slip tracking.

Mohamed [12] performed a mathematical simulation and implementation of slip control in antilock braking system in Matlab using a Bang-Bang control technique. It employs a quarter car vehicle's model undergoing a straight line braking maneuver. The model also incorporates a hydraulic brake valve dynamics and road-tyre interaction. The road-tyre interaction model is given in the form of an empirical function (Magic formula) describing the nonlinear relation between adhesion (rolling) coefficient and wheel slip. A Bang-Bang controller was implemented with the above model for controlling wheel slip at a given desired reference value. The braking performances in both assisted Antilock Braking System (ABS) mode and non-ABS mode were evaluated by simulations. Simulated results of stopping distances were confirmed using a road test setup. The results indicate that the braking performance of automotive assisted ABS was improved significantly, the braking time advanced, and the stopping distance shorten consequently, and the braking safety of vehicle can be improved. The problem with this type of controller is that its action is on two states and cannot achieve high performance or accuracy.

Otis et al [13] formulated a slip control model for purposes of performing slip tracking of target slip. Slip modeling in antilock braking system is performed to develop a quarter car vehicle deceleration model for braking without cornering. Input-state based feedback linearization is applied to the highly non-linear developed slip control model of the antilock braking system. Input-state feedback linearization is shown to provide a transformed linear ABS model while ensuring a verifiable stable state transformation. Lie algebra is used to formalize the analysis of the linearizing transformation. Simulation results of a quarter car vehicle’s braking dynamics demonstrate the validity of the approach along with the key development of an output to state transformation that facilitates the implementation of the linearization approach as a mechatronic technique to antilock braking system control. The problem with feedback linearized controller is that it is prone to chattering.

Chankit et al [5] formulated a slip control model in order to maintain a desire slip ratio. The effectiveness of the system during braking was obtained using Simulink models. A linear discrete PI controller was used to test the effectiveness of maintaining desired slip ratio at an optimum slip of 10%. The problem of this work is that the controller introduces spikes. Another challenge in the work is the fact that the wheel viscous force and the aerodynamic force of the car were not considered.

This paper seeks to improve on the work of [5]. They did not include some essential dynamics of vehicle that can affect the braking performance of the slip controller like the wheel viscous force and the aerodynamic drag force. These dynamics are taken care of in this work to make the system more robust. The discrete time proportional and integral controller used in [5], introduces spikes during braking hence this effect has

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been eliminated by the discrete time proportional integral derivative (PID) controller used in this paper. The approach for designing the PID in this paper is analytical. The designed controller is known as two degree of freedom (2DOF) PID. This work combines feedback linearization (FBL) and a 2DOFPID controller. The 2DOFPID controller provides a better tracking performance than a conventional PID. The controller is combined with a feedback linearized (FBL) vehicle system of the Matlab/Simulink model. The main objective of this paper is to design a slip tracking control system that will minimize slip using a feedback linearization approach and a two degree of freedom proportional integral and derivative (2DOF PID) controller.

III. Methodology

A. *Dynamic Equations of a Vehicle*

In this section, the dynamic equations of a vehicle are obtained using a quarter-car or single tyre model. Fig.1 represents a quarter-car model. It is required to make the following assumptions when using a quarter-car model for vehicle forward motion analysis during braking:

1. The longitudinal dynamics of a vehicle is considered
2. Motions in vertical and lateral direction are not considered
3. The analysis assumes that a vehicle considered is braking on a straight road with a longitudinal speed of 30m/s.

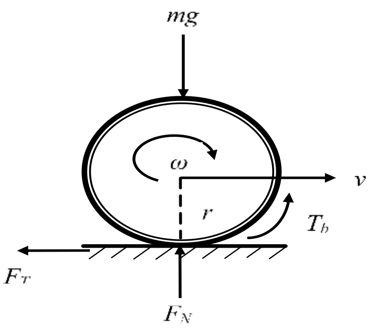

Fig. 1. Quarter-car model.

A quarter-car model is used to obtain the forward or longitudinal braking dynamics. It comprises a single tyre carrying a quarter mass, m, of the vehicle, such that the vehicle is moving with a longitudinal velocity $v(t)$ at any time, t. The wheel moves with an angular velocity of $\omega(t)$, driven by the mass, m in the direction of the longitudinal motion, before brakes are applied. Fig. 2 is an illustration of a tyre carrying a quarter mass of a car moving with forward velocity, $v(t)$ as the wheel rotates on the road surface.

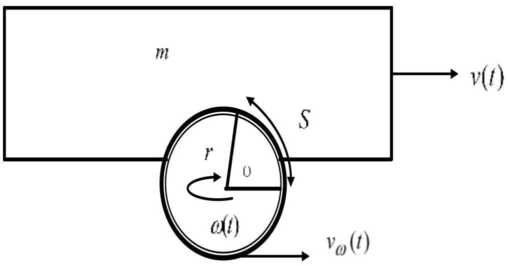

Fig. 2. A tyre carrying a quarter mass of a car.

The forward motion $v(t)$ causes the rotation of the wheel with angular velocity $\omega(t)$. The rotation of the wheel describes an arc of length, S making an angular displacement θ in radian at the centre. The entire rotational motion of the tyre is represented as follows:

$$S = r\theta \tag{1}$$

where r is the wheel radius in metre. Since $\theta = \omega t$ and $S = v_w t$, then:

$$v_w = \omega r \tag{2}$$

where v_w is the wheel forward velocity tangential to the road surface.

Tractive Force:

The opposition force to the Longitudinal (or forward) motion of the vehicle tyre generated due to friction between the tyre and the road surface is known as tractive force. It is given by:

$$F_T = \mu(\lambda) F_N \tag{3}$$

where F_T is the tractive force in Newton $\mu(\lambda)$ is the longitudinal friction coefficient which is a function of the wheel slip λ, and F_N is the normal reaction force exerted on the road.

Longitudinal Velocity of the Vehicle:

The Equation describing the vehicle forward motion can be obtained by using the laws of dynamic motion. The net force F_R acting on the vehicle is given by:

$$F_R \geq F_T + F_{drag} \tag{4a}$$

where F_R is the net force. Using the equality sign for analysis, we have:

$$F_R = F_T + F_{drag} \tag{4b}$$

The deceleration ($-a$) of the vehicle is

$$a = \frac{-1}{m}\left(F_{T} + F_{drag}\right) \tag{4c}$$

$$F_{drag} = \frac{1}{2} ADCv^2 \tag{4d}$$

where F_{drag} is the aerodynamic drag force of the vehicle A is the projected area of the vehicle, D is the density of air, v is the longitudinal velocity of the vehicle, C is the vehicle's aerodynamic friction coefficient. Equation (4c) can further be expressed as:

$$\dot{v} = -\frac{1}{m}\left[\mu(\lambda)F_N + \frac{1}{2}ADCv^2\right] \tag{4e}$$

Equation (4a) to (4e) established the effective force acting on the vehicle and the longitudinal velocity when braking.

Rotational Dynamics:

The rotational dynamic equation of the wheel is given by [9] as:

$$\dot{\omega} = \frac{1}{J}\left[r\mu(\lambda)F_N - F_w\omega - T_b(sign(\omega))\right] \tag{5}$$

where ω is the angular velocity of the wheel, J is the moment of inertia of the wheel, r is the radius of the wheel, F_w is the wheel viscous friction and T_b is the braking torque.

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Actuator Dynamic Model:

The dynamic model of hydraulic fluid lag of brake system is given by the first order transfer function [8]:

$$G(s)=\frac{k}{\tau s+1} \quad (6) \tag{6}$$

where k is the braking gain, which is a function of the brake radius, brake pad friction coefficient, brake temperature and the number of pads [1], and τ is the hydraulic torque time constant.

To compensate for the fluid lag or delay, a time delay function e^{-ST} is added to (6) and this yield:

$$T_b = e^{-ST}\frac{k}{\tau s+1}T_{ref} \tag{7}$$

Subject to the constraint: $0<T_b<T_{b\_max}$, a maximum braking torque limit T_{max} of *4000Nm* has been chosen [5].

Tyre-Friction Model:

The Pacejka friction model is very detailed, and it is the tyre-road friction description most commonly used in commercial vehicle simulators such as, for example, CarSim, Adams/Tyre, and Bikesim [14]. The Pacejka friction model is given in (8) and the parameters are defined in Table 1.

$$\mu_x = a(1-e^{-b\lambda}-c\lambda) \tag{8}$$

where a, b, c are constants

The Pacejka model describes the friction forces via static maps, which depend on different parameters. By changing the values of the parameters a, b, c in (8), many different tyre-road friction conditions can be modeled. The corresponding parameters values of $a=\vartheta_{r1}, b=\vartheta_{r2}$ and $c=\vartheta_{r3}$ are given in Table 1 for different road conditions. These values are substituted into (8).

TABLE 1. Values of the parameters for different road conditions [14].

Road condition	ϑ_{r1}	ϑ_{r2}	ϑ_{r3}
Dry asphalt	1.28	23.990	0.52
Wet asphalt	0.86	33.82	0.35
Cobblestone	1.37	6.46	0.67
Snow	0.19	94.13	0.06

B. Input-Output Linearization

The concept of feedback linearization is on the algebraic transformation of a nonlinear system dynamics into fully or partly linear one, so that linear control laws can be applied. Linear reference tracking of slip is not possible due to the nonlinearity of output slip.

$$\dot{x} = f(x)+g(x)u \tag{9}$$

$$y=h(x) \tag{10}$$

where the state variables $x=[x_1 \quad x_2]^T$ are the wheel angular velocity ω and the vehicle longitudinal velocity v (which are the state variables of the slip dynamics equation) respectively. $f,g:R^n\to R^n$ a function g which associate a plane of pairs of real numbers called vector field in R^n and y is the output slip function.

Differentiating the Output y: The feedback linearization approach is applicable to a class of nonlinear systems described by the canonical form given by (9) [3]. The basic approach is to differentiate the output until the input u appears, then design u to cancel nonlinearities.

Lie Derivative: The Lie transformation analysis is used for the linearization in this work. The preliminary mathematics of Lie derivative is presented as follows: Lie derivative of h with respect to f is a scalar function defined by $L_f h=\nabla hf$, where $\nabla h=\frac{\partial h}{\partial x}$ is the gradient of a smooth scalar function $h(x)$. The Lie derivative of h with respect to g is also defined as a scalar function given by $L_g h=\nabla hg$.

The Lie derivative is applied to (9) and (10). Differentiating y and expressing it in form of Lie derivative yields:

$$\dot{y}=h(\dot{x}) \tag{11}$$

In Lie derivative form:

$$\dot{y}=\nabla h(f+gu) \quad \text{or}$$

$$\dot{y}=L_f h(x)+L_g h(x)u \tag{12}$$

Substituting V into (12) in place of $\dot{y}$ and expressing the equation in terms of the input u gives the input transformation as:

$$u=\frac{1}{L_g h(x)}(-L_f h(x)+V) \tag{13}$$

where V is called the new input. This results in a linear relationship between y and V, that is $\dot{y}=V$.

In Antilock-braking system, the effectiveness of the braking performance of a vehicle on the road surface is enhanced by the braking torque T_b with respect to the wheel slip. The longitudinal wheel slip equation is given by:

$$\lambda=\frac{v-\omega r}{v}, v\neq 0 \tag{14}$$

Equation (14) shows no direct relationship between the output (wheel slip) λ and the input (braking torque), T_b. In order to develop a direct relationship between the wheel slip and the braking torque, (14) is differentiated with respect to time, assuming that the radius of the tyre remains constant. This provides a state equation that can be represented in canonical form so as to perform the linear transformation.

Differentiating (14) using the quotient rule from the first principles yields:

$$\frac{d\lambda}{dt}=\frac{v\frac{d(v-r\omega)}{dt}-(v-r\omega)\frac{dv}{dt}}{v^2} \tag{15}$$

$$\dot{\lambda}=\frac{r\omega}{v^2}\dot{v}-\frac{r}{v}\dot{\omega} \tag{16}$$

Substituting (3), (4e) and (5) into (16) yields the following:

$$\dot{\lambda}=-\frac{r}{v}\left(\frac{rF_T-T_b}{J}\right)-\frac{r\omega}{v^2}\left(\frac{F_T}{m}\right) \tag{17}$$

Equation (17) can be further simplified, knowing that $F_T=\mu(\lambda)F_N$, this yields the slip dynamics as

$$\dot{\lambda} = -\frac{1}{v}\left(\frac{\omega}{mv} + \frac{r^2}{J}\right)\mu(\lambda)F_N + \frac{r}{Jv}T_b \tag{18}$$

Equation (18) can be expressed in terms of the state variables to give its equivalent state Equation as:

$$\dot{x} = -\frac{1}{x_2}\left(\frac{x_1}{mx_2} + \frac{r^2}{J}\right)\mu(x)F_N + \frac{r}{Jx_2}T_b \tag{19}$$

Equation (19) provides an initial transformation in which the output is represented in state equation. This was obtained after the input-output linearization had been achieved.

The input-output linearization is achieved as follows:

I. Differentiate the output *y* until the input *u* appears.
II. Choose input *u* to cancel the nonlinearities and proving tracking performance.
III. Study the stability of the internal dynamics.

It can be seen that a comparison of (9) and (19) shows that (19) can be represented in a canonical form as (9). where $f(x)$ and $g(x)$ are defined as:

$$f(x) = -\frac{1}{v}\left(\frac{\omega}{mv} + \frac{r^2}{J}\right)\mu(\lambda)F_N,\ g(x) = \frac{r}{Jv} \text{ and } u = T_b \text{ and } \lambda = x \tag{20}$$

Equation (20) shows that the slip dynamics of a slip control system is a single input-single output (SISO) nonlinear system of the canonical form.

Choosing control input *u*: The goal of the ABS is to track a referenced wheel slip λ_r. At this point, it can be assumed that $\dot{y} = \dot{\lambda}$ and provided $g(x) \neq 0$, the control input u can be chosen as [19]:

$$u = \frac{1}{g(x)}[-f(x) + V] \tag{21}$$

where $f(x) \equiv L_f h(x)$, $g(x) \equiv L_g h(x)$ and V is the new input, see (13).

$$\dot{\lambda} = V. \tag{22}$$

Equation (22) gives a linear relationship between the new input V and the output $\dot{\lambda}$. Therefore, the nonlinearity in (18) is cancelled and a simplified linear relationship between the output $\dot{\lambda}$ and the new (or equivalent) input V is obtained.

The design objective is to find a control input *u* that will ensure that the slip controller tracks the desired slip trajectory, while keeping all the states variables bounded. The following assumptions are necessary [18].

Assumption 1: The vehicle velocity V and wheel speed ω are measurable or observable.

Assumption 2: The desired (or reference) trajectory vector defined within a compact subset of $\mathbf{R}^{\mathbf{l}}$, is assumed to be continuous, available for measurement, and $\varepsilon \| l_d(t) \| \epsilon\ W_X$ with W_X as a known bound.

For tracking the desired output λ_r, the control law is defined by:

$$V = \lambda_r^{\,n} - k_0 e - k_1 \dot{e} \ldots\ldots - k_{n-1} e^{(n-1)} \tag{23}$$

Let the tracking error e be given as:

$$e = \lambda - \lambda_r \tag{24}$$

and let the new input be chosen as:

$$V = \dot{\lambda}_r - ke \tag{25}$$

where *k* is a positive constant. From (24) and (25), the closed-loop tracking error dynamics will be: $\dot{e} + ke = 0$ and this indicates convergence.

Internal Dynamics: If the output of an *nth* order system should be differentiated r times to generate an explicit (or linear) relationship between the output *y* and the input *u*, the system is said to have relative degree *r*.

When the relative degree of a system is the same as its order:

I. There is no internal dynamics.
II. The problem will be input-state linearization

In this paper, the first derivative of the wheel slip dynamics of (14) gives an explicit relationship between the output (wheel slip) λ and the input (braking torque T_b). Since the *nth* order of the slip control equation is equals to the relative degree *r*, it is assumed that the slip control system has no internal dynamics.

Controllability: For any controllable system of order *n*, by taking at most *n* differentiations, the control input will appear to any output (that is $r \le n$). If the control input never appears after more than *n* differentiations, the system would not be controllable [19]. The first derivative of the wheel slip in this context gives a clear relationship between the output λ and the input T_b. Hence the control input appears at the first differentiation of the output and as such the system is controllable.

C. System Configuration and Controller Design

Fig. 3 is the block diagram of the slip control system. It is a closed-loop control model which integrates a standard quarter car model, a brake actuator, and a two degree of freedom proportional plus integral plus derivative (2DOFPID) controller. Each component of the control loop represents the different dynamics and equation for the slip control implementation in Matlab/Simulink environment. The control loop is that of a standard single input-single output control system. The controller, actuator, and the quarter-car models are all in the forward path. The wheel slip, which is the output, is fed back and compared to a desired slip value, with error fed into the (2DOFPID) controller. The equation of the designed 2DOFPID controller is given in (26).

$$U(s) = k_p(\beta\lambda_r - \lambda) + k_i\frac{i}{s}(\lambda_r - \lambda) + \frac{Nk_d s}{s+N}(\gamma\lambda_r - \lambda) \tag{26}$$

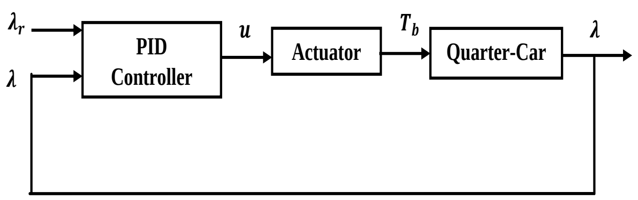

Fig. 3. Slip control system configuartion.

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The process of selecting the gains k_p, k_i and k_d of a PID controller is usually referred to as tuning. In this context, a numerical method is employed using the Simulink design optimization toolbox. The output of the plant, which is the slip (λ), is constrained to the desired step response signal with respect to design specifications in this context. The gain parameters are stated in Table 2.

The specifications from [10] and [15] have been adopted to suit the present work for the slip tracking as stated below:

i. Rise time measured between 10% to 90% of the final slip value should not be greater than 0.25secs,
ii. No wheel lock is allowed to occur for speeds above 4.0ms^{-1},
iii. Wheel lock for a period of 0.2sec is allowed.
iv. Over shoot should be ≤ 5%.
v. Stopping distance should not be greater than 60m.

TABLE 2. Parameters of the PID.

Parameter	value
k_p	2000
T_i	0.02
T_d	0.0005
N	10

D. Optimal Performance Evaluation

The main objective of slip control in antilock braking system is to maintain the maximum brake force to reduce the brake distance. The braking distance can be chosen as the cost function to define the optimality of the system. In this context, the braking distance is compared from the initial speed braking v_0 to final speed of t_f, and then the cost function can be defined as below:

$$d = \int_{v0}^{t_f} v dt \tag{27}$$

The performance of the controller was evaluated based on the integral squared error [ISE] of the slip, the integral squared control input and the stopping distance.

It is expected that performance should be a small deviation from the reference slip, less effective braking torque and a reduced stopping distance.

$$\text{ISE of slip} = \int_{v_0}^{t_f} (\lambda_r - \lambda)^2 dt \tag{28}$$

$$\text{ISCI} = \int_{v0}^{t_f} T_b^2 dt \tag{29}$$

Equations (28) and (29) are the mathematical expression for integral square error and integral square control input, ISCI.

System Parameters and Numerical Values:

In this paper, the definitions and the numerical values of the parameters used for this work are stated in Table 3. The parameters are obtained from [12], [5] and [9] and are adopted for simulation performed in this context.

TABLE 3. System parameters and numerical values.

Symbol	Description	Value	Unit
M	Quarter-car mass	447.5	Kg
J	Moment of inertia	1.7	Kgm^2
r	Wheel Radius	0.308	m
F_w	Wheel friction coefficient	0.08	Nms/rad
τ	Hydraulic time constant	0.0143	S
G	Gravitational acceleration	9.81	m/s^2
λ_r	Desired slip	0.1	Dimensionless
v_0	Initial vehicle speed	30	m/s
K	Hydraulic gain	1.0	Constant
C	Drag coefficient	0.539	Dimensionless
A	Projected area	2.04	m^2
D	Air density	1.225	Kg/m^3
A_p	Actuator pole	70	Dimensionless

IV. RESULTS AND DISCUSSION

A. Results Overview

In order to evaluate the performance of the 2DOFPID controller on different road conditions, simulations were implemented in Matlab/Simulink environment. The model parameters used in the simulations are listed in Table 3. Simulations are performed on a straight-line braking function, braking started at an initial forward or longitudinal speed of 30m/s respectively.

Simulations are conducted for dry asphalt and wet asphalt road surfaces. The friction coefficients corresponding to dry asphalt and wet asphalt road conditions are presented in Table1. The simulations are terminated at speed of about 0.5m/s and the braking torque limited to 4000*Nm*. This is because as the wheel speed approaches zero, the slip becomes unstable, therefore the antilock braking system (ABS) should disengage at low speeds to allow the vehicle to come to stop.

Simulation on Dry Asphalt Road Condition:

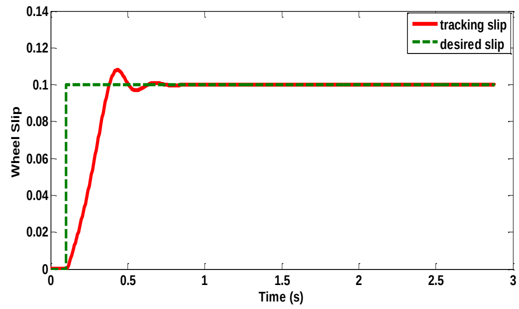

Fig. 4. Wheel slip against time.

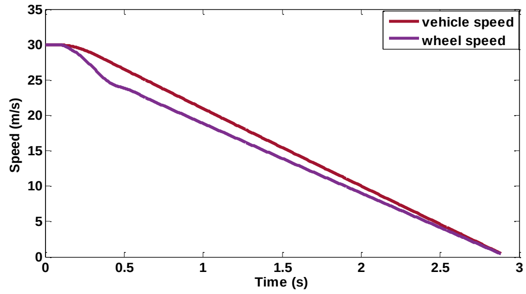

Fig. 5. Speed against time.

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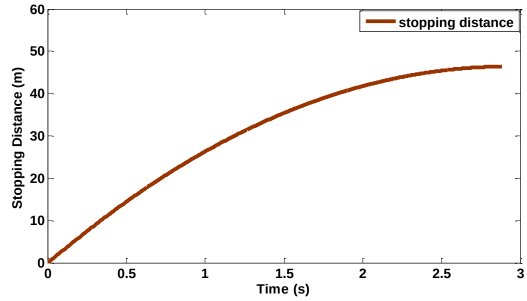

Fig. 6. Stopping distance against time.

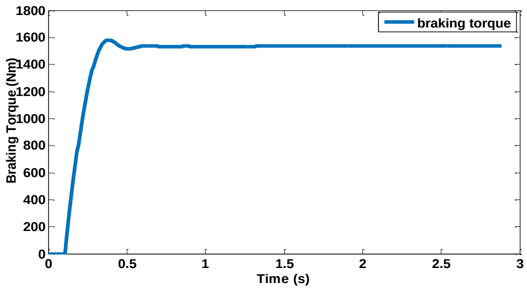

Fig. 7. Braking torque against time.

TABLE 4. Simulation results on dry asphalt road condition.

Performance index	Specification	Controller performance
Integral square error (ISE)	Minimum	0.00113
Integral square input $(Nm)^2 \times 10^6$	Minimum	6.15
Stopping distance (m)	≤ 60	44.75

Simulation on Wet Asphalt Road Condition:

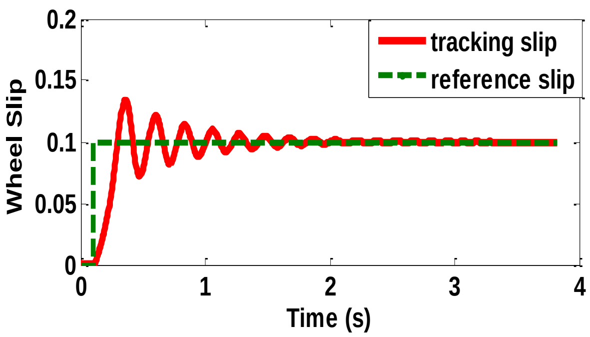

Fig. 8. Slip against time.

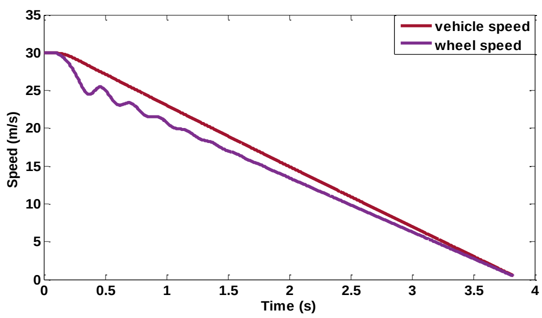

Fig. 9. Speed against time.

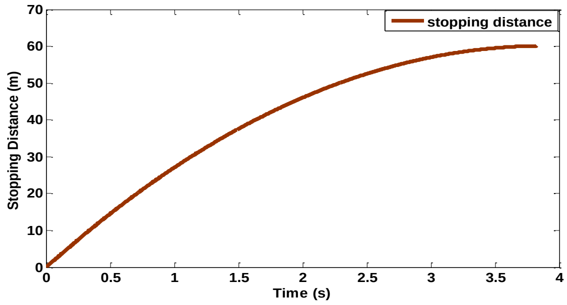

Fig. 10. Stopping distance against time.

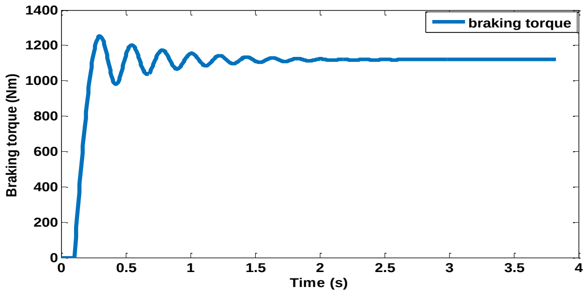

Fig. 11. Braking torque against time.

TABLE 5. Simulation results on dry asphalt road condition.

Performance index	Specification	Controller performance
Integral square error (ISE)	Minimum	0.00113
Integral square input $(Nm)^2 \times 10^6$	Minimum	4.453
Stopping distance (m)	≤ 60	57.38

TABLE 6. Overall simulation results.

Quantity	Dry asphalt road condition	Wet asphalt road condition
Wheel slip ratio	10%	10%
Rise time	0.2s	0.2s
Stopping time	2.813s	3.709s
Stopping distance	44.75m	57.38m
Braking torque	1537Nm	1121Nm

Performance Analysis:

The performance analysis is performed in this work with regard to the stopping distances achieved on different road surface conditions and when the slip controller is not integrated with the antilock braking system (ABS).The performance of the system with and without controller can be evaluated by letting the ABS-assisted stopping distance to be compared to those simulated with non-ABS assisted. In order to make easy this comparison, the performance improvement is calculated as:

$$ABS-(SDI)=\frac{SD_{non-ABS}-SD_{ABS}}{SD_{non-ABS}}\times 100\% \tag{30}$$

The result obtained from the analysis is presented in Table 7.

TABLE 7. Performance analysis with respect to stopping distance.

Road surface condition	SD_{ABS} (m)	$SD_{non-ABS}$ (m)	Improvement (%)
Dry Asphalt	44.75	135.2	70
Wet Asphalt	57.38	135.2	58

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B. Discussion

For the purpose of simulation of the system with the selected gains, dynamic model of the plant and controller was analyzed in Simulink. The simulation was performed to determine the degree to which the system operation correlates with the required performance specification.

In general, the two degree of freedom proportional integral and derivative controller (2DOFPID) is able to maintain an optimal wheel slip ratio of 10% for both road surface conditions. A comparison of the slip plots, Fig.4 and 8, for dry asphalt road surface and wet asphalt road surface conditions reveals cycling in the response plot of the controller on wet asphalt road surface. This is so because the wet asphalt road surface is a low friction surface. In both road surface conditions, the controller achieves a rise time of 0.2s.

In terms of the stopping distance, the controller achieved distance of lower value on dry asphalt road surface than the wet asphalt road surface. On dry asphalt road surface, the stopping distance is 44.75m with an improvement of 70%. On wet asphalt road surface, the stopping distance is 57.38m with an improvement of 58%.

The slip tracking and stopping distance are result of the effective braking torque. The braking torque is limited to 4000Nm. The 2DOFPID controller accomplished the braking using effective torques of 1537Nm and 1121Nm on dry asphalt road surface and wet asphalt road surface respectively which are of lower values than the maximum allowable torque for the road conditions considered in this context. This braking force is expected because the dry asphalt road is a high friction road surface and the wet asphalt road surface is a low friction road surface.

V. CONCLUSION

This paper has presented linear slip control for improved antilock braking system. In order to realize the objective of this paper, the dynamic equations of a braking vehicle were obtained using a quarter-car model. The dynamic equations were modeled using Simulink blocks. A two degree of freedom proportional integral and derivative (2DOFPID) controller was designed in Matlab/Simulink environment. The selection process for the parameters of the controller used in this context was quite time-consuming. However, a numerical method was employed to select the gains of the controller which were constrained to the required step response signal of the output using the Simulink design optimization toolbox. The designed controller was integrated with the dynamic equations of a car. Simulation of the braking performance of a car was conducted in Matlab/Simulink environment.

The simulation includes the standard dynamics of a quarter-car, the dynamics of the hydraulic brake system, and the control algorithm. The quarter car and the actuator are modeled in continuous time, while the controller is implemented in discrete time. This is referred to as hybrid system. Furthermore, the slip control system was improved to provide a robust tracking performance of a referenced slip and thereby maintaining an optimal slip ratio of 10%.

The main importance of slip control in antilock braking system is to minimize slip by improving the traction force between the tyre and the road surface. This invariably reduces the vehicle stopping distance when braking under severe road condition or wheel lock. In fact, the simulation results show that the braking performance of the assisted antilock braking system was significantly improved, the braking time reduced, and the stopping distance reduced consequently. The braking safety of the vehicle can be improved considering the fact that a 2DOFPID controller provides better tracking performance, eliminate large disturbances and noise such as chattering. It also improve system accuracy, steady state error and handle the large transport delays in the system.

The purpose for each road surface considered was to keep the output of the system on the trajectory. In each case the controller was able to bring the system back to the trajectory in few seconds, which signifies a robust control system.

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