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IFπGP Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

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Abstract*— *In this paper we have introduced intuitionistic fuzzy π generalized pre continuous mappings and some of their basic properties are studied.

Keywords*— *Intuitionistic fuzzy topology, intuitionistic fuzzy π generalized pre closed sets, intuitionistic fuzzy π generalized pre continuous mappings.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [13] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduce intuitionistic fuzzy π generalized pre continuous mappings and studied some of their basic properties. We arrive at some characterizations of intuitionistic fuzzy π generalized pre continuous mappings.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x) \rangle / x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x) \rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$. Also for the sake of simplicity, we shall use the notation $A = \{\langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle\}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1 \rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$\text{int}(A) = \cup\{G / G \text{ is an IFOS in X and } G \subseteq A\}$,

$\text{cl}(A) = \cap\{K / K \text{ is an IFCS in X and } A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition 2.5:[7] An IFS $A = \{\langle x, \mu_A, \nu_A \rangle\}$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$,

(ii) *intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,

(iii) *intuitionistic fuzzy regular open set* (IFROS in short) if $A = \text{int}(\text{cl}(A))$.

The family of all IFOS (respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IFαO(X), IFRO(X)).

Definition 2.6:[7] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) *intuitionistic fuzzy semi closed set* (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,

(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $\text{cl}(\text{int}(\text{cl}(A)) \subseteq A$,

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(iii) *intuitionistic fuzzy regular closed set* (IFRCS in short) if A = cl(int((A).

The family of all IFCS (respectively IFSCS, IFαCS, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IFαC(X), IFRC(X)).

Definition 2.7:[11] An IFS A in an IFTS (X, τ) is an
(i) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X.
(ii) *intuitionistic fuzzy regular generalized closed set* (IFRGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFROS in X.

Definition 2.8:[10] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

Definition 2.9: [10] An IFS A is said to be an *intuitionistic fuzzy generalized semi open set* (IFGSOS in short) in X if the complement A^c is an IFGSCS in X.

The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.10:[8] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy π generalized semi closed set* (IF GSCS in short) if scl(A) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ).

Result 2.11:[8] Every IFCS, IFGCS, IFRCS, IFαCS , IFGSCS is an IFπGSCS but the converses may not be true in general. Every IFαGCS is IFGSCS but the converse is need not be true.

Definition 2.12: [9] An IFS A is said to be an *intuitionistic fuzzy alpha generalized open set* (IFαGOS in short) in X if the complement A^c is an IFαGCS in X.

The family of all IFαGCSs (IFαGOSs) of an IFTS (X, τ) is denoted by IFαGC(X) (IFαGO(X)).

Definition 2.13:[5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B)$ ∈ IFO(X) for every B ∈ σ.

Definition 2.14: [7] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be
(i) *intuitionistic fuzzy semi continuous* (IFS continuous in short) if $f^{-1}(B)$ ∈ IFSO(X) for every B ∈ σ.
(ii) *intuitionistic fuzzy α- continuous* (IFα continuous in short) if $f^{-1}(B)$ ∈ IFαO(X) for every B ∈ σ.
(iii) *intuitionistic fuzzy pre continuous* (IFP continuous in short) if $f^{-1}(B)$ ∈ IFPO(X) for every B ∈ σ.

Result 2.15:[7] Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping.

Definition 2.16:[6] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy γ continuous* (IFγ continuous in short) if $f^{-1}(B)$ is an IFγOS in (X, τ) for every B ∈ σ.

Definition 2.17:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an *intuitionistic fuzzy generalized continuous* (IFG continuous in short) if $f^{-1}(B)$ ∈ IFGCS(X) for every IFCS B in Y.

Result 2.18:[10] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.17:[9] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ).

III. INTUITIONISTIC FUZZY π GENERALIZED PRE CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy π generalized pre continuous mapping and studied some of its properties.

Definition 3.1: A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy π generalized pre continuous* (IFπGP continuous in short) if $f^{-1}(B)$ is an IFπGPCS in (X, τ) for every IFCS B of (Y, σ).

Example 3.2: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.3, 0.1), (0.7, 0.6) ⟩, T_2 = ⟨ y, (0.8, 0.7), (0.2, 0.1) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGP continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence f is an IFπGP continuous mapping.

Example 3.4: Let X = {a, b}, Y = {u, v} and T_1 = ⟨ x, (0.2, 0.3), (0.8, 0.7) ⟩, T_2 = ⟨ y, (0.8, 0.7), (0.2, 0.2) ⟩. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = { $0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

The IFS $A = \langle y, (0.2, 0.2), (0.8, 0.7) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFCS in X. Therefore, f is an IFπGP continuous mapping but not an IF continuous mapping.

Theorem 3.5: Every IFα continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let $f: (X, \tau) \to (Y,\sigma)$ be an IFα continuous mapping. Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence, f is an IFπGP continuous mapping.

Example 3.6: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $T_1 = \langle x, (0.1, 0.2), (0.5, 0.6) \rangle$, $T_2 = \langle y, (0.6, 0.5), (0.2, 0.3) \rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.2, 0.3), (0.6, 0.5) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFαCS in X. Then f is IFπGP continuous mapping but not an IFα continuous mapping.

Theorem 3.7: Every IFG continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let $f: (X, \tau) \to (Y,\sigma)$ be an IFG continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGCS in X. Since every IFGCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence f is an IFπGP continuous mapping.

Example 3.8: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $T_1 = \langle x, (0.2, 0.4), (0.4, 0.6) \rangle$, $T_2 = \langle y, (0.7, 0.8), (0.1, 0.2) \rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.1, 0.2), (0.7, 0.8) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFGCS in X. Then f is IFπGP continuous mapping but not an IFG continuous mapping

Theorem 3.9: Every IFR continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let $f: (X, \tau) \to (Y,\sigma)$ be an IFR continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence f is an IFπGP continuous mapping.

Example 3.10: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $T_1 = \langle x, (0.1, 0.1), (0.8, 0.9) \rangle$, $T_2 = \langle y, (0.7, 0.8), (0.1, 0.2) \rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.1, 0.2), (0.7, 0.8) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFRCS in X. Then f is IFπGP continuous mapping but not an IFR continuous mapping

Theorem 3.11: Every IFP continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let $f: (X, \tau) \to (Y,\sigma)$ be an IFP continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFPCS in X. Since every IFPCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence f is an IFπGP continuous mapping.

Example 3.12: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $T_1 = \langle x, (0.2, 0.3), (0.6, 0.7) \rangle$, $T_2 = \langle y, (0.5, 0.6), (0.3, 0.4) \rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.3, 0.4), (0.5, 0.6) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFPCS in X. Then f is IFπGP continuous mapping but not an IFP continuous mapping

Theorem 3.13: Every IFαG continuous mapping is an IFπGP continuous mapping but not conversely.
Proof: Let $f: (X, \tau) \to (Y,\sigma)$ be an IFαG continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFαGCS in X. Since every IFαGCS is an IFπGPCS, $f^{-1}(A)$ is an IFπGPCS in X. Hence, f is an IFπGP continuous mapping.

Example 3.14: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $T_1 = \langle x, (0.5, 0.6), (0.5, 0.4) \rangle$, $T_2 = \langle y, (0.6, 0.5), (0.4, 5) \rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.4, 0.5), (0.6, 0.5) \rangle$ is IFCS in Y. Then $f^{-1}(A)$ is IFπGPCS in X but not IFαGCS in X. Then f is IFπGP continuous mapping but not an IFαG continuous mapping.

The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts' means continuous.

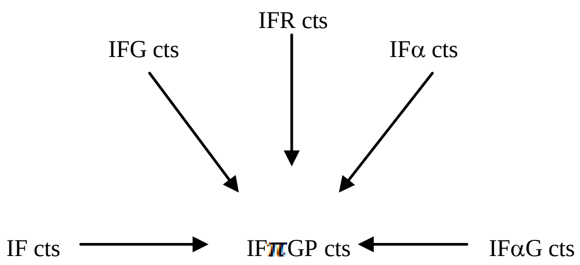

Fig.1 Relation between intuitionistic fuzzy generalized b continuous mappings and other existing intuitionistic fuzzy mappings.
None of them is reversible.

IV. CONCLUSION

In this paper we have introduced intuitionistic fuzzy πgeneralized Pre continuous mappings and studied some of its basic properties. Also we studied the relationship between intuitionistic fuzzy generalized continuous mappings and some of the intuitionistic fuzzy continuous mappings already exist.

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