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Unsteady MHD Flow between Two Parallel Plates with Slip Flow Region and Uniform Suction at One Plate

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Abstract— *The effects of Unsteady MHD flow between two parallel plates with slip flow region and uniform suction at one plate is studied here. A similar solution of the governing equations for fully developed flow is obtained. The axial and transverse velocity of fluid, and pressure distribution were presented. Analytical expression is given for velocity field and effects of various parameters entering the problem are discussed with the help of graphs.*

Keywords— *Uniform suction, slip flow region, porous medium, MHD and pressure gradient.*

I. INTRODUCTION

Magnetohydrodynamics plays an important role in power generation, space propulsions, cure of diseases, control of thermonuclear reactor, boundary layer control in the field of aerodynamics. In past few years several simple flow problems associated with classical hydrodynamics have received new attention within the more general context of hydrodynamics. To study such applications which are closely associated with magneto-chemistry requires a complete understanding of the equation of state and transfer properties such as diffusion, the shear stress, thermal conduction, electrical conduction, radiation. Some of these properties will undoubtedly be influenced by the presence of external magnetic field. The flow through porous media has become an important topic because of the recovery of crude oil from pores of reservoir rocks. From the primitive years mass transfer plays an impotent role in vaporization of ocean, burning of pool of oil, spray drying, leaching and abolition of a meteorite. Rapits (1) have studied hydrodynamic free convection flow through a porous medium between two parallel plates. Ananda Rao, Satyanarayana Murthy and Ramana Rao (2) have studied hydrodynamic flow and heat transfer in a saturated porous medium between two parallel porous walls in a rotating system. Hassaninen et.al [3] has investigated the magnetic flow through porous medium between two infinite plates. S.M.Cox [4] has considered the two dimensional flow of viscous fluid in a channel with porous wall. Many research works concerning the magnetic flow has been obtained under different physical conditions [5-7].

In this paper, Unsteady MHD flow between two parallel plates with slip flow regions and uniform suction at one plate has been studied.

II. MATHEMATICAL FORMULATION

Let us consider an unsteady incompressible viscous fluid flow between two parallel porous plates with slip flow region in the presence of magnetic field strength H_0 applied perpendicular to the plate. The origin is taken at the centre of the channel and let x-axis and y-axis are along the plates and perpendicular to the plates respectively. 2h is the distance between the plates, one at y = -h and other at y = h. It is assumed that the plates are non-conducting and both the plates are under rest with constant suction in one plate. The fluid is having constant viscosity. The fluid is driven by a uniform pressure gradient parallel to the plate of the channel. So the governing equations of the flow are as follows:
The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The equation of motion is

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial P}{\partial x}=\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2 u}{\rho}-\frac{\mu u}{K'\rho} \tag{2}$$

$$\frac{\partial v}{\partial t}+\frac{1}{\rho}\frac{\partial P}{\partial y}=\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

With the boundary conditions:

$$\begin{aligned} u(x,y)&=h_1\frac{\partial u(x,y)}{\partial y}, \quad v(x,y)=0, at\ y=h \\ u(x,y)&=h_2\frac{\partial u(x,y)}{\partial y}, \quad v(x,y)=-V_0,\ at\ y=-h \end{aligned} \tag{4}$$

III. METHOD OF SOLUTION

To solve the above equations from (1) to (3) with the boundary conditions, let us assume the following solutions:

$$\left.\begin{aligned} u&=u(x,y)e^{-i\omega t}, \\ v&=v(x,y)e^{-i\omega t}, \\ \text{and } P&=P(x,y)e^{-i\omega t}, \end{aligned}\right\} \tag{5}$$

Then the equations (1) to (3) reduce to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

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$$i\omega u = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu u}{K'\rho} \quad (7)$$

$$i\omega v = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad (8)$$

With the boundary conditions:

$$\left.\begin{aligned} u(x,h) &= h_1 \frac{\partial u(x,y)}{\partial y}, \qquad v(x,h) = 0 \\ u(x,-h) &= h_2 \frac{\partial u(x,-h)}{\partial y}, v(x,-h) = -V_0, \end{aligned}\right\} \quad (9)$$

Let $\eta = \frac{y}{h}$ is the dimensionless distance, $v = \frac{\mu}{\rho}$ and stream function $\psi(x,y)$ satisfying equation (6) is defined by

$$\left.\begin{aligned} u(x,y) &= \frac{\partial \psi}{\partial y} \text{ and } v(x,y) = -\frac{\partial \psi}{\partial x} \\ \psi(x,y) &= (hu_0 - v_0 x) f(\eta) \end{aligned}\right\} \quad (10)$$

Then equations (7) to (8) reduce to

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = (hu_0 - v_0 x)\left(i\omega f' - \frac{v}{h^2} f''' + \frac{\sigma B_0^2}{\rho} f' + \frac{v}{K'} f'\right) \quad (11)$$

$$-\frac{1}{\rho h}\frac{\partial P}{\partial \eta} = \left(i\omega v_0 f' - \frac{v v_0}{h^2} f''\right) \quad (12)$$

Differentiating equation (11) w.r.t η and equation (12) w.r.t x, and then eliminating P, we get

$$\frac{\partial}{\partial \eta}\left(i\omega f' - \frac{v}{h^2} f''' + \frac{\sigma B_0^2}{\rho} f' + \frac{v}{K'} f'\right) = 0 \quad (13)$$

Integrating equation (13), we have

$$f''' - \left(\alpha^2 h^2 + M^2 + \frac{1}{K}\right) = Q \quad (14)$$

with boundary conditions

$$\left.\begin{aligned} f'(1) &= h_1 f''(1), \qquad f'(1) = 0 \\ f'(-1) &= h_{12} f''(-1), \quad f'(-1) = -1 \end{aligned}\right\} \quad (15)$$

where Q is a constant, α is the oscilating parameter, M is the magnetic parameter, K is the porosity parameter and $\alpha = \frac{i\omega}{v}$, $M = \frac{\sigma B_0^2 h^2}{\mu}$, $\frac{1}{K} = \frac{h^2}{K'}$.

Solving equation (14) by using boundary condition (15), we get

$$f = c_1 + c_2 e^{-\sqrt{M_1}\eta} + c_3 e^{\sqrt{M_1}\eta} - c_4 \frac{\eta}{M_1} \quad (16)$$

where

$$c_1 = -\frac{1}{2} - \left(\frac{a_{11} - a_{12}}{a_{14}a_{11} - a_{12}a_{13}}\right)\cosh\left(\sqrt{M_1}\right),$$

$$c_4 = M_1\left(\frac{1}{2} - \left(\frac{a_{11} - a_{12}}{a_{14}a_{11} - a_{12}a_{13}}\right)\sinh\left(\sqrt{M_1}\right)\right)$$

$$c_2 = \left(\frac{a_{12}}{a_{12}a_{13} - a_{14}a_{11}}\right), c_3 = \frac{a_{11}}{a_{14}a_{11} - a_{12}a_{13}}$$

$$a_{13} = e^{\sqrt{M_1}} - e^{-\sqrt{M_1}} - \left(h_2 M_1 + \sqrt{M_1}\right)e^{\sqrt{M_1}} + \left(h_1 M_1 + \sqrt{M_1}\right)e^{-\sqrt{M_1}}$$

$$a_{12} = \left(h_1 M_1 - \sqrt{M_1}\right)e^{\sqrt{M_1}} + \left(h_2 M_1 - \sqrt{M_1}\right)e^{-\sqrt{M_1}}$$

$$a_{11} = \left(h_2 M_1 + \sqrt{M_1}\right)e^{\sqrt{M_1}} + \left(h_1 M_1 + \sqrt{M_1}\right)e^{-\sqrt{M_1}}$$

$$a_{14} = e^{-\sqrt{M_1}} - e^{\sqrt{M_1}} - \left(h_2 M_1 - \sqrt{M_1}\right)e^{\sqrt{M_1}} + \left(h_1 M_1 - \sqrt{M_1}\right)e^{-\sqrt{M_1}}$$

So the Axial velocity

$$u = \frac{1}{h}(hu_0 - v_0 x)e^{-i\omega t}\left(c_3\sqrt{M_1}e^{\sqrt{M_1}\eta} - \sqrt{M_1}c_2 e^{-\sqrt{M_1}\eta} - \frac{c_4}{M_1}\right) \quad (17)$$

The Transverse velocity

$$v = v_0 e^{-i\omega t}\left(c_1 + c_2 e^{-\sqrt{M_1}\eta} + c_3 e^{\sqrt{M_1}\eta} - c_4\frac{\eta}{M_1}\right) \quad (18)$$

Integrating $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial \eta}d\eta$ and by using equations (11) to (12), we get the Pressure Distribution as follows:

$$P = \left(xhu_0 - v_0\frac{x^2}{2}\right)\left[\begin{aligned} &\frac{\rho v}{h^2}\left(M_1^{\frac{3}{2}}c_3 e^{\sqrt{M_1}\eta} - M_1^{\frac{3}{2}}c_2 e^{-\sqrt{M_1}\eta}\right) \\ &-\left(\sigma B_0^2 + i\omega\rho + \frac{\rho v}{K}\right)f'(\eta) \end{aligned}\right] + \left[\begin{aligned} &-i\omega\rho h v_0\left\{2c_1\eta + \frac{2}{\sqrt{M_1}}(C_3 - C_2)\sinh\left(\eta\sqrt{M_1}\right)\right\} \\ &+\frac{v\rho v_0}{h}f' \end{aligned}\right] \quad (19)$$

IV. Graphical Results and Discussion

In this paper, the Unsteady MHD flow between two parallel plates with slip flow region and uniform suction at one plate has been discussed. Analytical solutions of this problem are obtained and the outcome is illustrated graphically and flow characteristics have been studied. In order to have physical correlations, we have to choose suitable values of flow parameters.

Axial Velocity profiles: The axial velocity profiles are depicted in Figs 1-4. Figure 1 shows the effect of the parameter M on axial velocity at any point of the fluid between the plates, when $u_0 = 0.5$, $x = 1$, $h_1 = 0.2$, $h_2 = 0.2$ and $K = 0.1$. It is noticed that the axial velocity decreases with the increase of magnetic parameter (M) from plate one to plate two.

Figure 2 shows the effect of the parameters u_0 *and* x on axial velocity at any point of the fluid between the plates, when $M = 2$, $h_1 = 0.2, h_2 = .2$ and $K = 0.1$. It is noticed that the axial velocity increases with the increase of initial velocity (u_0) and initial position (x) from plate one to plate two.

Figure 3 shows the effect of the parameters h_1 and h_2 on axial velocity at any point of the fluid between the plates,

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when $u_0 = 0.5$, $x = 1$, $M = 2$ and $K = 0.1$. It is noticed that the axial velocity decreases with the increase of slip flow parameters h_1 and h_2 from plate one to plate two.

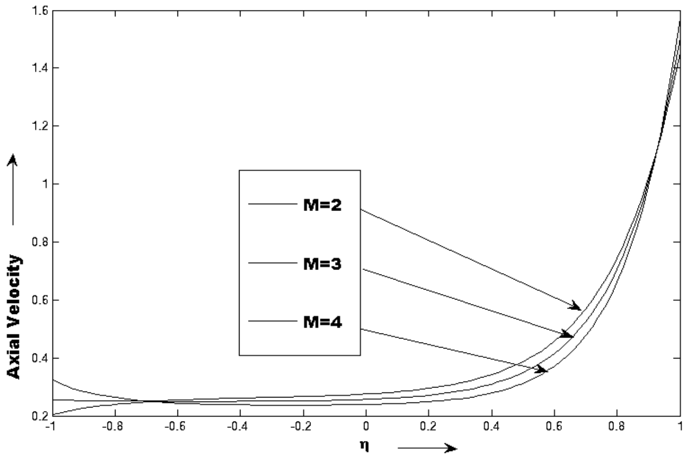

Fig. 1. Effect of M on axial velocity profile, when $u_0 = 0.5, x = 1, h_1 = 0.2, h_2 = .2$ and $K = 0.1$

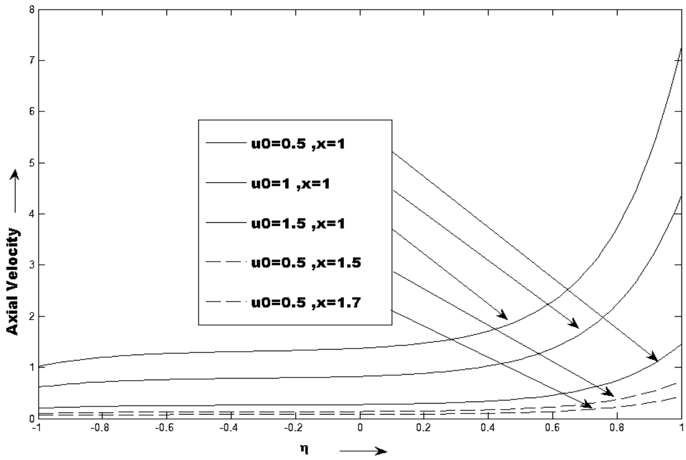

Fig. 2. Effect of u_0 and x on axial velocity profile, when $M = 2, h_1 = 0.2, h_2 = .2$ and $K = 0.1$

Figure 4 shows the effect of the parameter K on axial velocity at any point of the fluid between the plates, when $M = 2$, $h_1 = 0.2$ and $h_2 = .2$. It is noticed that the axial velocity increases with the increase of porous parameter (K) from plate one to plate two.

Transverse Velocity Profiles:

Figure 5 shows the effect of the parameter M on transverse velocity at any point of the fluid between the plates, when $h_1 = 0.2$, $h_2 = 0.2$ and $K = 0.1$. It is noticed that the transverse velocity initially decreases and then increases with the increase of magnetic parameter (M) from plate one to plate two.

Figure 6 shows the effect of the parameter K on transverse velocity at any point of the fluid between the plates, when $h_1 = 0.2$, $h_2 = .2$ and $M = 2$. It is noticed that the transverse velocity initially decreases and then increases with the increase of porous parameter (K) from plate one to plate two.

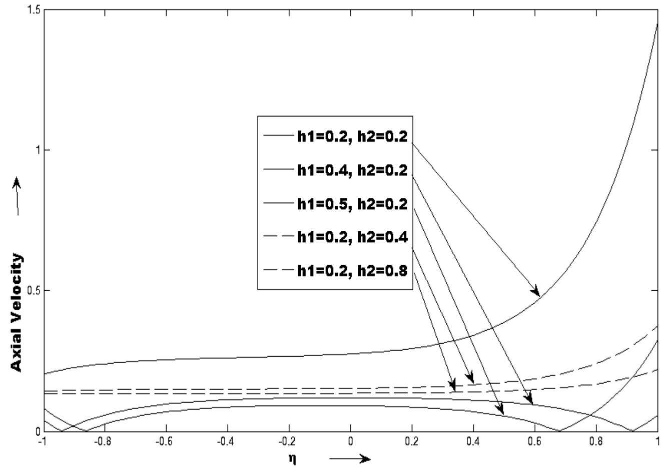

Fig. 3. Effect of h_1 and h_2 on axial velocity profile, when $M = 2, u_0 = 0.5, x = 1$ and $K = 0.1$

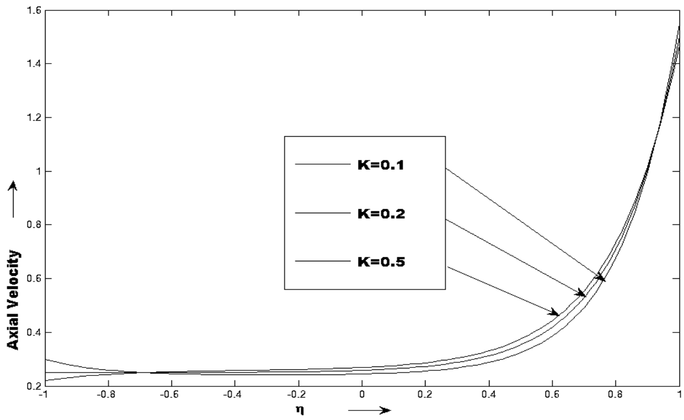

Fig. 4. Effect of K on axial velocity profile, when $M = 2, u_0 = 0.5, x = 1$ and $M = 2$

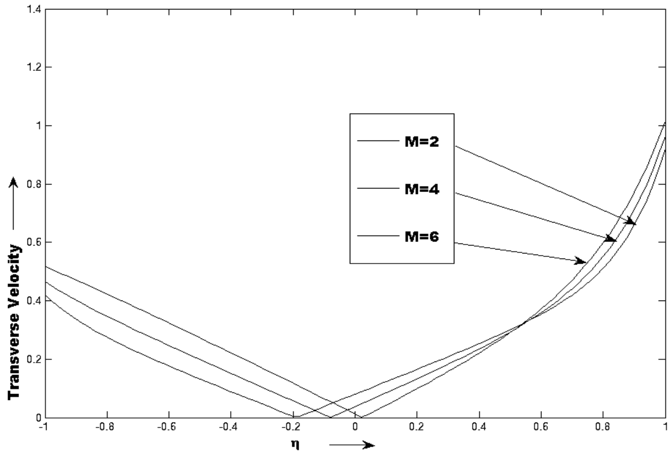

Fig. 5. Effect of M on transverse velocity profile, when $h_1 = 0.2, h_2 = .2$ and $K = 0.1$

Figure 7 shows the effect of the parameters h_1 and h_2 on transverse velocity at any point of the fluid between the plates,

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when $M = 2$ and $K = 0.1$. It is noticed that the transverse velocity initially decreases and then increases with the increase of slip flow parameters h_1 and decreases with the increase of slip flow parameter h_2 from plate one to plate two.

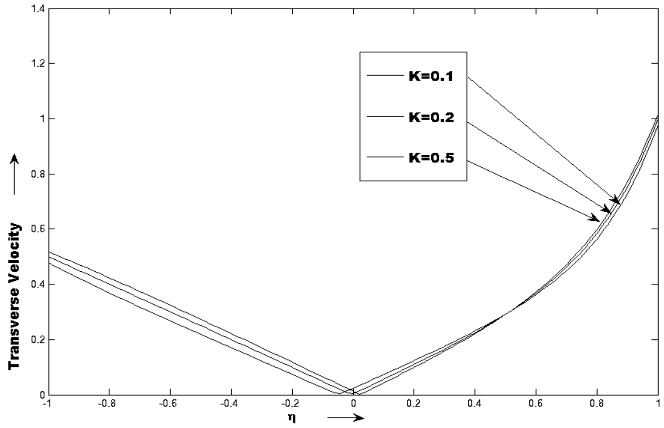

Fig. 6. Effect of K on transverse velocity profile, when $h_1 = 0.2, h_2 = .2$ and $M = 2$

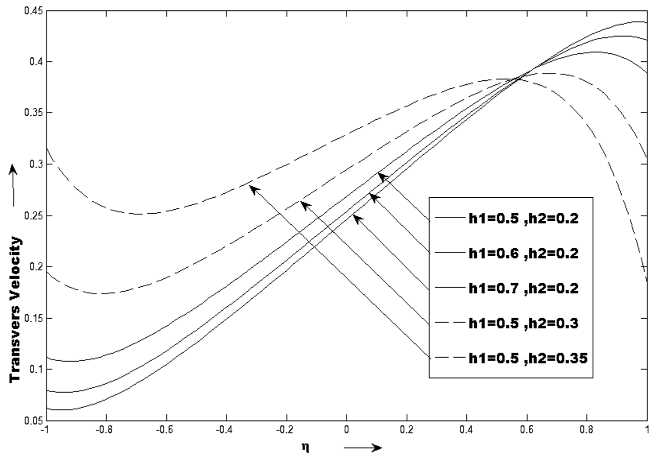

Fig. 7. Effect of h_1 and h_2 on transverse velocity profile, when $M = 2$ and $K = 0.1$

V. Conclusion

From this paper study, the following conclusions are set out:

1. It is observed that the axial velocity increases with the increase of all parameters except Magnetic parameter (M), slip flow parameters (h_1 and h_2).
2. The Lorenz Force is dominated by the force of inertia.
3. Increase in pores of the plates increases the axial velocity whereas the transversal velocity decreases initially and increases after wards.
4. The axial velocity decreases with the increase of slip flow parameters h_1 and h_2 and Magnetic Parameter M from plate one to plate two.
5. The transverse velocity initially decreases and then increases with the increase of slip flow parameter h_1, Magnetic parameter M and Porosity parameter K, whereas decreases with the increase of slip flow parameter h_2 from plate one to plate two.

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