

Well Trajectory Optimization of Homogeneous and Heterogeneous Reservoirs by the Use of Adjoint-Based Optimization Technique

A. A. Shamsuddeen¹, H. Ismail², Z. Z. Ibrahim³

^{1, 2, 3}Petroleum Engineering Department, ATBU., Bauchi, Nigeria- 740272

Email address: ¹ashurah2013@gmail.com

Abstract— It is essential to delineate well placement and trajectory of production or injection wells, which is an important step of any field development program. Drilling a well is an expensive process in terms of money, time and effort invested. Being an expensive item, wells must be carefully studied before being drilled. In order to make this process more efficient, this study presents a computer-assisted approach of determining the optimal well trajectory using adjoint-based optimization technique.

The algorithm is based on surrounding the injection or production wells with dummy wells; a technique proposed in earlier studies. These dummy wells have a minimal rate of injection or production so as to minimize their influence on the simulated output. The sum of the gradients of the objective function with respect to the flow rate in each dummy well over the life-time of the reservoir is used to define an improved well trajectory. The whole process is repeated until the maximum net present value is achieved. In order to ensure that the newly optimized well trajectory is drillable, the algorithm restricts the curvature of the trajectory. The optimization algorithm was applied to synthetically generated homogeneous and heterogeneous reservoirs to improve a single well trajectory.

Depending on the reservoir, fluid and economic parameters, the algorithms display the adeptness to predict optimized well type and trajectory of wells. Although the optimization results showed a considerable improvement in the net present value of the project, the algorithm can get stuck in local optima.

Keywords— Adjoint- based optimization, algorithm, dummy wells, homogeneous and heterogeneous reservoirs and well trajectory.

I. INTRODUCTION

Well number, location and trajectory design is one of the crucial aspects of Field Development Program (FDP). Optimal well trajectories, with a good connectivity between the well and the reservoir, are essential to maximize the objective(s) of the FDP.

Well trajectory design is an arduous task to estimate the best ways to drill and perforate the desired reservoir layers to achieve pre-defined company objective(s). Besides, wells are an expensive part of the development phase and thus, they must be carefully studied and planned. Hence, designing a well trajectory demands an interdisciplinary team effort calling for inputs from geologists, geophysicists, reservoir engineers and drilling engineers. Many parameters affect the well trajectory placement including but not limited to field location (offshore/onshore), location of the rig, cost effectiveness, total drilling length, target zones, dogleg severity, step-out distance, basic well design, and the objectives of maximizing Recovery Factor (RF), economic

values and/or production. Based on these parameters, the well trajectory design can either be vertical, deviated or horizontal. Usually, a well trajectory is determined after numerous discussions about the aforementioned cost and efficiency factors.

Since the well locations are usually represented as integer well grid block indices in reservoir simulators, well path design is treated as a discrete optimization problem.

A. Gradient-based Well Placement Optimization

There are two notable methods of well placement optimization: gradient-based and non-gradient-based. It is advantageous to use a gradient-based optimization technique to improve the computational speed albeit, it cannot differentiate between global and local optimal solution. At the same time, it demands writing complicated computer-based codes.

The gradient-based well placement optimization was established by Zandvliet et al. (2008). The proposed method finds the optimal location of vertical wells using eight 'pseudo-wells' surrounding the well whose location has to be optimized. These pseudo-wells produce (or inject) at very low rate compared to their corresponding main production (or injection) well. The adjoint method is used to calculate the gradients of the Net Present Value (NPV) over the prescribed life span of the reservoir with respect to flowrates in pseudo-wells. This is utilized to calculate the improving direction to achieve better well placement.

Besides demonstrating an approach to determine the optimal well locations using a gradient-based optimization method, the authors also exhibited that production constraints significantly influence the well placement optimization problem. Three different production constraints were run for three different terminal times and the results demonstrated the need to use as realistic production constraints as possible to obtain the best well placement results. Although this adjoint-based optimization technique may get stuck in local optimal solution, the paper presents the significance of using adjoint method to compute improving directions for all wells in only one forward and one backward simulation.

Inspired by the work of Handels et al. (2007), Sarma et al. (2008) presented an efficient well placement method with the direct application of gradients. Contrary to Handels et al. (2007), the gradients of the objective function were calculated with respect to well location indices. Instead of the original

discrete-parameter well placement problem, a continuous approximation was defined in order to compute the gradients and thus obtain the optimal well locations with standard gradient-based algorithms and adjoint models. This continuous approximation was achieved and displayed by using a bivariate Gaussian function instead of the Dirac-delta function in the governing equations. Discretizing the equations resulted in modification to the well terms (and approximations to the original well terms) in the mass balance equations. Similar to Handels et al. (2007), the original wells were surrounded by pseudo-wells but the search direction and the step size weren't limited.

Keeping the production optimization in mind, an alternate method of optimal well placement was proposed by Wang et al. (2007). The algorithm suggested placing an injector well in each grid block (except the grid blocks containing producer wells) and then optimizing the NPV (objective function in this case). To ensure the elimination of the injector wells during the NPV optimization process, the authors added a drilling well costs term in the NPV calculation function. As the cost of drilling a well detracts from the NPV, elimination of most of the non-optimally located wells becomes inevitable. In order to further decrease the number of wells, the authors also included a bound constraint and hence constrained the problem by postulating a constant total injection rate. This constraint accompanied by a steepest ascent algorithm is used to redistribute the injection rates among the wells remaining at each iteration of the NPV optimization process to maximize NPV over a specified reservoir lifetime. Although the algorithm take numerous iteration steps to converge (only one well can be deleted in each iteration step), it demonstrated consistency and exactitude in two simple 2D reservoir cases.

B. Gradient-based Well Trajectory Optimization

A method to automate the process of optimizing the trajectory of production well was first proposed by Vlemmix et al. (2009). Motivated by the advancements in the adjoint-based well location optimization, the authors extended the method of Handels et al. (2007) to determine an optimal well trajectory in a three-dimensional reservoir model. Since optimal well trajectory is crucial to avoid gas cusping and water coning, the authors verified the method on a thin oil rim reservoir with a relatively large gas cap and aquifer.

The Vlemmix et al. (2009) method is based on surrounding each trajectory point with 'pseudo-side tracks'. These fictional 'pseudo-side tracks' have very small perforations and thus production rates. The reason that this approach was chosen over placing vertical pseudo wells in each grid block is that the effect of the side tracks on the total well behaviour including lift and well bore friction can be taken into account. The gradients of the objective function with respect to a dimensionless multiplication factor for perforations (interpreted as a 'pseudo valve representation' of an Inflow Control Valve) are used to find the improving directions for the trajectory points.

Two of the main disadvantages of the method were restriction of the trajectory movement to only two directions and restriction to optimize the well length. Additionally, the

adjoint-based optimization technique may get stuck in a local optimal solution. The method employs drillability/smoothing algorithm to ensure dogleg severity is below the predefined limit. This ensures that the final well trajectory is drillable and realistic.

C. Gradient-free Optimization

Gradient-free optimization technique is another widely used method of optimizing well placements and trajectories. Although it is a computationally demanding methodology and numerous reservoir simulations have to be executed, it can in theory, capture the global solution. Several efficient algorithms have been established so far by means of Genetic Algorithm (GA), Artificial Neural Network (ANN), Simulated Annealing, etc. to compute the optimal well placement and trajectory.

Yeten et al. (2002) presented a novel method for the optimization of the type (number of laterals), location and trajectory of nonconventional wells (NCWs). The authors presented a procedure of optimizing NCW using GA. GAs are stochastic search algorithms based on the general principles of Darwinian evolution and hence require numerous simulation runs. So as to moderate the actual number of simulations and improve the efficiency, GAs are used in conjunction with three routines: ANN, hill climber and a near-well up scaling technique. While ANN is used as a proxy to the reservoir simulator, hill climbing procedure enhances the search in the immediate neighbourhood of the solution and near-well up scaling methodology speeds up the finite difference simulation runs. Besides, the authors also successfully attempted to account to the effects of reservoir uncertainty in few cases by including numerous equiprobable geostatistical realizations of the reservoir in the calculations of the objective function.

Another method of optimizing well location was presented by Badru et al. (2003) to maximize the NPV in a field development plan (FDP). The paper presented the use of Hybrid Genetic Algorithm (HGA) in conjunction with a reservoir simulator. HGA includes GA, polytope algorithm and a kriging proxy. Although the algorithm optimizes the number and location of wells, the produced results were strongly dependent on process variables (completion, recovery process, production/injection rates, project life, etc.). Various field-scale example reservoirs presented in the paper demonstrate its worth of judging locations as good as engineering judgment.

Lee et al. (2009) presented a paper on designing economically optimized wells by GA. The authors proposed using GA with a node-based configuration to get robust and more realistic well-designs considering location, trajectories and interwell interference. The developed algorithm is capable of designing both vertical and horizontal wells (with several kick-off points) while improving the objective of the FDP. It was also duly presented that the interwell interference yields a comparatively lesser NPV than a non-interwell interference scenario.

Motivated by the proposal of Vlemmix et al. (2009), the research paper intends to further extend his proposal of using a computer-assisted process of optimizing the well trajectory. In

order to do so, the thesis establishes a new method to assist the determination of the optimal well trajectory with the aid of adjoint-based technique. The algorithm has been developed to predict improved well trajectories in three-dimensional reservoirs.

Given the initial well number, location and perforations, the first part of the paper aims at maximizing the objective function by improving the well trajectory. To maintain practicality, it has been ensured that the well trajectories are drillable by bringing dogleg severity into play.

II. METHODOLOGY

A. Formulation

This chapter introduces the concept of optimizing the trajectory of any well using the gradient method. Adjoint-based optimization has been implemented to obtain the best location for the well(s). To keep things simple, we presumed that the production well location and trajectory is known while the optimal trajectory of an injector well needs to be determined in order to optimize the objective function. The following are the principal assumptions and the overview of the whole optimization process.

(a) Assumptions

Following are the key assumptions:

- Fluid Properties: Two-phase fluid (oil-water).
- Rock Properties: Incompressible.
- Producer wells: Operated on prescribed Bottom-hole Pressure (*BHP*).
- Injector wells: Operated on prescribed rate while ensuring that the injection pressure does not exceed a predefined maximum injection pressure. If the injection pressure exceeds the maximum allowed pressure, the injector starts operating on prescribed maximum *BHP*.
- Injection rate: Pre-defined.
- Well Index: Projection well index has been used.

(b) Overview

This section gives us an overview of the process of optimizing well trajectory. As illustrated by Fig. 2.1, the whole process is started by initializing the reservoir parameters (reservoir geometry, rock and fluid property, etc.) and the simulation parameters (number of simulations, etc.). The algorithm helps us to define the number of producers and perforated grid blocks, radius of the wellbore, reservoir simulation steps, initial reservoir pressure and phase saturation. Besides, kick-off point can also be honoured by fixing the heel position as per the requirement of the user.

Once the parameterization has been completed, an initial well trajectory is obtained from the user and dummy wells are introduced. These dummy wells are used to calculate an improving direction by calculating the change in the NPV for a unit change in injection rate. In order to calculate the gradients, a forward reservoir simulation and a backward adjoint simulation are run. The maximum gradient values are used to ascertain the improving direction. It must be noted that the minimum gradient values are used to define the improving direction for the producers since the flowrate is conventionally negative in magnitude. Although the improving direction

points toward the best location for the corresponding step, it is necessary to make sure that the new trajectory is drillable. This is guaranteed by using the dogleg severity (DLS) as drillability constraint. The whole process is reiterated until a trajectory with highest objective function is reached or the termination/convergence criteria is applicable.

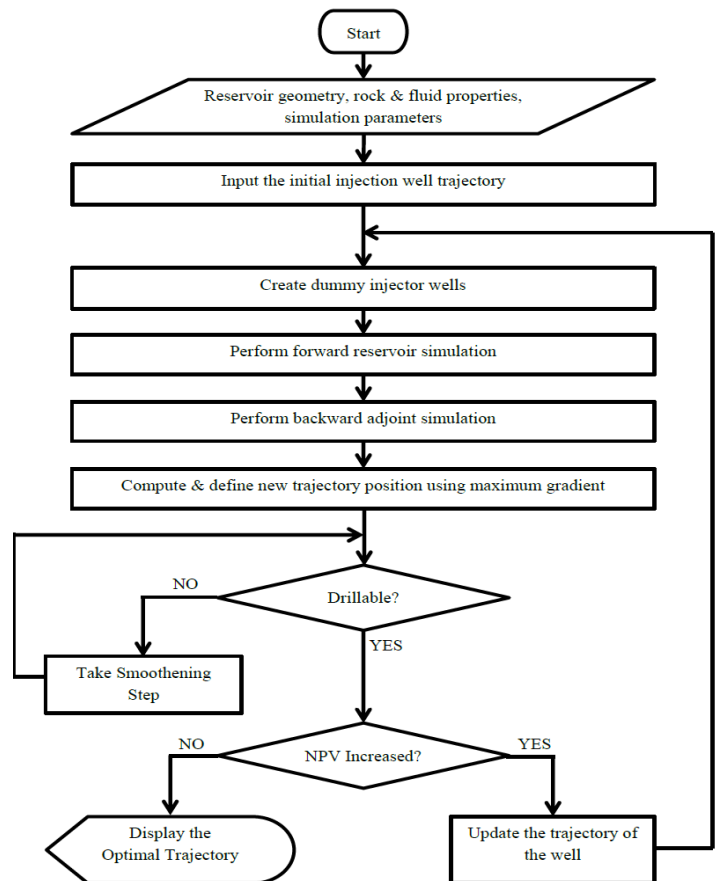


Fig. 2.1. Overview of the injection well trajectory optimization process.

The main goal of the research paper is to find the best trajectory for a well so that the objective of the FDP is maximized. The objective function can be either economic value of the project (NPV) or (RF). In the developed algorithm, the NPV of the project has been considered as the objective:

$$J = \text{NPV} \tag{2.1}$$

B. Concept of Dummy Well

In Fig. 2.1, dummy wells were introduced as an important component of the whole process. This section explains dummy wells, their implementation and purpose.

In order to find the optimal location for a trajectory, Vlemmix et al. (2009) surrounded the main production well by an imaginary pseudo-side track to all adjacent grid blocks in each direction. This helped the authors to move the trajectory points (based on the gradients) and hence move the whole trajectory towards the optimal solution. Quite similar to their approach, this paper presents a method for optimizing the trajectory of a well using dummy wells. A dummy well is

nothing but a replica of the original trajectory segment in six directions, i.e. $\pm x$; $\pm y$ and $\pm z$ as shown in Fig. 2.2. Please note that two more dummy wells are created in a three-dimensional reservoir. These imaginary dummy wells are placed at a certain distance parallel to the main trajectory well segments. This approach is advantageous to optimize both the producer and injector well trajectory in a three dimensional environment.

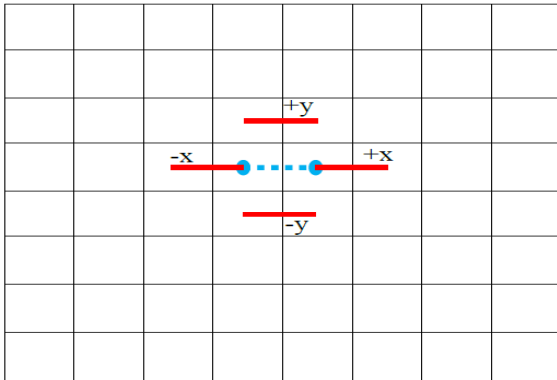


Fig. 2.2. Four dummy wells (red) surrounding two consecutive nodes of the main horizontal injector (dashed blue) well in a two-dimensional reservoir.

The dummy wells inject (or produce) at a negligible rate and hence hardly affect the overall flow behaviour of the reservoir. The beauty of using the dummy well is that it helps to find the improving direction which in turn helps to change the well trajectory in such a way that the objective function is improved. These efficient dummy wells also assist in changing the direction of the trajectory of the well.

C. Improving Direction and Revised Position

After accepting an initial well trajectory from the user, the algorithm intends to delineate a new well trajectory so that a higher NPV can be achieved. As the name suggests, an improving direction guides the well in the direction of the highest objective function value. Once the dummy wells are generated for each well segment, the sum of the gradients of the objective function with respect to the flow rate in each dummy well (parallel to a particular trajectory segment) over the life-time of the reservoir is calculated as:

$$g = \sum_{k=1}^K \frac{\partial J}{\partial u_{d,k}} \tag{2.2}$$

Here g denotes the summed gradient, $u_{d,k}$ denotes the flow rate u in dummy well d at time-step k and K denotes the total number of simulation time steps. The computed gradients are used to figure out the improving direction. The highest summed gradient value and the corresponding dummy wells are nominated (for each individual direction). These selected dummy wells and their positions are then used to define the new and improved location of the well trajectory with the help of a weighing factor β :

$$X_i^{j+1} = \beta P_j^i + (1 - \beta) X_i^j, \tag{2.3}$$

where \mathbf{x} denotes the position vector of the i^{th} trajectory point, j denotes the iteration and the weight factor, β defines the step size towards the improving direction. A weight factor of one

would imply that the new well trajectory is same as the revised position p .

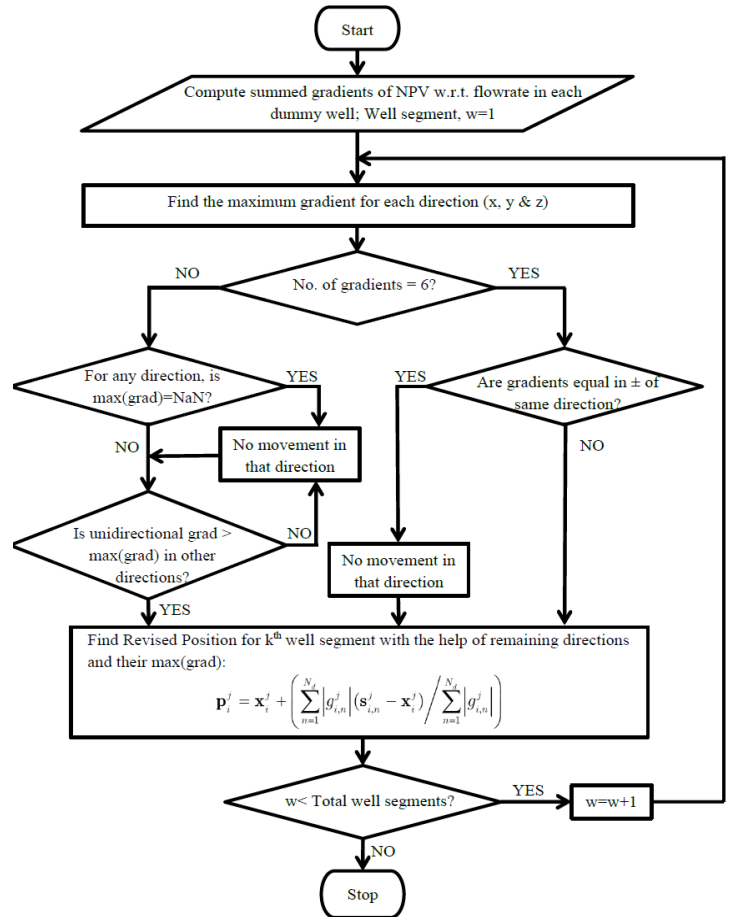


Fig. 2.3. An overview of the computation of the revised position for all the well segments.

The revised position vector of the i^{th} trajectory point is represented by \mathbf{p} :

$$p_i^j = x_i^j + \frac{\sum_{n=1}^{N_d} |g_{i,n}^j| (s_{i,n}^j - x_i^j)}{\sum_{n=1}^{N_d} |g_{i,n}^j|} \tag{2.4}$$

where N_d signifies the number of dummy wells taken into account to evaluate the revised position \mathbf{p} and \mathbf{s} represents the position vector of the dummy well. Fig. 2.3 illustrates an overview of the revised position computation.

As discussed earlier, six dummy wells are generated for each trajectory segment. Since the dummy wells are present in \pm directions, only three of those point in the direction of improvement. Considering the improvement of injector well trajectory, the maximum gradient always indicates an improved location. The summed gradients may or may not be positive depending on the prescribed flowrate. If the flowrate is higher than the optimized flowrate, negative gradients are observed. Similarly, a flowrate lower than the optimized flowrate yields positive gradients. Hence, the absolute value of the gradient is taken in account to assure that the revised position is unaffected by the positivity/negativity of the gradients. If a dummy well pair does not exist in a direction,

the existing gradient in that direction is compared with the maximum gradient values in the other two directions. If it is higher than at least one of those, the lone gradient value and its corresponding direction are taken in account to compute the revised position. One must also note that equal gradients in the \pm direction indicate local optimal solution. In short, this approach of taking three maximum gradients of each direction is credible as it does not have a tendency to get stuck except in a local optimum.

D. Dogleg Severity

It is important to ensure that the proposed well trajectory is drillable. The drilling industry uses Dogleg Severity (DLS) to define the drillability of a well. DLS provides the wellbore curvature between two consecutive directional surveys. Usually, it is expressed in degree/100 feet or degree/30 meter. In this thesis, DLS has been used to improve and predict a realistic trajectory using Jansen (1993) and Vlemmix et al. (2009) method.

The curvature (C) is inversely proportional to the radius of curvature (R) and measures the curvature of a circular arc:

$$C = \frac{1}{R} \tag{2.5}$$

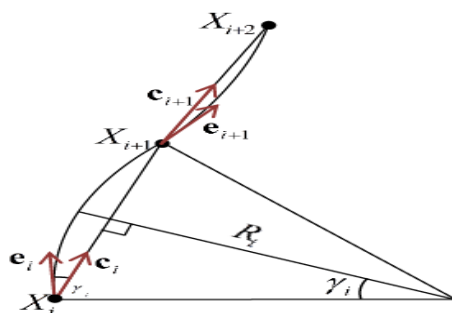


Fig. 2.4. The figure displays trajectory represented as a sequence of circular arcs and the associated variables essential to calculate the radius of curvature using the minimum curvature method.

The direction vectors of the tangents to the borehole trajectory in the trajectory points are assumed to be initially unknown except for the first point. A ‘marching algorithm’ has been used to compute the curvatures sequentially. The Cartesian coordinates of the trajectory points are known and are given by X_i for a trajectory point X_i (a point located at the i^{th} location). Using the Cartesian coordinates of the trajectory points, a unit-length direction vector c_i along the chord between the trajectory points X_i and X_{i+1} is computed first:

$$c_i = \frac{X_{i+1} - X_i}{|X_{i+1} - X_i|} \tag{2.6}$$

Further, it is assumed that the unit-length direction vector e_i is equal to c_i for $1 \leq i$. In order to use dogleg severity, the radius of curvature is defined between the 2 trajectory points X_i and X_{i+1} (Fig. 2.4) as:

$$R_i = \frac{|X_{i+1} - X_i|}{2 \tan \gamma_i} \tag{2.7}$$

where the angle γ_i is given by

$$\gamma_i = \arcsin(e_i \cdot c_i) \tag{2.8}$$

The next unit-length direction vector e_{i+1} at trajectory point X_{i+1} can be computed as:

$$e_{i+1} = 2c_i \cos \gamma_i - e_i \tag{2.9}$$

If the computed radius of curvature is less than a pre-defined radius obtained by dogleg severity, an iterative process follows to increase the radius of curvature. This iterative process was defined as smoothing process by Vlemmix et al. (2009). The smoothing process, in general, smoothens all the points except the heel and toe points:

$$X_i^s = (1 - \delta)X_i + \delta \left(\frac{X_{i-1} + X_{i+1}}{2} \right) \tag{2.10}$$

where δ is a dimensionless weighing factor and $0 < \delta \leq 1$. The smoothing process is reiterated until a drillable trajectory is obtained as illustrated in Fig. 2.5. The figure shows a non-drillable multi-point trajectory being converted into a drillable trajectory using the algorithm described.

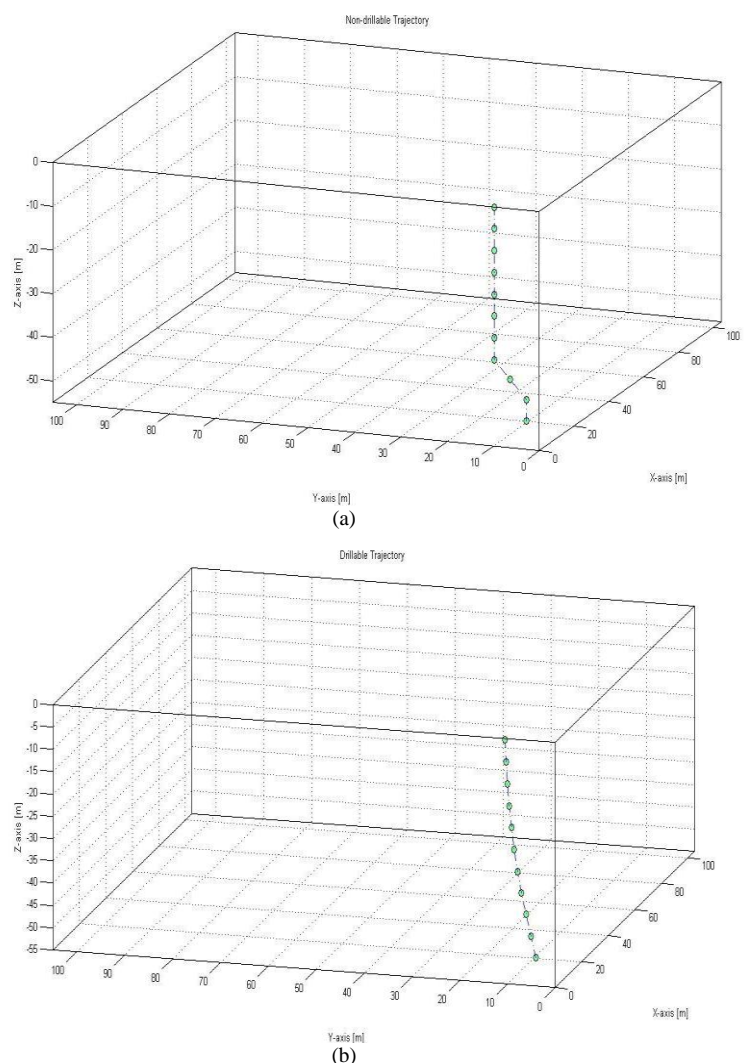


Fig. 2.5. Smoothing of the trajectory: A non-drillable trajectory (a) followed by a drillable trajectory (b) produced by the discussed algorithm.

Figure 2.6 displays a flow chart of the process of checking and if necessary, smoothing the trajectory points to achieve a drillable well trajectory. For the future cases, a DLS of $10^\circ/30$ m has been assumed unless stated otherwise. This corresponds to a radius of curvature of approximately 172m.

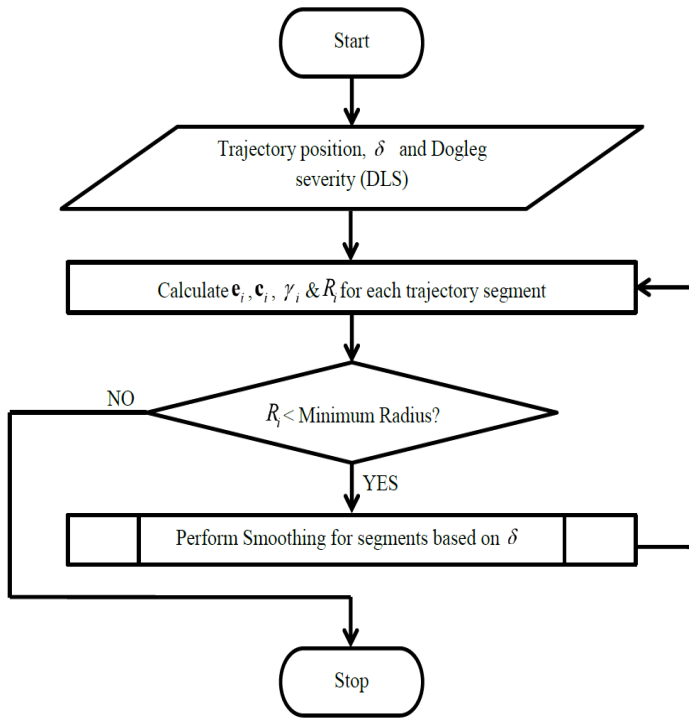


Fig. 2.6: Flowchart – Using DLS to define a drillable trajectory.

E. Drilling Cost

As mentioned earlier, the main objective of the paper is to find the best well trajectory so that the NPV of the project is maximized. The following equation has been used in order to calculate the NPV of the project:

$$J = \sum_{k=1}^K \left[\frac{\sum_{i=1}^{N_{inj}} r_{w,inj} \cdot (u_{w,i})_k + \sum_{j=1}^{N_{pr}} (r_{w,pr} \cdot |y_{w,j}|_k + r_{o,pr} \cdot |y_{o,j}|_k)}{(1+b)^{\frac{E_k}{\tau}}} \times \Delta t_k \right] + \sum_{i=1}^{N_{inj} + N_{pr}} C_i \cdot L_i \quad (2.11)$$

Where,

- b - Discount Rate, 1/year
- C_i - Drilling cost (of well trajectory i) per unit meter, \$/m
- k - Simulation time step, -
- L_i - Length of the trajectory inside reservoir gridblocks (of well trajectory i), m
- N_{inj} - Number of Injection wells, -
- N_{pr} - Number of Production wells, -
- $r_{o,pr}$ - Produced oil price per unit volume, \$/m³
- $r_{w,inj}$ - Injected water cost per unit volume, \$/m³
- $r_{w,pr}$ - Produced water cost per unit volume, \$/m³
- Δt_k - Time Interval of time step k , day
- τ - Reference time interval (taken as a year)
- $u_{w,i}$ - Water injection rate of well i , m³/day
- $y_{o,j}$ - Oil production rate of well j , m³/day
- $y_{w,j}$ - Water production rate of well j , m³/day

F. Convergence

The algorithm can be terminated by either of the following five criteria:

1. Principal criterion: Lower relative increase of the objective can be used to terminate the algorithm using:

$$\frac{J^j - J^{j-1}}{J^{j-1}} < \epsilon \quad (2.12)$$

Here, ϵ can range as per the requirement of the user. Higher value of ϵ means saving a lot more iterations albeit, it comes with a trade-off with best optimal well trajectory assignment.

2. Decrement criterion: The algorithm can also be concluded if the NPV does not increase with each iteration. The maximum number of decrements can be adjusted as per the needs of the user. This criterion ensures that the highest NPV and the equivalent trajectories are displayed as the final output.

3. Oscillating criterion: In some cases, it may be observed that the trajectory starts oscillating alternately between two nearly equal solutions. In such case, the oscillating criterion calculates the maximum objective function and displays the corresponding trajectory.

4. Equal gradients criterion: In some scenarios, it is possible to observe equal gradients of the objective function over the lifespan of the reservoir with respect to the flow rates in the dummy wells. In such scenarios, one can presume the achievement of either a local or global optimal solution. The algorithm is terminated under such a circumstance.

5. Maximum iterations criterion: The user can define the maximum number of iterations and the algorithm is run till the maximum value of the iteration unless this criterion is interrupted by any of the above mentioned auxiliary criterion.

III. RESULTS AND DISCUSSIONS

A. Results

This section covers a detailed outcome of the first attempt on optimizing the well trajectory in two and three-dimensional homogeneous and heterogeneous reservoirs. In order to understand the behaviour of the algorithm and the nature of the output well trajectory, the base case of both two and three-dimensional reservoirs is followed by few altered cases. Note that the reservoirs represented in the following sections were synthetically generated using reservoir simulation packages; Mat lab reservoir simulation toolbox (MRST) (Lie, 2014) and ECLIPSE (Eclipse 2013.1). Following reservoir and fluid parameters (Table 3.1) were initialized for both cases of homogeneous and heterogeneous reservoirs. Please note that all the reservoirs have Vertical Transverse Isotropy (VTI), with $k_z = 0.1k_x$ (or k_y). Table 3.2 shows the economic parameters.

TABLE 3.1- Reservoir and fluid properties.

Symbol	Value	Unit
ρ_w	1014	Kg/m ³
ρ_o	859	Kg/m ³
μ_w	1x 10 ⁻³	Pa s
μ_o	0.5x 10 ⁻³	Pa s
c_w	1x 10 ⁻¹⁰	Pa ⁻¹
c_o	1x 10 ⁻¹⁰	Pa ⁻¹
c_r	0	Pa ⁻¹
K_{rw}^0	1	-
K_{ro}^0	1	-
S_{we}	0.15	-
$S_{w,0}$	0.15	-
S_{or}	0.2	-
P_R	4x 10 ⁷	Pa
R_{well}	0.1	m
ϕ	0.3	-
k	9.869x 10 ⁻¹³	m ²

TABLE 3.2- List of economic parameters.

Symbol	luc	Unit
r_o	314.5	\$/m ³
$r_{w,pr}$	18.9	\$/m ³
$r_{w,inj}$	6.3	\$/m ³
c_{wi}	500	\$/m

The two phase relative permeability is depicted in Fig. 3.1.

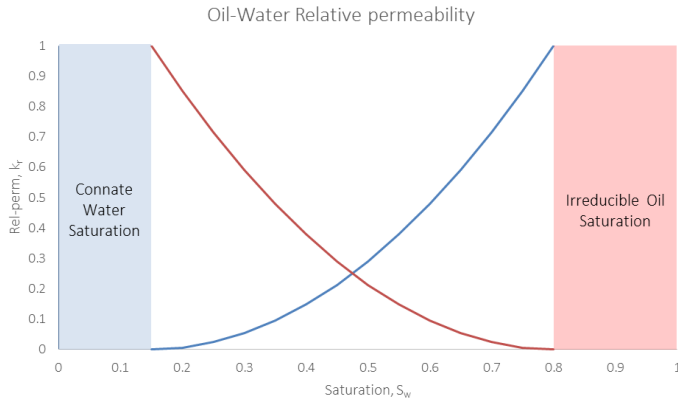
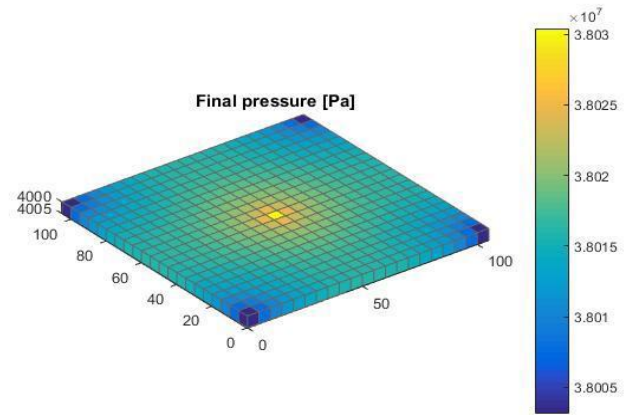


Fig. 3.1. Oil-Water relative permeability.

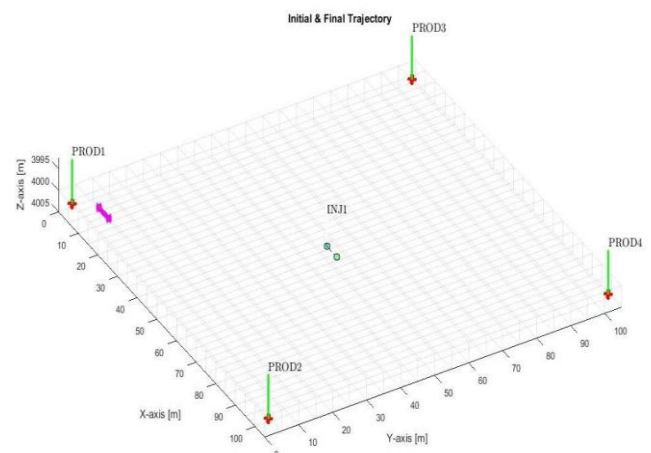
A. Homogeneous Reservoir

(a) Two-dimensional reservoir

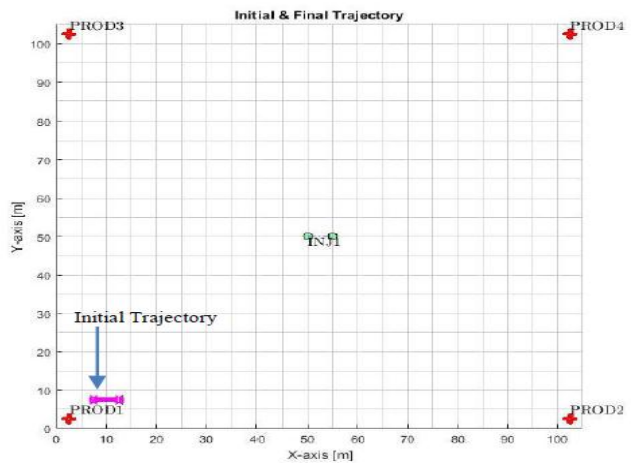
For the first test case, the trajectory of an injector well was optimized in a 21 x 21 x 1 grid blocks horizontal reservoir model with grid block dimensions of 5 x 5 x 5 m³. Four production wells, fixed at the corners of the reservoir, deplete the reservoir for an uninterrupted period of 1,000 days. Whereas the producers operate under a prescribed pressure of 380 x 10⁵ Pa, the injector operates under a rate constraint of 10 m³/day while ensuring that injection pressure does not exceed 420 x 10⁵ Pa (and frack the reservoir). The initial reservoir pressure of 4,000 m deep reservoir was taken as 400 x 10⁵ Pa. Intuitively, one would expect the injectors to move towards the centre of this designed homogenous reservoir. The reservoir simulation was run for twenty five simulation time steps. It must be noted that gravity plays no role in the single-layered reservoir and hence, can be turned off for the simulation.



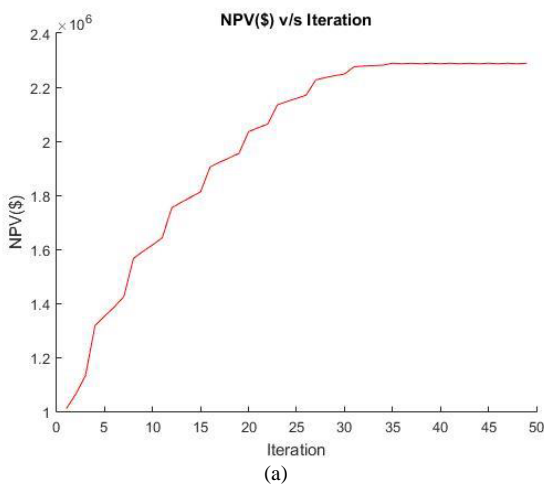
(b)



(c)



(d)



(a)

Fig. 3.2. NPV graph, pressure map and corresponding trajectory map when the initial trajectory is a horizontal well (c) (grid blocks [23;24]). The figure (d) illustrates the top view (XY) of the trajectory map.

In the first scenario, a two-point trajectory was optimized. Since neither of the ends was fixed, the trajectory was permitted to move in any direction. As anticipated, the trajectory moved towards the centre of the reservoir (Fig. 3.2). The NPV was increased by approximately 127% using this

method and it takes about 35 iterations to achieve the maximum NPV.

As discussed earlier, the number of iterations can be decreased by either changing the weight factor/step size β to increase the movement in improving direction or constraining the algorithm to stop converging by using any of the termination/convergence criteria. Keeping the iterations in mind, β was increased from 0.5 to 1. This step helped to reduce the number of iterations significantly (-49% compared to the base case) while moving the well trajectory to its optimal solution at the centre (Fig. 3.3). The final outcome of the scenario was identical to the base case. Moreover, the whole process can be further improved in terms of iterations by concluding the algorithm with the aid of principal criterion of termination. In that scenario, the algorithm can be stopped at about 17 iterations (depending upon the definition of ϵ).

wells were fixed at the corners of a 21 x 21 x 7 grid blocks reservoir (with grid block dimensions of 5 x 5 x 5 m³). The life span of the matchbox shaped reservoir was assumed to be an uninterrupted duration of 1,500 days. Similar to the two-dimensional scenario, the producers were operated under a prescribed pressure of 380 x 10⁵ Pa and the injector was operated under a rate constraint of 300 m³/day while ensuring that injection pressure does not exceed 420 x 10⁵ Pa. The initial reservoir pressure of 4,000 m deep reservoir was taken as 400 x 10⁵ Pa.

The reservoir simulation was run for thirty five simulation time steps. To guarantee a drillable and optimized well trajectory, the maximum allowed DLS was set to 10⁹/30 m. Gravity plays an important role in a three-dimensional reservoir. Depending on the net thickness and the lateral distance between the injector-producer pair, it may strongly affect the sweep efficiency of the reservoir layers at the top. In order to capture and appreciate this effect, all the scenarios assume gravity in the reservoir. The first case scrutinised the behaviour of the injector well trajectory while it is free to move in any direction since neither of the ends was fixed. In this scenario, a multi-point trajectory was tested (Fig. 3.4). The injector well was initially located vertically with perforations in all the seven layers. Four vertical production wells, placed at the edges of the reservoir, also perforated all the layers at their respective position. The weight factor/step size β was defined as unity. As anticipated, the trajectory moved towards the centre of the reservoir. NPV was increased by approximately 17% using this method and it took 19 iterations to achieve the maximum NPV.

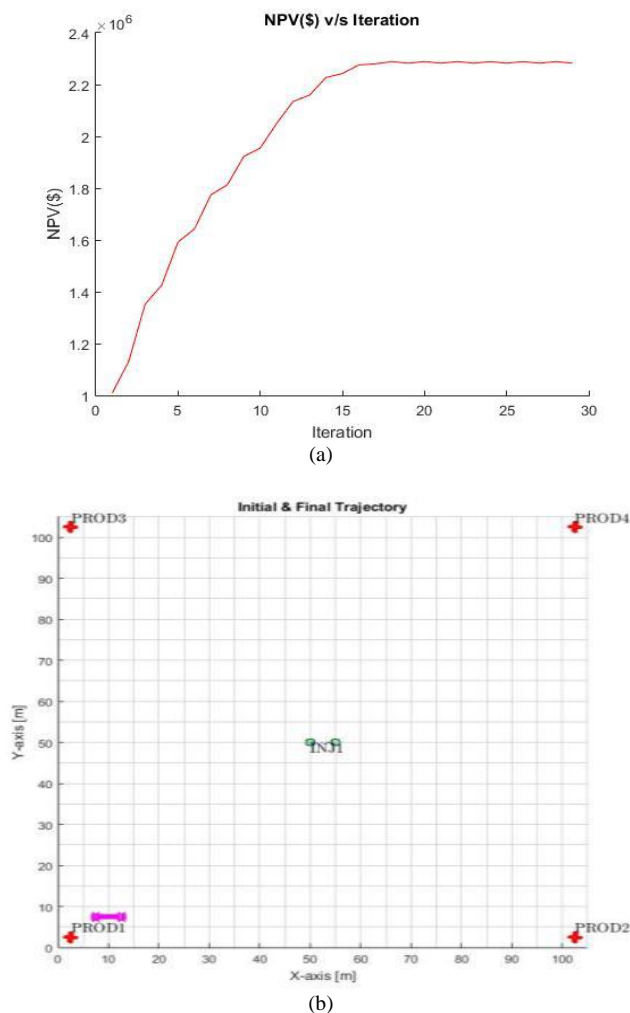
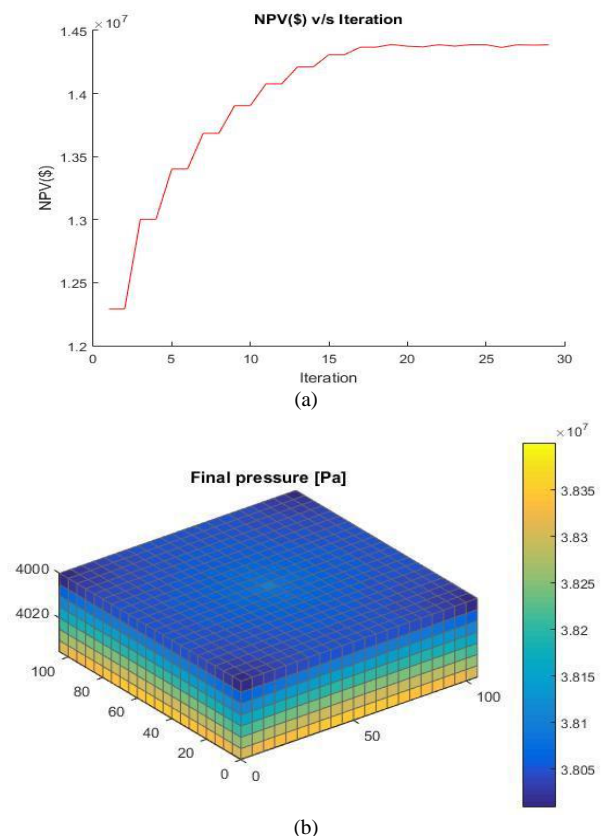


Fig. 3.3. NPV graph (a) and the optimal trajectory of injection well (top-view) when the initial trajectory is a horizontal well (b) (grid blocks [23;24]) and $\beta=1$.

(b) Three-dimensional reservoir

Before proceeding to the heterogeneous reservoirs, it is vital to take a quick look at the resultant trajectory in a three-dimensional homogenous reservoir. In order to test the algorithm in a three dimensional reservoir, four production



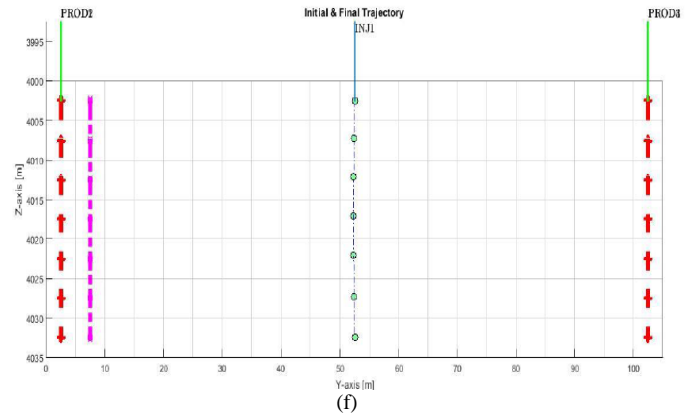
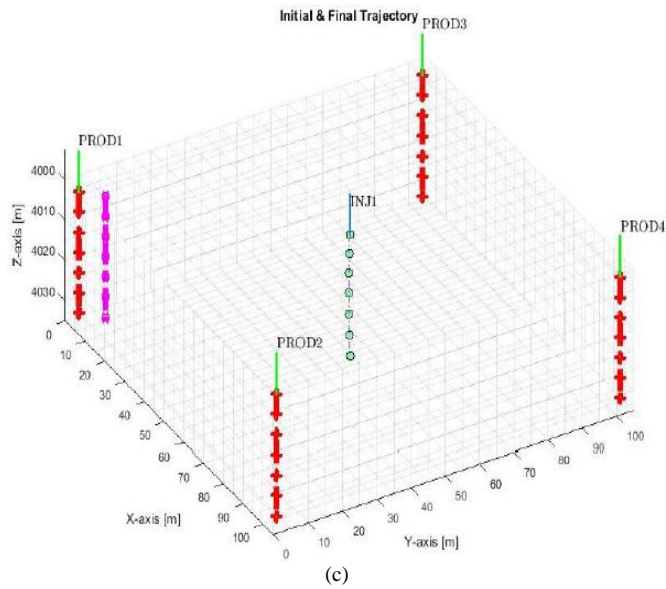
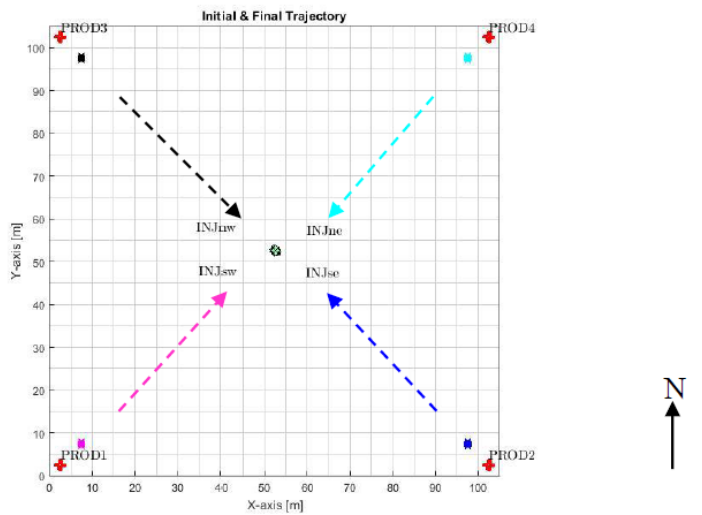
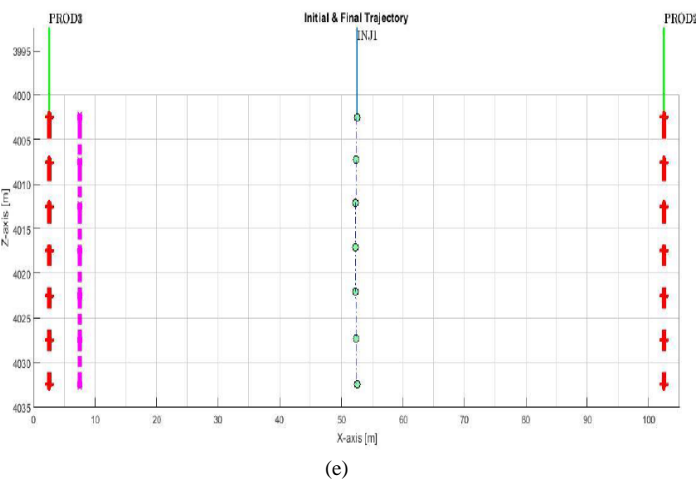
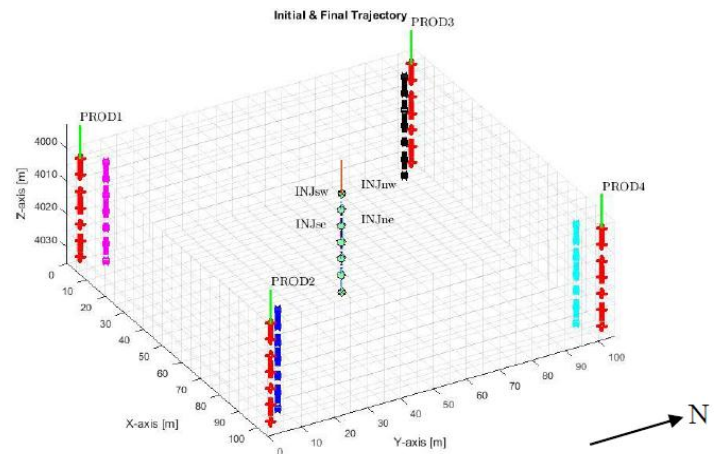
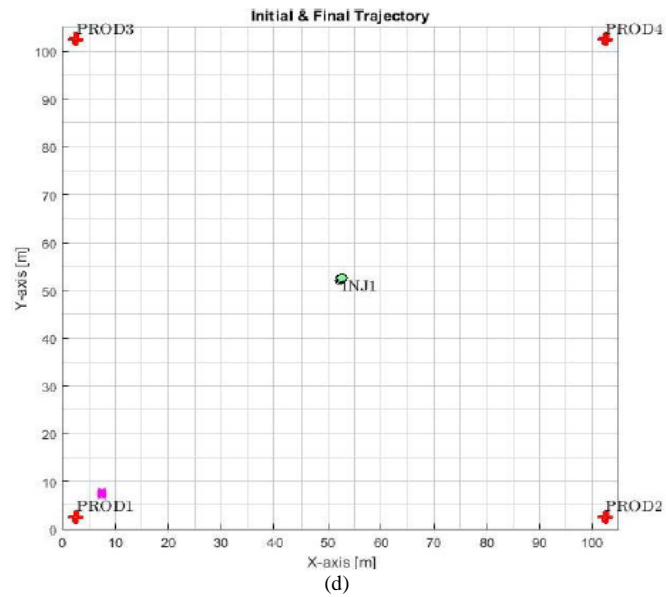


Fig. 3.4. NPV graph (a), pressure map (b) and corresponding trajectory map when the initial trajectory (magenta) is a vertical well perforating all 7 layers (c). The top view (d) (XY), front view (e) (XZ) and side view (f) (YZ) of the trajectory map.

The base case was repeated with different initial trajectories. As illustrated in Fig. 3.5, the final solution more or less converges to the same position:



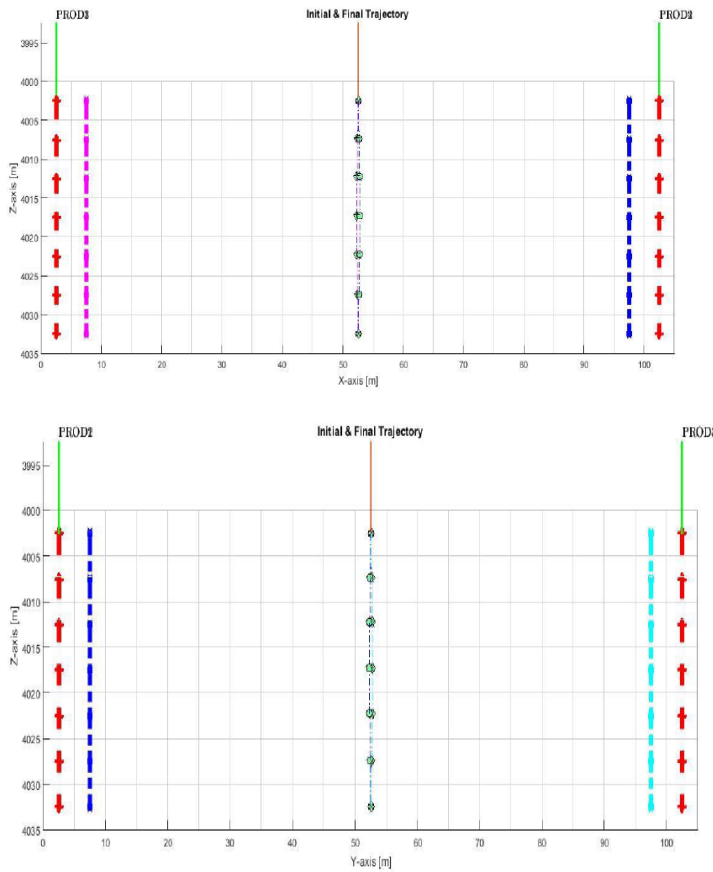


Fig. 3.5. Different initial trajectories converging to same grid blocks to produce comparable results.

Moreover, the base case mentioned above was also re-examined by placing a horizontal injection well initially (Fig. 3.6). Although it took 28 iterations in this case, the final trajectory and the NPV were almost identical to the ones produced with the base case.

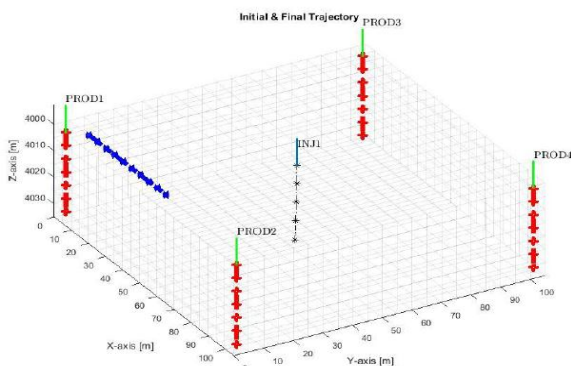


Fig. 3.6. Final trajectory (Black) when the initial trajectory (Blue) is a horizontal well.

B. Heterogeneous Reservoir

(a) Two-dimensional reservoir

In order to test the algorithm in a two-dimensional reservoir, four production wells were fixed at the corners of a 21 x 21 x 1 grid block reservoir with grid block dimensions of 5 x 5 x 5 m³. The life of the reservoir was assumed to be 1,000

days. The producers were operated under a prescribed pressure of 380×10^5 Pa and the injector was operated under a rate constraint of 10 m³/day while ensuring that injection pressure does not exceed 420×10^5 Pa. The initial reservoir pressure of 4,000 m deep reservoir was taken as 400×10^5 Pa. The reservoir simulation was run for twenty five simulation time steps. Since gravity plays no role in the single-layered reservoir, it was turned off for the simulation.

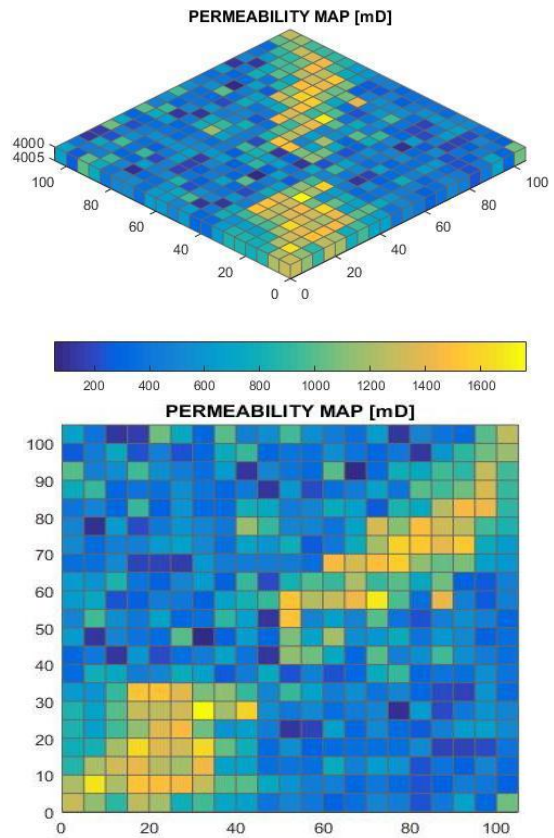
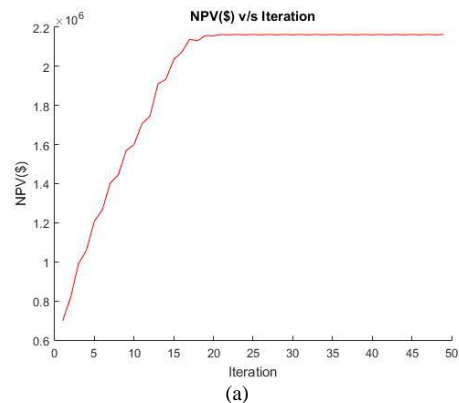


Fig. 3.7. Permeability map of single-layered heterogeneous reservoir.

A freely moving two-point trajectory was tested in this heterogeneous reservoir. As shown in Fig. 3.8, the trajectory reached an optimal location using 21 iterations. A 210% increase in the NPV was observed. The trajectory tried to position itself in such a fashion that the water production was as delayed as possible.



presumed to be constant, the permeability of each layer was depicted by the permeability map of Fig. 3.7.

The reservoir simulation with 64 time steps depletes the ‘match-box’ shaped reservoir for an uninterrupted period of 1,500 days. DLS was assumed to be $10^0/30$ m. Whereas the producers operated under a prescribed pressure of 398×10^5 Pa, the injector operated under a rate constraint of $300 \text{ m}^3/\text{day}$ while ensuring that injection pressure does not exceed 435×10^5 Pa (and frack the reservoir). Contrary to the homogeneous reservoir, predicting the optimized well trajectory in a three-dimensional heterogeneous reservoir is a challenging task. To observe the progress of improving well trajectories and study the behaviour of the algorithm, the step size β was reduced to 0.5 in this case.

During the first case, a freely moving multi-point trajectory was tested. The trajectory was initially located vertically perforating all the seven layers. The trajectory progressed to an optimal location near the centre of the reservoir to maximize the NPV. As anticipated, the trajectory does not move all the way towards the centre because of the heterogeneity in the reservoir. NPV was increased by approximately 28% using this method and it took 45 iterations to achieve the maximum NPV. The number of iterations can be reduced by either using a bigger step size or using principal termination criterion.

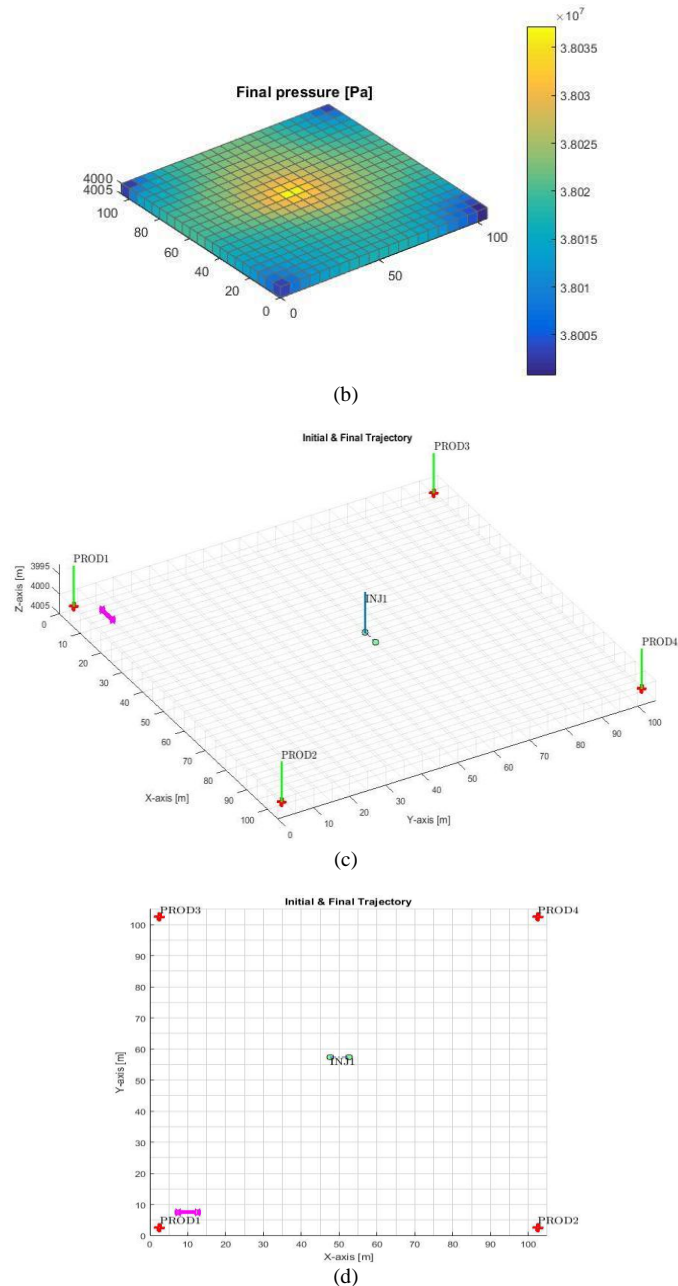
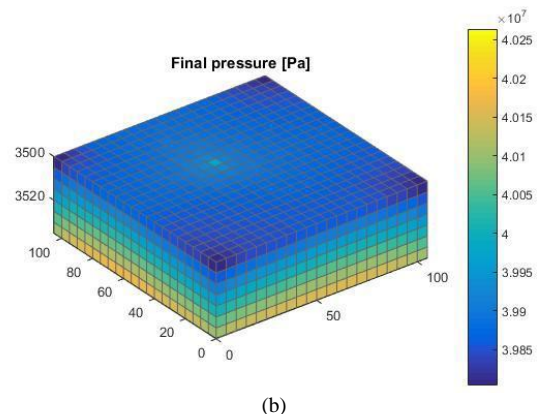


Fig. 3.8. NPV graph (a), pressure map (b) and corresponding trajectory map when the initial trajectory is a horizontal well (c) (grid blocks [23;24]). The figure (d) illustrates the top view (XY) of the trajectory map.

An altered base case with step-size $\beta = 0.5$ suggested that it takes about 40 iterations to reach a similar result in terms of NPV and well trajectory, compared to 21 iterations with $\beta = 1$.

(b) Three-dimensional reservoir

To examine the working of the algorithm in a three-dimensional heterogeneous reservoir, four production wells were fixed at the corners of a $21 \times 21 \times 7$ grid blocks reservoir with grid block dimensions of $5 \times 5 \times 5 \text{ m}^3$. The initial reservoir pressure of the 3,500 m deep reservoir was assumed to be 413×10^5 Pa. While the porosity of the reservoir was



Similar to the homogeneous three-dimensional case, the base case was repeated with different initial trajectories. To ensure equal time-steps and to avoid any convergence issue, the time-steps of the reservoir simulation were increased to 155 and the step size was defined as 0.5. As illustrated by Fig. 3.10, the final solution more or less converges to the same position.

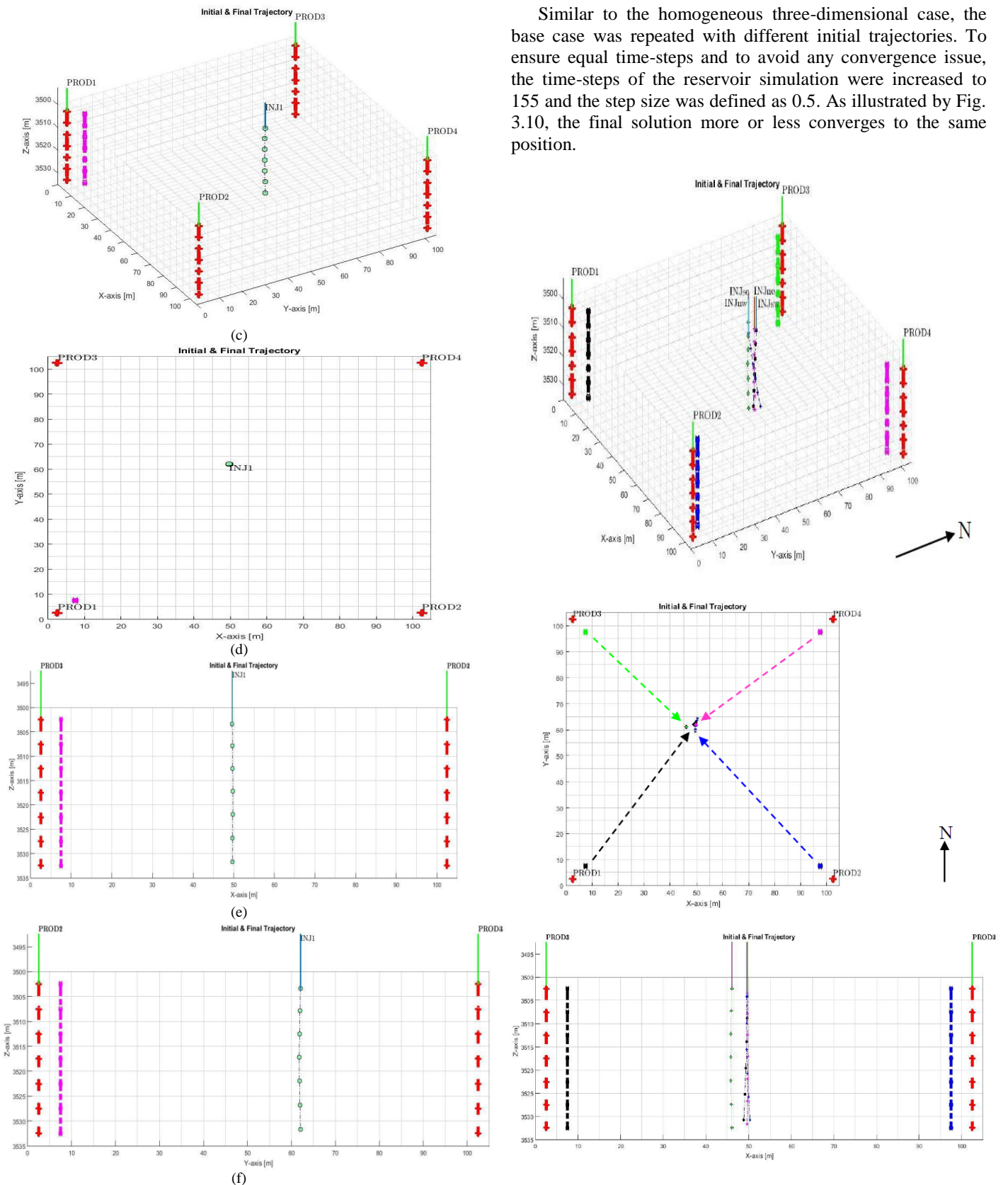


Fig. 3.9. NPV graph (a), pressure map (b) and corresponding trajectory map when the initial trajectory is a vertical well perforating all 7 layers (c) . The top view (d) (XY), front view (e) (XZ) and side view (f) (YZ) of the trajectory map.

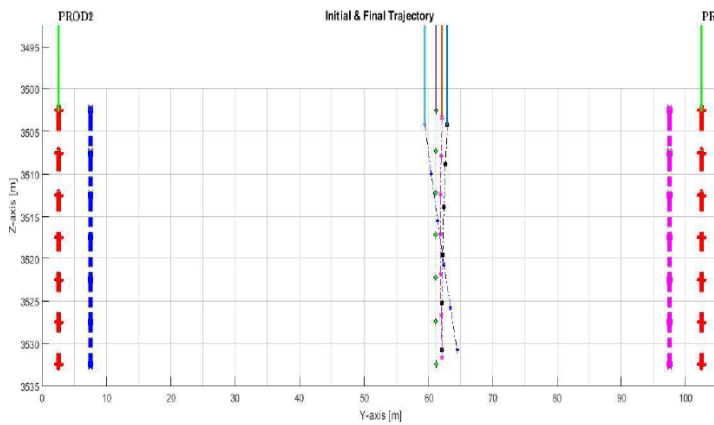


Fig. 3.10. Different initial trajectories converging to almost same grid blocks to produce comparable results.

Although, the optimized trajectory for the case with initial injector at the ‘south-east’ corner ended up slightly deviated, all the four optimized trajectories ended up within the same grid blocks. The whole process, probably, can be further improved by taking grid refinement process and reducing the step-size near the local/global optimum. The NPV achieved in all the cases was essentially the same while the number of iterations varied.

B. Discussions

1. Flow rate, NPV and gradients are linked closely. This relationship is depicted in Fig. 3.11 and Fig. 3.12. The flow rates influence the NPV which in turn also affects the sign and magnitude of the gradients.

a) NPV versus injection rate: This relationship is depicted in Fig. 3.11. Lower injection rates imply positive gradients or in other terms, an indication that the reservoir needs more injection to displace oil, maintain pressure and improve the NPV. Higher injection rates, on the other hand, imply negative gradients or in other words, an indication that the reservoir does not need more injection and lowering flow rate will help to achieve a higher NPV.

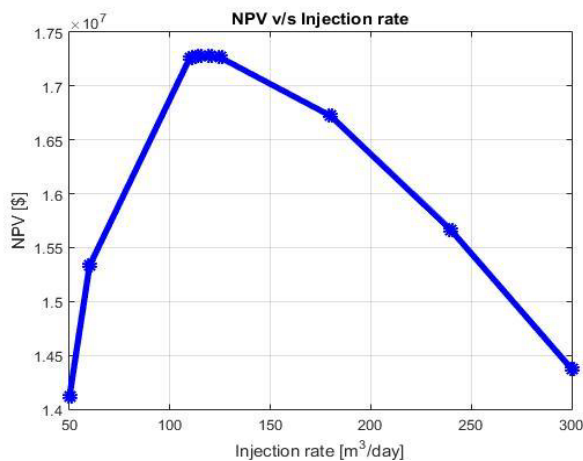


Fig. 3.11. Variation of NPV with change in injection rate of a well.

b) NPV versus production rate: This relationship is described in Fig. 3.12. As depicted in Fig. 3.12, lower production rates

lead to negative gradients which in turn suggests that the producer is being under-utilized and increasing the production rate would help to achieve a higher NPV. On the contrary, higher production rates imply positive gradients and an indication that lowering the production rate can improve the NPV.

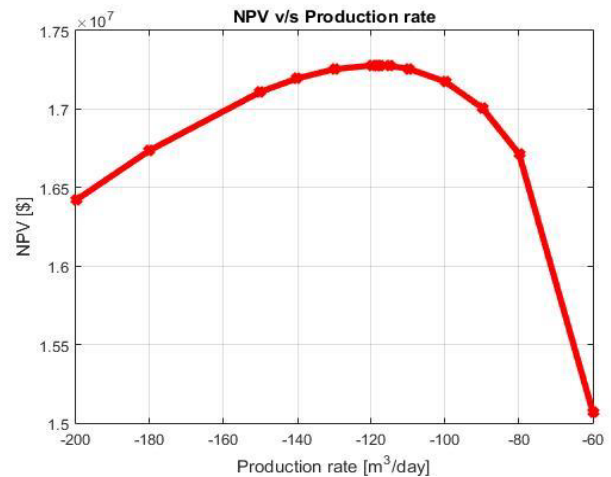


Fig. 3.12. Variation of NPV with change in production rate of a well.

c) Trajectory optimization and gradients: Whereas, the maximum of the gradients always points in the direction of the better injector well trajectory, the minimum gradients point towards the direction of the better production well trajectory. The signs of the gradients do not affect the optimization procedure.

d) Trajectory optimization and flowrate: Flowrate plays a governing role in deciding an optimized well trajectory. One should bear in mind that lower-than-optimized flowrate may yield slightly different optimal well trajectory compared to a higher-than-optimized flowrate.

2. Global Solution: In order to achieve global optimum, the trajectory optimization algorithm can be augmented with non-gradient-based methods like ANN, GA, Simulated Annealing, etc.

3. Flowrate Optimization: As inferred earlier, the NPV obtained by optimizing the well number and/or trajectory is still not optimal and there is an extensive opportunity for improving it based on various parameters, especially flowrate. An optimized flowrate of the perfect number of wells with best trajectories can guarantee an optimal FDP. Flowrate optimization can be pursued once the wells have reached the appropriate location. The project can be improved further by utilizing smart well technology to control the flowrate over the time.

4. Two-point trajectory optimization and dummy wells: Even though the well trajectory can be easily rotated with the help of multi-point trajectory, delineating the type of well with a two-point well trajectory is not feasible. For such a case, well trajectories can be developed by generating dummy wells in more than six directions. Furthermore, a multi-directional dummy well configuration can definitely improve the NPV during the last few iterations of the optimization process.

5. Geological Uncertainty: Imaging sub-surface is a difficult task and hence Robust Optimization (RO) is vital to capture the geological uncertainty. Given the lower number of iterations, RO can be carried out to understand the best well trajectory under the given uncertainty in the reservoir properties.

6. Production constraint: Production constraints strongly affect the final outcome. So far, BHP constrained producers and flowrate (and/or BHP) constrained injectors have been tested. Enforcing more constraints like average reservoir pressure, water-cut, field production/injection rate etc. may further improve the output.

7. Optimizing FDP: The multi-stage algorithm needs to be combined with well scheduling and well controls optimization to develop an optimal FDP. Although well controls optimization seems to be straightforward, optimizing the well schedule using gradients seems to be a challenging job.

8. Realistic Reservoir Model: To understand the reliability and the loopholes in the current algorithm, a realistic reservoir needs to be investigated. Moreover, the algorithm can also be tested with different reservoir fluids including aquifer and gas cap.

9. Since an engineer's judgement and experience cannot be ignored, the performance of the trajectory optimization algorithm is much appreciated when the optimization process is started from the engineer's best estimate of the well trajectory. Similarly, including engineer's judgement in the multi-stage optimization process can not only help in demarcating the best wells and their placements but also reducing the number of iterations. A more successful field development plan can be generated using this method.

IV. CONCLUSIONS

Several two and three-dimensional reservoir models were examined during the process of this research work which helps to appreciate the performance of the algorithms presented in the work. Based on the various observations and results obtained for different scenarios, it can be concluded that:

1. Computer assisted algorithm:

a) Local v/s Global Optimal solution: Even though the optimization algorithm functions pretty well, sometimes the well trajectories may get stuck in a local optimum. The algorithm cannot discriminate between global and local optimal solution. Predominantly for large problems, it is highly likely that the method may converge to a local optimal solution. In order to solve this problem, one can repeat the optimization routine from different starting points to select the best trajectory.

b) NPV Increment: Even if the trajectory optimization algorithm may fail to find the globally optimal solution, any NPV increment is more than welcome. The cases presented in the results displayed a wide range of NPV increment under the provided reservoir and production parameters (6 - 216%).

c) Automated process: This study suggests that the computer-assisted field development optimization is feasible. While well number optimization does not need any initial knowledge of

wells, well trajectory optimization can be accomplished with either the engineer's best estimate or random guess.

2. Drilling optimal well trajectory:

a) Well type: The optimization algorithm assists delineating the well types (vertical, horizontal or deviated). As seen, a vertical well can be easily transformed to a horizontal well and vice-versa.

b) Drilling costs and well length: The additional term included in the objective function plays a significant role to curb and optimize the well length, especially in the case of fixed heel trajectory where the trajectory may extend to a large extent. Realistic information supplied for the drilling costs can help to accurately curb the costs of drilling while ensuring higher NPV.

3. Weight factor/step size β :

Step size plays a crucial role. Increasing the step size can reduce the number of iterations while reducing it increases the number of iterations.

REFERENCES

- [1] Badru, O., and Kabir, C. S. (2003). Well placement optimization in field development. Paper SPE 84191 presented at the Annual Technical Conference and Exhibition, Denver, Colorado, U.S.A., 5-8 October. DOI: 10.2118/84191-MS.
- [2] Beckner, B. L., and Song, X. (1995). Field development planning using simulated annealing - optimal economic well scheduling and placement. Paper SPE 30650 presented at the SPE Annual Technical Conference & Exhibition, Dallas, U.S.A., 22-25 October. DOI: 10.2118/30650-MS.
- [3] ECLIPSE (2013.1). *ECLIPSE Reference Manual 2013.1*. Schlumberger.
- [4] Emerick, A. A., Silva, E., Messer, B., Almeida, L. F., Szwarcman, D., Pacheco, M. A. C., and Vellasco, M. M. B. R. (2009). Well placement optimization using a genetic algorithm with nonlinear constraints. Paper SPE 118808 presented at the 2009 SPE Reservoir Simulation Symposium, The Woodlands, Texas, U.S.A., 2-4 February. DOI: 10.2118/118808-MS.
- [5] Forouzanfar, F., Li, G. and Reynolds, A.C. (2010). A two-stage well placement optimization method based on adjoint gradient. Paper SPE 135304 presented at the 2010 SPE Annual Technical Conference and Exhibition, Florence, Italy, 20-22 September. DOI: 10.2118/135304-MS.
- [6] Forouzanfar, F. and Reynolds, A.C. (2014). Joint optimization of number of wells, well locations and controls using a gradient-based algorithm. *Chemical Engineering Research and Design* **92** (7): 1315-1328. DOI:10.1016/j.cherd.2013.11.006.
- [7] Handels, M., Zandvliet, M.J., van Essen, G.M., Brouwer, D.R. and Jansen, J.D. (2007). Adjoint based well-placement optimization under production constraints. Paper SPE 105797 presented at the 2007 SPE Reservoir Simulation Symposium, Houston, Texas, U.S.A., 26-28 February. DOI: 10.2118/105797-MS.
- [8] Jansen, J.D. (1993). *Nonlinear dynamics of oilwell drillstrings*. PhD dissertation, Delft University of Technology, Delft, The Netherlands.
- [9] Jansen, J.D. (2014). *Gradient-based optimization of flow through porous media*. Course notes for Advanced Reservoir Simulation, Delft University of Technology, Delft, The Netherlands.
- [10] Jansen, J.D., Bosgra, O.H. and Van den Hof, P.M.J. (2008). Model-based control of multiphase flow in subsurface oil reservoirs. *Journal of Process Control* **18** (9): 846-855. DOI: 10.1016/j.jprocont.2008.06.011.
- [11] Jansen, J.D. (2013). The Egg Model - data files. TU Delft. <http://dx.doi.org/10.4121/uuid:916c86cd-3558-4672-829a-105c62985ab2>.
- [12] Lee, J.W., Park, C., Kang, J. M., and Jeong, C. K. (2009). Horizontal well design incorporated with interwell interference, drilling location, and trajectory for the recovery optimization. Paper SPE 12239 presented at the 2009 SPE/EAGE Reservoir Characterization and Simulation

- Conference, Abu Dhabi, U.A.E., 19-21 October. DOI: 10.2118/125539-MS.
- [13] Lie, K.A. (2014). An introduction to reservoir simulation using MATLAB: User guide for the Matlab Reservoir Simulation Toolbox (MRST). SINTEF ICT, Department of Applied Mathematics, Oslo, Norway. <http://www.sintef.no/Projectweb/MRST/Publications>.
- [14] Peaceman, D.W. (1978). Interpretation of well-block pressures in numerical reservoir simulation. Paper SPE 6893 presented at the 1977 SPE Annual Fall Technical Conference and Exhibition, Denver, Colorado, U.S.A., 9-12 October. DOI: 10.2118/6893-PA.
- [15] Sarma, P. and Chen, W.H. (2008). Efficient well placement optimization with gradient-based algorithms and adjoint models. Paper SPE 112257 presented at the 2008 SPE Intelligent Energy Conference and Exhibition, Amsterdam, The Netherlands, 25-27 February. DOI: 10.2118/112257-MS.
- [16] Shu, J. (2005). *Comparison of various techniques for computing well index*. Master's thesis, Petroleum Engineering Department, Stanford University, California.
- [17] Van Essen, G.M., Zandvliet, M.J., Van den Hof, P.M.J., Bosgra, O.H. and Jansen, J.D. (2006). Robust waterflooding optimization of multiple geological scenarios. Paper SPE 102913 presented at the 2006 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 24-27 September. DOI: 10.2118/102913-MS.
- [18] Vlemmix, S., Joosten, G.J.P., Brouwer, D.R. and Jansen, J.D. (2009). Adjoint-based well trajectory optimization in a thin oil rim. Paper SPE 121891 presented at the 2009 SPE EUROPEC/EAGE Annual Conference and Exhibition, Amsterdam, The Netherlands, 8-11 June. DOI: 10.2118/121891-MS.
- [19] Wang, C., Li, G. and Reynolds, A.C. (2007). Optimal well placement for production optimization. Paper SPE 111154 presented at the 2007 SPE Eastern Regional Meeting, Lexington, Kentucky, U.S.A., 11-14 October. DOI: 10.2118/111154-MS.
- [20] Yeten, B., Durllofsky, L.J. and Aziz, K. (2002). Optimization of Nonconventional Well Type, Location and Trajectory. Paper SPE 77565 presented at the 2002 SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 29 September-02 October. DOI: 10.2118/77565-MS.
- [21] Jansen, J.D., Brouwer, D.R., Naevdal, G. and van Kruijsdijk, C.P.J.W. (2005). Closed-loop reservoir management. *First Break* 23 (1): 43-48. DOI: 10.3997/1365-2397.2005002.
- [22] Jansen, J.D., Douma, S.D., Brouwer, D.R., Van den Hof, P.M.J., Bosgra, O.H. and Heemink A.W. (2009). Closed-loop reservoir management. Paper SPE 119098 presented at the 2009 SPE Reservoir Simulation Symposium, The Woodlands, Texas, U.S.A., 2-4 February. DOI: 10.2118/119098-MS.
- [23] Zandvliet, M.J., Handels, M., van Essen, G.M., Brouwer, D.R. and Jansen, J.D. (2008). Adjoint based well-placement optimization under production constraints. *SPE Journal* 13 (4): 392-399. DOI: 10.2118/105797-PA.
- [24] Zhang, K., Li, G., Reynolds, A.C., Yao, J. and Zhang, L. (2010). Optimal well placement using an adjoint gradient. *Journal of Petroleum Science and Engineering* 73 (3-4): 220-226. DOI: 10.1016/j.petrol.2010.07.002.