International Research Journal of Advanced Engineering and Science
ISSN: 2455-9024

Modification to Heywood's Equations to Estimate the Stress Concentration Factors for Unidirectional Glass Epoxy Laminates with Circular and Square Holes

H. Zamanian[1], M. Salehi[2]
[1, 2]Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

***Abstract*— *In this work considering the glass/epoxy laminate as a unidirectional long fiber composite and an orthotropic plate, its average stress, maximum stress, and stress concentration factor have been obtained around a circular hole using ABAQUS and then the result of this part have been verified by Heywood's constitutive equations. In the next step, we considered the stress concentration factor of the orthotropic laminate around a square hole, as a function of different variables: the dimension of the hole, width of plate, orthotropic elastic coefficients, fiber orientation angle, the stress concentration factor for a plate with infinite width, and a magnification factor. Then same procedure was utilized to obtain a formula such as the Heywood formulation which estimates the stress concentration factor of glass/epoxy laminate around square holes. After obtaining this formula, to investigate its accuracy again, ABAQUS software was used to extract the stress concentration factor for different materials such as steel, aluminum, e-glass/epoxy, and s-glass/epoxy. These results were then compared to the obtained formula. The statistics data showed a 10 percent difference between the stress concentration factor around the square hole using ABAQUS and the obtained formula. It can be concluded that, in this special case, instead of using ABAQUS, it's better to use the formula to estimate the stress concentration factor because it saves calculation time. Additionally, for this kind of Glass/epoxy laminate, two smaller holes have been designed around the larger hole to show the deduction of the stress concentration factor*.

***Keywords*— *Heywood's constitutive equations, stress concentration factor, unidirectional glass epoxy laminates, circular and Square holes.*

I. INTRODUCTION

Glass/epoxy laminates are unidirectional long fiber composites with two or more layers of Glass/epoxy laminas with different orientation angles. These kinds of laminates not only have lower density in respect to metallic plates, but also have more capacity to resist corrosion. Because of these characteristics, glass/epoxy laminates are of interest for many industries such as aerospace, marine, and vehicle. The stress concentration factor of laminates around the holes is required for structural designing, such as estimation of the fatigue life

To date, many grateful attempts have been done to investigate the quality and quantity of stress concentration around different holes, such circular and square holes. Toubal et al. used a non-contact measurement method, namely Electronic Speckle Pattern Interferometer (ESPI), to investigate the tensile strain field of a composites plate in the presence of stress concentrations caused by a geometrical defect consisting of circular hole at the center of the laminate [1]. Howland analyzed the stresses in the neighborhood of a circular hole within a strip under tension. He suggested a formulation for calculation of stress concentration factors for isotropic plates like commercial stainless steel with circular holes [2]. Different kinds of FE models have been developed to extract the value of stress concentration factors. Shastry and Rao studied stress concentration in tensile strips with large circular holes at the center of the plate [3]. Paul and Rao analyzed stress distribution around an elliptical hole in a thick FRP laminate under transverse loading by using FEM [4]. Nurettin et al. used FEM to predict the elastic-plastic behavior of thermoplastic composite laminated plates with square holes at the center of the laminate [5]. Hwai-Chung and Bin proposed an empirical calculation method which can estimate the stress concentration factor of isotropic plate around the circular holes as well as the estimation of the stress concentration factor for orthotropic laminates around same holes [6]. Ukadgaonker and Rao suggested a general solution for stresses around holes in symmetric laminates under in-plane loading by adapting the formulation given by Savin [7-8]. Jain and Mittal investigated the ratio of the hole diameter to plate width upon stress concentration factor (SCF) and deflection of isotropic plates as well as orthotropic lamination under different transverse static loading condition [9]. Mohammadi et al. analyzed the stress concentration facotr around a hole in a radially inhomogeneous plate by deriving and then solving differential equations of the stress function for plane stress conditions [10].

In the previously completed work, considering the SMC-R65 as anisotropic plate, we investigated the stress concentration factor of a SMC-R65 lamina with different holes, including circular and square holes at the center of the plate; finally, we presented a formula to estimate the stress concentration factor for an isotropic plate with a square hole [11]. The aim of this study is to extract average stress, maximum stress and stress concentration factor around the circular hole for a glass/epoxy laminate using ABAQUS and verifying its result with the Heywood formulation. The innovation of this study is to investigate the stress concentration factor for a glass/epoxy laminate around square holes and try to present a formula similar to Heywood's formulation in order to estimate this kind of stress concentration factor. The final attempt of the present study is to design two smaller holes around the larger hole in order to show the deduction of the stress concentration factor around the larger hole for this kind of Glass/epoxy laminate.

H. Zamanian and M. Salehi, “Modification to heywood's equations to estimate the stress concentration factors for unidirectional glass epoxy laminates with circular and square holes,” *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 4, pp. 18-25, 2016.

II. Material and Method

A. Stress Concentration Around A Circular Hole

In this part, considering the glass/epoxy lamina as a unidirectional long fiber composite and an orthotropic plate, its average stress, maximum stress and stress concentration factors have been obtained around a circular hole using ABAQUS. These results have been verified by Heywood constitutive equations' results, which have been obtained by using MATLAB. An orthotropic lamina like Glass/epoxy lamina with a circular hole at the center can be modeled as Figure 1, considering the fibers with the angle of θ to the x axis. In this model, r is the radius of the circular hole, W is the width of the laminate which is 0.5 m in this study, and d is the difference between the width of the laminate and the radius of the circular hole. The thickness of this model is t and has been selected as 1mm for a Glass/epoxy lamina.

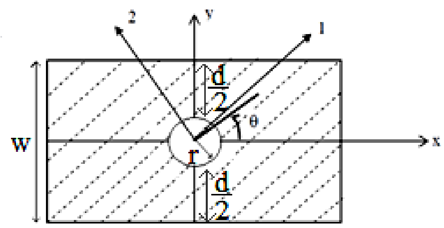

Fig. 1. Orthotropic laminate with circular hole under uniaxial loading.

Stress concentration factor for a plate with infinite width has been given in Equation 1 [12].

$$k^{\infty} = 1 + \frac{1}{\mu}\sqrt{\frac{2}{A_{66}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)} \qquad (1)$$

Where A_{ij}s denote the effective laminate in-plane stiffness with 1 and 2 parallel and perpendicular to the loading directions, respectively. So, the stress concentration factor for a plate with infinite width can be written as Equation 2 by substituting A_{ij} s.

$$k_t^{\infty} = 1 + \frac{1}{\mu}\sqrt{2\left(\sqrt{\frac{E_x}{E_y}} - v_{xy} + \frac{E_x}{2G_{xy}}\right)} \qquad (2)$$

The parameters k_t^{∞} and μ in equation 1and 2 are equal to 3 and 1 for circular holes (Peterson, 1996). Haywood has presented a formulation to calculate the stress concentration factor around a circular hole for orthotropic material as well as isotropic material. This formulation is a function of the radius of the circular hole r, the width of the laminate W, the stress concentration factor for a plate with infinite width k_t^{∞}, and a magnification factor which is called M in this study. Gross concentration factor has been given by Heywood in Equation 3.

$$\frac{k_t^{\infty}}{k_{tg}} = \frac{3\left(1-\frac{2r}{w}\right)}{2+\left(1-\frac{2r}{w}\right)^3} + \frac{1}{2}\left(\frac{2r}{w}M\right)^6\left(k_t^{\infty}-3\right)\left[1-\left(\frac{2r}{w}M\right)^2\right] \qquad (3)$$

Magnification factor M and the net stress concentration factor k_{tn} are both a function of $\left(1-\frac{2r}{w}\right)$ and they have been given bellow in Equations 4.

$$M^2 = \frac{\sqrt{1-8\left[\frac{3\left(1-\frac{2r}{w}\right)}{2+\left(1-\frac{2r}{w}\right)^3}-1\right]}}{2\left(\frac{2r}{w}\right)^2} \qquad (4)$$

$$k_{tn} = k_{tg}\left(1-\frac{2r}{w}\right)$$

In orthotropic laminate, considering the longitudinal Young's modulus of the laminate as E_1 when all the fibers are parallel to x axis and, also, considering the transverse Young's modulus of the laminate as E_2 when all the fibers are perpendicular to X axis, relations between E_x, E_y, v_{xy}, G_{xy} with E_1, E_2, v_{12} and G_{12} can be written as Equations 5 [13].

$$\frac{1}{E_x} = \frac{C^4}{E_1} + \frac{S^4}{E_2} + \left(\frac{1}{G_{12}} - \frac{2v_{21}}{E_1}\right)C^2S^2$$

$$\frac{1}{E_y} = \frac{S^4}{E_1} + \frac{C^4}{E_2} + \left(\frac{1}{G_{12}} - \frac{2v_{21}}{E_1}\right)C^2S^2$$

$$\frac{v_{xy}}{E_y} = \frac{v_{yx}}{E_x} = \frac{v_{21}}{E_1} - \left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{2v_{21}}{E_1} - \frac{1}{G_{12}}\right)C^2S^2$$

$$\frac{1}{G_{XY}} = 4\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{2v_{21}}{E_1}\right)C^2S^2 + \frac{1}{G_{12}}\left(C^2 - S^2\right)^2 \qquad (5)$$

In equation 5, the parameters c and s are the cosine and sine of the angle between the fibers and the x axis. Considering Glass/epoxy lamina as an orthotropic lamina, elastic properties of glass/epoxy lamina have been given in Table I.

TABLE I. Property of glass/epoxy lamina.

E_1	E_2	V_{12}	G_{12}
43 Gpa	10 Gpa	0.25	7 Gpa

In order to simulate uniaxial tension on the Glass/epoxy lamina, a constants load as 10000 N/m has been applied to both sides of the lamina. Because the width and thickness of lamina are both constant and are equal to 0.5 and 0.001 m, the average stress is just a function of the radius of the hole and has been obtained by considering normal stress by using Equation 6.

$$\sigma_{avg} = \frac{F}{(W-2r).t} \qquad (6)$$

Finite element mesh was generated by using 5802 triangular elements with three nodes for the analysis. The triangle meshes which have been created by ABAQUS are shown in figure 2 for Glass/epoxy lamina.

H. Zamanian and M. Salehi, "Modification to heywood's equations to estimate the stress concentration factors for unidirectional glass epoxy laminates with circular and square holes," *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 4, pp. 18-25, 2016.

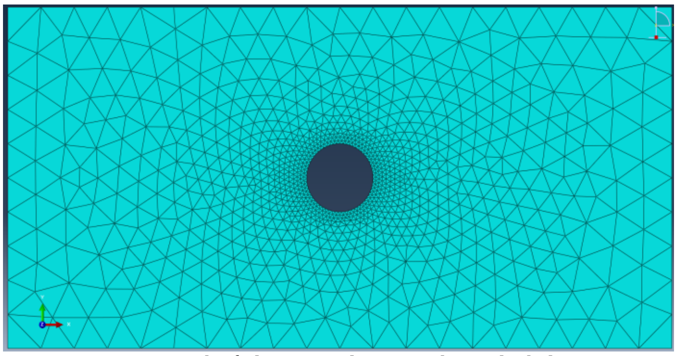

Fig. 2. Mesh of glass/epoxy lamina with circular hole.

Maximum stress and strain distribution have been obtained by considering Von Misses stress distribution criteria by using ABAQUS. In each step of calculating stress concentration, maximum stress has been obtained from Von Misses stress distribution and divided by the average stress. In the next step, by using equations 2 up to 5, the constitutive equations have been generated in MATLAB and then the net stress concentration factor has been extracted and presented with respect to the ratio of the radius of the circular hole over the difference between the width of the laminate and the radius of the circular hole $\left(\frac{r}{d}\right)$ and the stress concentration factor for a plate with infinite width k_t^∞.

III. Results and Discussion

Figure 3 illustrates the net stress concentration factor of a Glass/epoxy lamina around the circular hole in respect to the ratio of the radius of the circular hole over the difference between the width of the laminate and the radius of the circular hole $\left(\frac{r}{d}\right)$. Stress concentration factors for a plate with infinite width k_t^∞, were calculated for different orientation angles including 0,30,45,60, and 90 degrees by generating constitutive equations in MATLAB.

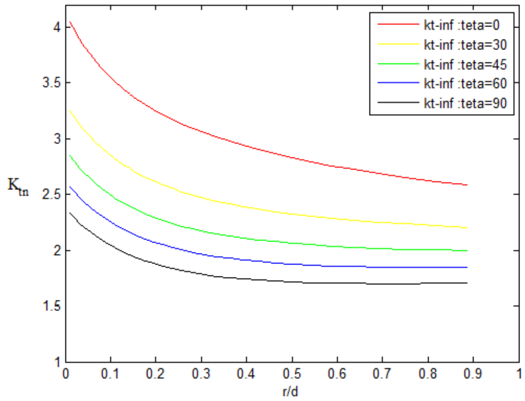

Fig. 3. The net stress concentration factor of a Glass/epoxy lamina around the circular hole by generating the constitutive equations in MATLAB.

Von misses stress distribution (at the top) and strain distribution (at the bottom) for glass/epoxy laminate under uniaxial loading have been shown in figure 4 for the orientation angle of 30 degree, radius of 5 cm, width of 0.5 m, and thickness of 1mm.

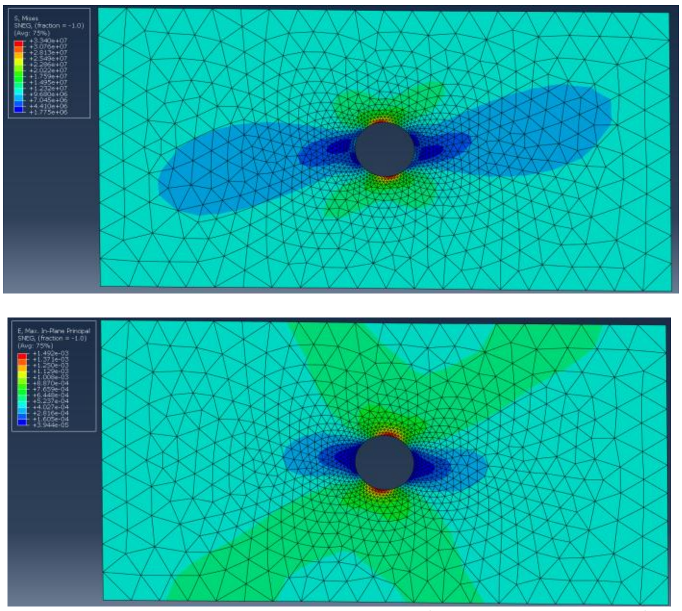

Fig. 4. Von misses stress and strain distribution for glass/epoxy laminate for θ=30, r=5cm, w=0.5m and thickness=1mm.

TABLE II. The net stress concentration factor for glass/epoxy laminate with a circular hole for orientation angle 0, 30, and 45 degrees(* All the stresses are with respect to MPa).

R(m)	d/2(m)	D(m)	r/d	A $(m)^2$	σ_{avg}	σ_{max} θ=0	σ_{max} θ=30	σ_{max} θ=45	K θ=0	K θ=30	K θ=45
0.005	0.245	0.49	0.01	0.00049	10.2	32.96	33.4	28.44	3.230	3.27	2.787
0.01	0.24	0.48	0.02	0.00048	10.42	34.66	32.03	29.16	3.327	3.075	2.799
0.02	0.23	0.46	0.04	0.00046	10.87	34.77	32.37	29.56	3.199	2.978	2.720
0.03	0.22	0.44	0.07	0.00044	11.36	35.26	32.54	29.78	3.103	2.864	2.621
0.04	0.21	0.42	0.10	0.00042	11.9	35.89	33.44	30.45	3.015	2.809	2.558
0.05	0.2	0.4	0.13	0.0004	12.5	35.82	33.61	30.93	2.866	2.689	2.474
0.07	0.18	0.36	0.19	0.00036	13.89	36.78	34.74	31.82	2.648	2.501	2.291
0.11	0.14	0.28	0.39	0.00028	17.86	45.57	42.23	38.54	2.552	2.365	2.158
0.12	0.13	0.26	0.46	0.00026	19.23	48.45	45.27	40.98	2.519	2.354	2.131
0.13	0.12	0.24	0.54	0.00024	20.83	50.21	48.45	43.78	2.410	2.326	2.101
0.14	0.11	0.22	0.64	0.00022	22.73	55.45	52	46.99	2.440	2.288	2.068
0.15	0.1	0.2	0.75	0.0002	25	60.52	54.85	50.58	2.421	2.194	2.023
0.16	0.09	0.18	0.89	0.00018	27.78	65.58	60.71	55.33	2.361	2.186	1.992

TABLE III. Net stress concentration factor for glass/epoxy laminate with circular hole for orientation angle 60 and 90 degrees (* All the stresses are with respect to MPa).

r(m)	d/2(m)	D(m)	r/d	A$(m)^2$	σ_{avg}	σ_{max} θ=60	σ_{max} θ=90	K θ=60	K θ=90
0.005	0.245	0.49	0.01	0.00049	10.2	25.75	23.06	2.523	2.260
0.01	0.24	0.48	0.02	0.00048	10.42	26.25	23.56	2.520	2.262
0.02	0.23	0.46	0.04	0.00046	10.87	26.57	23.7	2.444	2.180
0.03	0.22	0.44	0.07	0.00044	11.36	26.79	24.07	2.358	2.118
0.04	0.21	0.42	0.10	0.00042	11.9	27.17	24.51	2.282	2.059
0.05	0.2	0.4	0.13	0.0004	12.5	27.84	24.88	2.227	1.990
0.07	0.18	0.36	0.19	0.00036	13.89	28.86	26.29	2.078	1.893
0.11	0.14	0.28	0.39	0.00028	17.86	35.34	33.23	1.979	1.861
0.12	0.13	0.26	0.46	0.00026	19.23	37.68	35.68	1.959	1.855
0.13	0.12	0.24	0.54	0.00024	20.83	40.42	38.16	1.940	1.832
0.14	0.11	0.22	0.64	0.00022	22.73	43.73	42.11	1.924	1.853
0.15	0.1	0.2	0.75	0.0002	25	47.95	46.46	1.918	1.858
0.16	0.09	0.18	0.89	0.00018	27.78	52.7	51.38	1.897	1.850

Table II and III present calculation data of the net stress concentration factor for glass/epoxy lamina with a circular hole for different orientation angles including 0,30,45,60, and 90 degrees. In Table II and III, σ_{max} s have been obtained by using ABAQUS

In order to compare statistics data to the constitutive equations, the net stress concentration factor has been depicted in respect to the ratio of the radius of the circular hole over the difference between the width of the laminate and the radius of the circular hole $\left(\frac{r}{d}\right)$ and the stress concentration factor for a plate with infinite width k_t^{∞}, and different orientation angles including 0,30,45,60, and 90 degrees in figure 5 using results of tables II and III.

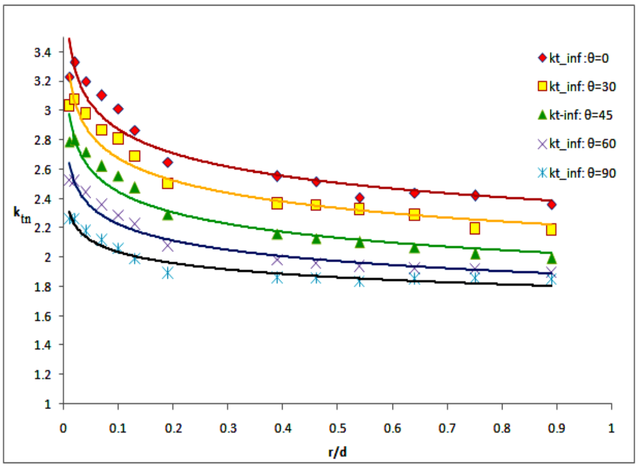

Fig. 5. The net stress concentration for glass/epoxy lamina with circular hole under uniaxial loading by using FEM.

As can be seen, results of FEM, which have been presented in figure 5, are in good agreement with the constitutive equations' results which are presented in figure 3. It can be concluded that, the method of using ABAQUS and creating statistics data, and then extracting the stress concentration factor, is acceptable and can be used for extracting the stress concentration factor around a square hole.

A. Stress Concentration for Square Hole

In this step, by considering the net Stress concentration factor of the orthotropic laminate as a function of length of the hole 2a, width of plate w , orthotropic elastic coefficients $E_{x,}E_{y,}v_{xy,}G_{xy}$, fiber orientation angle θ, the stress concentration factor for a plate with infinite width k_t^{∞} , and a magnification factor M, the same procedure has been utilized to obtain a formula, such as Heywood formulation to estimate the stress concentration factor for glass/epoxy laminates around a square hole. Glass/epoxy lamina with a square hole at the center can be modeled as figure 6, considering the fibers with the angle of θ to the x axis. In this model, 2a is length of square hole, W is the width of the laminate, which is 0.5 m in this study. The thickness of this model is t and has been considered as 1mm for this Glass/epoxy lamina.

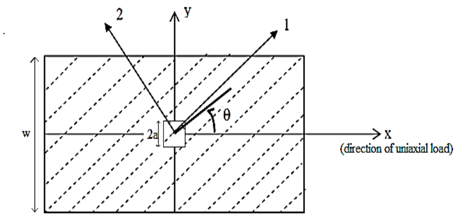

Fig. 6. Orthotropic laminate with square hole under uniaxial loading.

Similar to the Heywood method, the net stress concentration factor for an isotropic plate with a square hole and finite width has been obtained as a function of $\left(1-\frac{2a}{w}\right)$ by using FEM results along with fit data as shown in Equation 7. The procedure of derivation of this formula was proved in our previous article [11].

$$k_{tn} = 0.389 + 4.619\left(1-\frac{2a}{w}\right) - 3.326\left(1-\frac{2a}{w}\right)^2 \qquad (7)$$

Similar to the Heywood method, by comparing the stress concentration factor for a plate with infinite width k_t^{∞} and the net stress concentration factor for an isotropic plate with a square hole k_{tn} together it can be concluded when the width of the plate is infinite, the ratio of (2a/w) is very small. So, if we

H. Zamanian and M. Salehi, "Modification to heywood's equations to estimate the stress concentration factors for unidirectional glass epoxy laminates with circular and square holes," *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 4, pp. 18-25, 2016.

replace zero as (2a/w) in the net stress concentration factor, its value is equal to the stress concentration factor for a plate with infinite width and, finally, we can obtain the value of the coefficient k_t^{∞} and μ for the square hole. For a isotropic plate young's modules in all direction are equal to each other, so we can replace 1 as the ratio of $\frac{E_x}{E_y}$. Considering the relation between the young's module and shear module as $G_{xy}=\frac{E_x}{2(1-v_{xy})}$, we can replace all of these parameters in equation 2 and 7 and obtain k_t^{∞} and μ as 1.682 and 2.93 for a square hole. Consequently, similar to Heywood's formulation, the gross stress concentration factor for an isotropic plate with a square hole can be considered as Equation 8.

$$\frac{k_t^{\infty}}{k_{tg}}=\frac{k_t^{\infty}.\left(1-\frac{2a}{w}\right)}{k_{tn}}=\frac{1.682\left(1-\frac{2a}{w}\right)}{0.389+4.619\left(1-\frac{2a}{w}\right)-3.326\left(1-\frac{2a}{w}\right)^2} \tag{8}$$

The magnification factor should be zero when the width of the plate is infinite. Consequently, similar to the Heywood method, we can drive the magnification factor for a square hole as shown in Equation 9.

$$M^2=\frac{\sqrt{1-8\left[\frac{k_t^{\infty}}{k_{tg}}-1\right]}-1}{2\left(\frac{2a}{w}\right)^2}=\frac{\sqrt{1-8\left[\frac{1.682\left(1-\frac{2a}{w}\right)}{0.389+4.619\left(1-\frac{2a}{w}\right)-3.326\left(1-\frac{2a}{w}\right)^2}-1\right]}-1}{2\left(\frac{2a}{w}\right)^2} \tag{9}$$

In order to create a formula like the Heywood's formulation, we can consider the gross concentration factor of a Glass/epoxy lamina with a square hole as Equation 10 with 3 unknown coefficients, including α, β, and γ and then try to determine these coefficients via statistics data.

$$\frac{k_t^{\infty}}{k_{tg}}=\frac{1.682\left(1-\frac{2a}{w}\right)}{0.389+4.619\left(1-\frac{2a}{w}\right)-3.326\left(1-\frac{2a}{w}\right)^2}+\alpha\left(\frac{2r}{w}M\right)^{\beta}\left(k_t^{\infty}-1.682\right)\left[1-\left(\frac{2r}{w}M\right)^{\gamma}\right] \tag{10}$$

In the next step, finite element mesh was generated by using 10144 triangular elements with three nodes for the analysis of Glass/epoxy lamina with a square hole using ABAQUS, similarly to circular hole. The triangle meshes which have been created by ABAQUS have been shown in figure 7 for Glass/epoxy lamina.

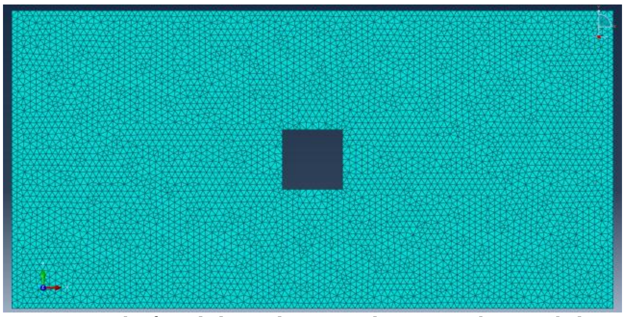

Fig. 7. Mesh of single layer glass/epoxy laminate with square hole.

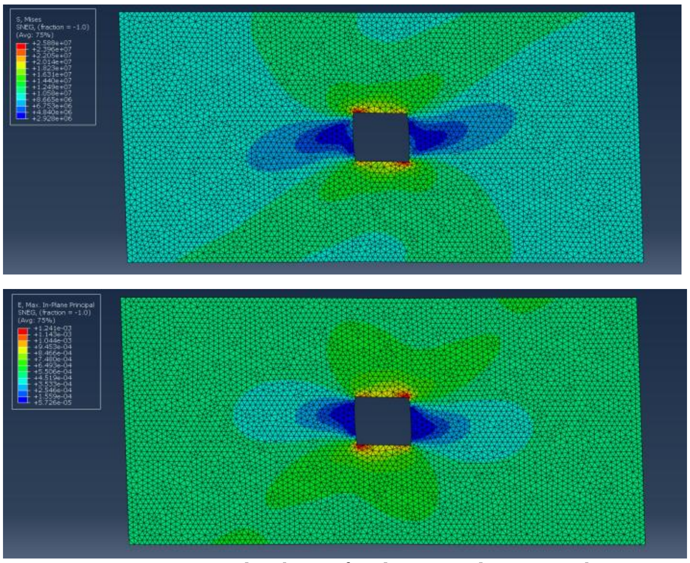

Fig. 8. Von misses stress distribution for glass/epoxy laminate with square hole.

Similar to the circular hole, Von misses stress distribution for glass/epoxy laminate under uniaxial loading has been extracted for different sized square holes, different orientation angles including 0,30,45,60, and 90 degrees, and then maximum stresses are used for obtaining statistics data. Figure 8 shows the Von misses stress distribution (at the top) and strain distribution (at the bottom) of a glass/epoxy lamina with a square hole under uniaxial loading for the orientation angle of 30 degree, width of 0.5 m, thickness of 1mm, and a hole with dimension of 10 cm.

TABLE IV. The net stress concentration factor for glass/epoxy lamina with a square hole for orientation angle 0, 30, 45 degrees (* All the stresses are with respect to MPa).

a(m)	1-2a/w	σ_{avg}	σ_{max} θ=0	σ_{max} θ=30	σ_{max} θ=45	k θ=0	k θ=30	k θ=45
0.025	0.9	11.11	22.82	25.88	20.14	2.054	2.32	1.813
0.05	0.8	12.5	29.11	26.71	25.46	2.329	2.137	2.037
0.075	0.7	14.29	33.43	31.16	29.68	2.34	2.181	2.078
0.1	0.6	16.67	36.89	36.5	33.91	2.213	2.19	2.035
0.125	0.5	20	44.31	41.82	38.82	2.216	2.091	1.941
0.15	0.4	25	52.07	49.04	44.5	2.083	1.962	1.78
0.165	0.34	29.41	57.65	53.6	48.76	1.96	1.822	1.658

Table IV and V present statistics data for calculation of the net stress concentration factor for glass/epoxy lamina with a square hole for different orientation angles, including 0,30,45,60, and 90 degrees. In table IV and V, σ_{max}s have been obtained by using ABAQUS.

TABLE V. The net stress concentration factor for glass/epoxy lamina with a square hole for orientation angle 60 and 90 degrees (* All the stresses are with respect to MPa)

a(m)	1-2a/w	σ_{avg}	σ_{max} θ=60	σ_{max} θ=75	K θ=60	K θ=90
0.025	0.9	11.11	19.3	18.67	1.737	1.654
0.050	0.8	12.5	24.2	23.22	1.936	1.829
0.075	0.7	14.29	27.9	26.57	1.957	1.830
0.100	0.6	16.67	31.7	30.03	1.903	1.772
0.125	0.5	20	36.4	34.49	1.825	1.675
0.150	0.4	25	41.3	39.03	1.653	1.512
0.165	0.34	29.41	46	43.92	1.566	1.451

In the next step, results of stress concentration factors, which have been presented in Table IV and V, have been inserted in MATLAB toolbox and then a curve fitting task was done and, finally, 3 unknown coefficients in equation 12 including α, β, and γ have been obtained as α=0.4 , β= -0.5, and γ=0.5. So, by replacing α, β, and γ in Equation 10, the final formula to estimate the stress concentration factor of a glass/epoxy lamina around square holes can be written as bellow in Equations 11.

$$\frac{k_t^\infty}{k_{tg}} = \frac{1.682\left(1-\frac{2a}{w}\right)}{0.389+4.619\left(1-\frac{2a}{w}\right)-3.326\left(1-\frac{2a}{w}\right)^2} + 0.4\left(k_t^\infty-1.682\right)\sqrt{\frac{\left[1-\left(\frac{2a}{w}M\right)\right]}{\left(\frac{2a}{w}M\right)}} \qquad (11)$$

$$k_{tn}=k_{tg}\left(1-\frac{2a}{w}\right)$$

As shown in figure 9, similar to circular hole equations 2, 5, 9, and 11 have been generated in MATLAB and the net stress concentration factor of a Glass/epoxy lamina around a square hole has been depicted in respect to $\left(1-\frac{2a}{w}\right)$ and the stress concentration factor for a plate with infinite width k_t^∞ for different orientation angles including 0,30,45,60, and 90 degrees.

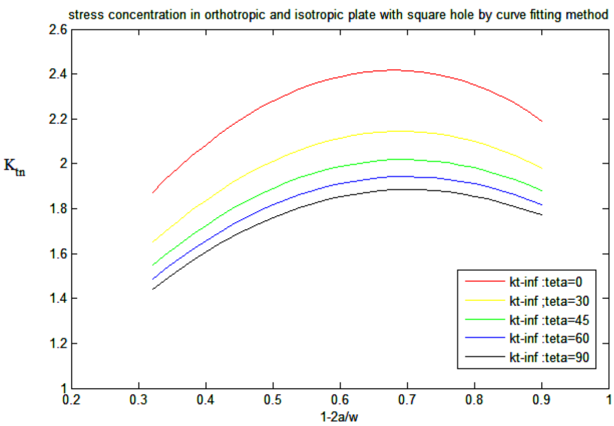

Fig. 9. The net stress concentration for glass/epoxy laminate with square hole under uniaxial loading by present.

In order to compare the statistics data to results of equations 2,5, 9, and 11, results of tables IV and V have been used and the net stress concentration factors have been depicted in respect to $\left(1-\frac{2a}{w}\right)$, and the stress concentration factor for a plate with infinite width k_t^∞ for different orientation angles, including 0,30,45,60, and 90 degrees in figure 10.

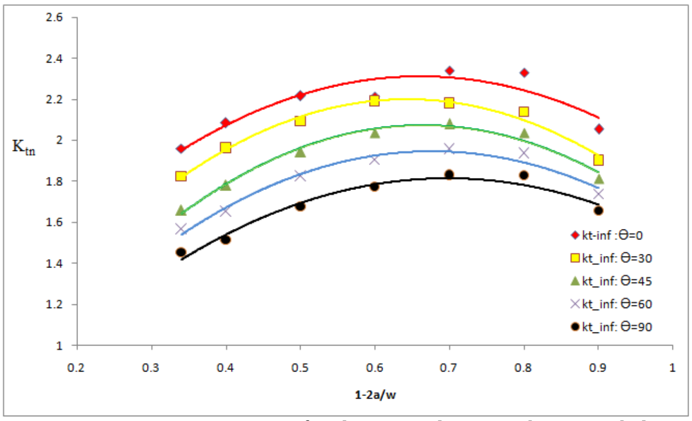

Fig. 10. Net stress concentration for glass/epoxy lamina with a square hole under uniaxial loading by FEM.

As can be seen, results of FEM, which have been presented in figure 10, are in good agreement with the constitutive equations results, which are presented in figure 9. It can be concluded that, the accuracy of equation 13, which has been created in this study, is acceptable. To do more investigation on the accuracy, and also the generality of the obtained formula again by using ABAQUS and MATLAB, results of the present formula were compared to FE results for different materials such as steel, aluminum, e-glass/epoxy, s-glass/epoxy laminas with a square hole at the center of them, and results are presented in Tables VII to X. The material properties, which have been used in order to analyze these material in MATLAB and ABAQUS, have been presented in Table VI.

TABLE VI. Material properties of steel, aluminum, e-glass/epoxy, s-glass/epoxy.

Material Property	Units	Steel	Aluminum	E-Glass/Epoxy	S-Glass/Epoxy
E_1	Gpa	207	75.8	39	43
E_2	Gpa	207	75.8	8.6	8.9
G_{12}	Gpa	79.6	0.33	3.8	4.5
ν_{12}	-	0.3	28.5	0.28	0.27

TABLE VII. Net stress concentration factor of steel.

2a	W	k_t^∞	k_{tn} (formula)	k_{tn} (FEM)	Difference%
0.1	0.5	1.682	1.95	1.94	0.5
0.15	0.5	1.682	1.99	1.97	1.0
0.3	0.5	1.682	1.7	1.7	0.0

TABLE VIII. Net stress concentration factor of aluminum.

2a	W	k_t^{∞}	k_{tn} (formula)	k_{tn} (FEM)	Difference %
0.1	0.5	1.682	1.95	1.93	1.0
0.15	0.5	1.682	1.99	1.98	0.5
0.3	0.5	1.682	1.7	1.67	1.7

TABLE IX. Net stress concentration factor of e-glass/epoxy.

2a	1-2a/w	θ	k_t^{∞}	k_{tn} (formula)	k_{tn} (FEM)	Difference %
0.1	0.8	30	1.7	1.972	2.12	7.0
0.1	0.8	45	1.57	1.846	1.96	5.8
0.1	0.8	60	1.55	1.83	1.85	1.1
0.1	0.8	90	1.59	1.879	1.84	2.1

TABLE X. Net stress concentration factor of s-glass/epoxy.

2a	1-2a/w	θ	k_t^{∞}	k_{tn} (formula)	k_{tn} (FEM)	Difference %
0.1	0.8	30	1.73	1.993	2.13	6.4
0.1	0.8	45	1.59	1.867	1.97	5.2
0.1	0.8	60	1.55	1.836	1.87	1.8
0.1	0.8	90	1.57	1.851	1.83	1.1

As can be seen in Tables VII to X, results of FEM are in good agreement with results of constitutive equations; these results have less than 10 percent difference. Additionally, another task has been done in order to show the fact that designing two smaller holes around the larger circular hole can decrease the stress concentration factor. A Glass/epoxy laminate with 8 layers and with orientation angles of [90,-45,45,0,0,45,-45,90], with the thickness of 1mm and width of 0.5 m, has been modeled in ABAQUS and its stress concentration factor has been extracted by utilizing the same procedure. Triangular elements with three nodes have been generated by using ABAQUS. Figure 11 shows the triangular meshes which have been created for this Glass/epoxy laminate with 3 circular holes under uniaxial loading with 8 layer, all having the same thickness, orientation angles [90,-45,45,0,0,45,-45,90] ,width of 0.5 m, thickness of 1mm, and two smaller holes with a dimension of 50% of the larger hole.

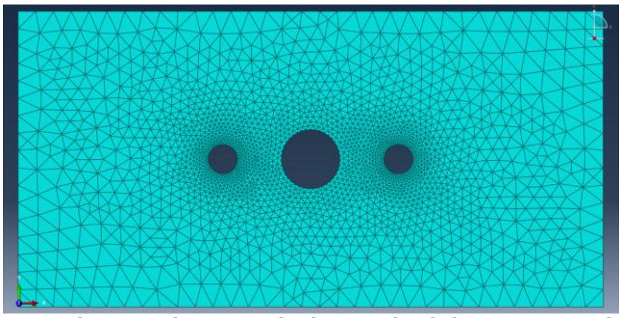

Fig. 11. Glass/epoxy laminate with 8 layers and with the orientation angles [90,-45,45,0,0,45,-45,90].

Similar to the previous procedures, Von misses stress and strain distribution for this glass/epoxy laminate have been extracted for different sized circular holes and then maximum stresses have been used to calculate the stress concentration factor. Figure 12 shows the Von misses stress and strain distribution of glass/epoxy laminate with and without two smaller holes, and a large circular hole at the center for Glass/epoxy laminate with 8 layers, and with the orientation angles [90,45,45,0,0,45,45,90].

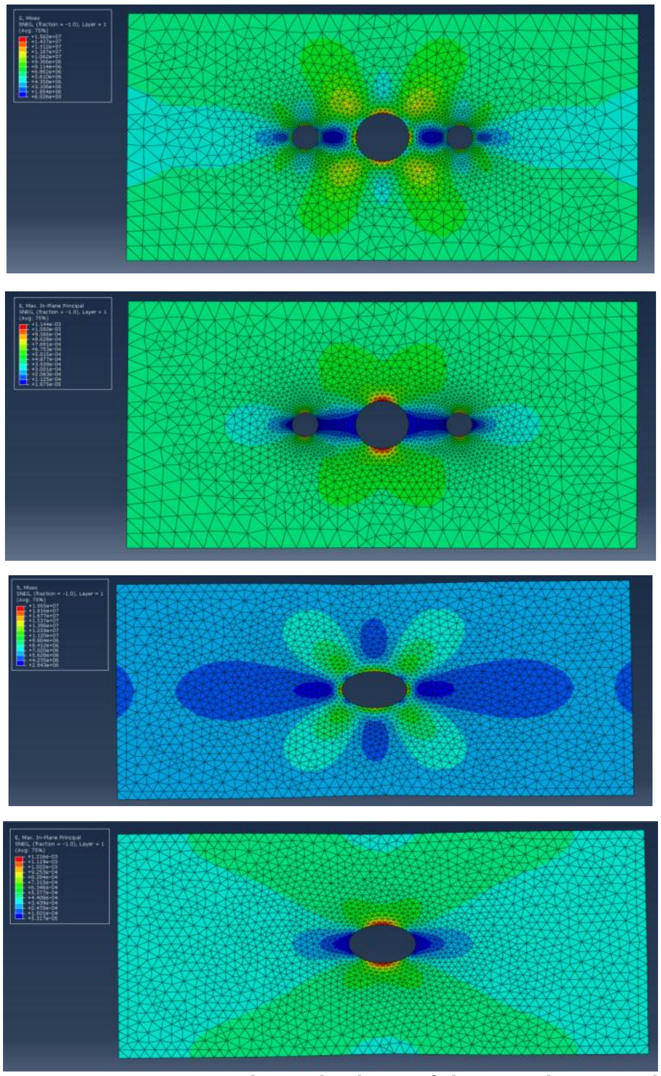

Fig. 12. Von misses stress and strain distribution of glass/epoxy laminate with and without two smaller holes and a large circular hole at the center for Glass/epoxy laminate with 8 layers and with the orientation angles [90,45,45,0,0,45,45,90].

TABLE XI. The stress concentration factor of the Glass/epoxy laminate with and without the two smaller holes.

r(m)	r/d	A(m²)	σ_{avg}	σ_{max} without two smaller holes	σ_{max} With two smaller holes	SCF without two smaller holes	SCF with two smaller holes
0.02	0.04	0.00046	10.87	1.784	1.507	1.641	1.386
0.03	0.07	0.00044	11.36	1.824	1.602	1.606	1.410
0.05	0.13	0.0004	12.5	1.955	1.562	1.564	1.250
0.07	0.19	0.00036	13.89	2.077	1.652	1.495	1.189
0.1	0.39	0.00028	17.86	2.207	1.753	1.236	0.982
0.11	0.46	0.00026	19.23	2.584	2.238	1.344	1.164
0.12	0.54	0.00024	20.83	2.993	2.302	1.437	1.105
0.13	0.64	0.00022	22.73	2.894	2.543	1.273	1.119

H. Zamanian and M. Salehi, "Modification to heywood's equations to estimate the stress concentration factors for unidirectional glass epoxy laminates with circular and square holes," *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 4, pp. 18-25, 2016.

Table X presents the calculation of the net stress concentration factor for glass/epoxy laminate with and without two smaller holes, and a large circular hole at the center for Glass/epoxy laminate with 8 layers and with the orientation angles [90,45,45,0,0,45,45,90]. In Table XI, σ_{max} s have been obtained by using ABAQUS.

Figure 13 shows the stress concentration factor of the Glass/epoxy laminate with and without the two smaller holes.

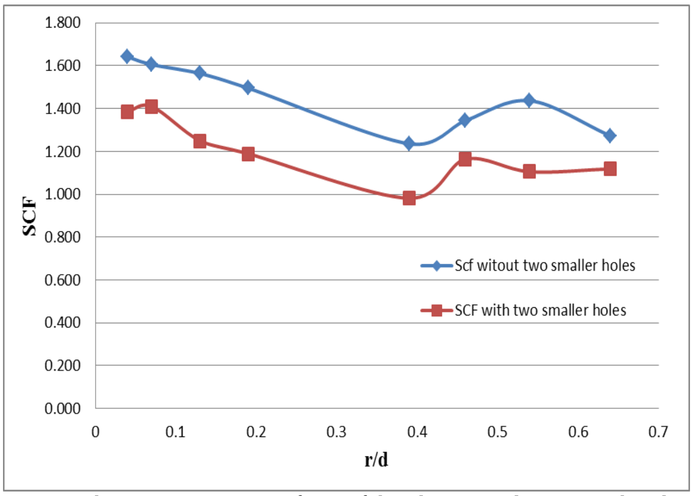

Fig. 13. The stress concentration factor of the Glass/epoxy laminate with and without the two smaller holes.

As can be seen in figure 13, designing two smaller holes around the main hole can decrease the stress concentration factor. Regarding the optimum size of the smaller holes, future investigations are needed to determine their optimum values.

IV. Conclusion

In this study we extracted the maximum stress and stress concentration factor around the circular hole for a glass/epoxy laminate using ABAQUS, and then we verified results of ABAQUS with Heywood's constitutive equations, which were generated in MATLAB. As an innovation, considering Heywood's constitutive equations and their variables, we showed the stress concentration factor for a Glass/epoxy lamina with a square hole can be obtained by using ABAQUS results and doing a curve fitting task in MATLAB. In addition, this study showed designing two smaller holes around the main hole can decrease the stress concentration factor around the main hole for Glass/epoxy laminates.

References

[1] L. Toubal, M. Karama, and B. Lorrain, "Stress concentration in a circular hole in composite plate," *Composite Structures*, vol. 68, issue 1, pp. 31–36, 2005.

[2] R. C. J. Howland, "On the stresses in the neighborhood of a circular hole in a strip under tension," *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, vol. 229, pp. 49-86, 1930.

[3] B. P. Shastry and G. Venkateswara Rao, "Studies on stress concentration in tensile strips with large circular holes," *Computers & Structures*, vol. 19, no. 3, pp. 345-349, 1984.

[4] T. K. Paul and K. M. Rao, "Stress analysis around an elliptical hole in thick FRP laminates under transverse loading," *Computers & Structures*, vol. 35, No. 5, pp. 553-561, 1990.

[5] N. Arslan, M. Çelik, and N. Arslan, "Prediction of the elastic-plastic behavior of thermoplastic composite laminated plates with square hole," *Composite Structures*, vol. 55, issue 1, pp. 37-49, 2002.

[6] H.-C. Wu and B. Mu, "On stress concentrations for isotropic/orthotropic plates and cylinders with a circular hole," *Composites Part B: Engineering*, vol. 34, issue 2, pp. 127–134, 2003.

[7] V. G. Ukadgaonker, and D. K. N. Rao, "A general solution for stresses around holes in symmetric laminates under in-plane loading," *Composite Structures*, vol. 49, issue 3, pp. 339-354, 2000.

[8] G. N. Savin, *Stress Concentration around Holes*, Oxford/London/New York/Paris: Pergamon Press, 1961.

[9] N. K. Jain and N. D. Mittal, "Finite element analysis for stress concentration and deflection in isotropic,orthotropic and laminated composite plates with central circular hole under transverse static loading," *Materials Science and Engineering: A*, vol. 498, issue 1-2, pp. 115–124, 2008.

[10] M. Mohammadi, J. R. Dryden, and L. Jiang, "Stress concentration around a hole in a radially inhomogeneous plate," *International Journal of Solids and Structures*, vol. 48, issue 3-4, pp. 483–491, 2011.

[11] H. Zamanian, B. Marzban, P. Bagheri, and M. Gudarzi, "On stress concentration factor for randomly oriented discontinuous fiber laminas with circular/square hole," *Journal of Science and Engineering*, Vol. 3, issue 1, pp. 7-18, 2013.

[12] R. Peterson, *Stress Concentration Design Factors*, New York: John Wiley and Sons, 1996.

[13] V. V. Vasiliev and E. V. Morozov, *Mechanics and Analysis of Composite Material*, Elsevier, 2007.

H. Zamanian and M. Salehi, "Modification to heywood's equations to estimate the stress concentration factors for unidirectional glass epoxy laminates with circular and square holes," *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 4, pp. 18-25, 2016.