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Nonlinear Analysis of the HIV/AIDS Control Pandemic Model within a Heterogeneous Population

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***Abstract*—*In this paper, the Homotopy Perturbation Method (HPM) is used to solve the system of non-linear deterministic models on the control of HIV/AIDS using the therapeutic dose within a heterogeneous population. A homotopy is constructed with an embedding parameter* $\rho \in [0,1]$ *which is considered as a small parameter. The HPM deforms a difficult problem into a more simple problem which can be easily solved. The HPM is implemented with appropriate initial condition. The approximate analytical solutions obtained are uniformly valid not only for small parameters but also good for a very large parameters. The method gets solution for non-linear models without any discretization linearization or any restrictive assumptions*.

***Keywords*—** *Homotopy Perturbation Method, Non-Linear, Differential Equation.*

I. HIV Therapy Introduction

There is currently no cure or effective HIV Vaccine. Treatment consists of highly active antiretroviral Therapy (HAART) which slows progression to HIV/AIDS. The management of HIV/AIDS normally includes the use of multiple antiretroviral drugs in an attempt to control HIV infection.

There are several classes of antiretroviral agents that act on different stages of the HIV life cycle. The use of multiple drugs that act on different viral targets is known as (HAART), HAART decreases the patient total burden of HIV, maintains the function of the immune system and there after prevent the opportunistic infection that often led to death. [1]

The progression to AID of an HIV Patient is very rare as at present because treatment has been successful in the past few years. An AIDS free generation is indeed within reach according to Natural Institute of Allergy and Infectious disease(NIAID). They noted that in 2010 alone, an estimated 700,000 lives were saved by antiretroviral therapy [2], [3].

The antiretroviral therapy is recommended to be offered to all patients with HIV because of the complexity of selecting the potential side effects and the importance of taking medication regularly to prevent viral resistance, emphasizing the important of involving patient in therapeutic choice and the potential benefits [4] [5].

The first effective therapeutic agent to HIV was the Nucleoside reverse transriptose inhibitor (NRTI), Zidovudine (AZT) approved by US.FDA in 1987 [6].

The homotopy perturbation method (HPM) has been applied to wide class of nonlinear problem in Engineering and Sciences. Some of these problems can usually be reduced to a system of integral equation [7]. Several methods has been proposed to obtain analytical solution to linear and nonlinear problems such as Adomian Decomposition Method (ADM), differential Transformation Method (DTM), and Decomposition method [8]. The homotopy perturbation method (HPM) has some significant advantages over numerical methods. It provides analytical, verifiable rapidly convergent approximation which yield insight into the character and the behavior of the solution. The homotopy perturbation method solve any linear or nonlinear equation in Sciences and Engineering. Homotopy Perturbation Method was used to solve some initial Boundary problems with local conditions [9]. The method, Homotopy Perturbation method was applied to solution of partial differential equation [10] and in [11], the homotopy perturbation method was used to solve some linear and non-linear parabolic method. We in this paper intend to solve systems of non-linear differential models using Homotopy perturbation method.

II. Basic Idea of Homotopy Pertubation Method

To illustrate the idea, we consider the following non-linear differential equation.

$$A(u) - f(r) = 0 \quad r\varepsilon\Omega --------------- (i)$$

With the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \quad r\varepsilon\tau --------------- (ii)$$

Where, A= Differential equation
B= Boundary operation
$f(r)$ = Analytical function and
τ = The boundary of the domain Ω.

An operation A can be divided into two parts L and N where L is the Linear part and N is the nonlinear part. Equation (1) can be written as

$$L(u) + N(u) - f(r) = 0 ------------ (iii)$$

By the homotopy technique [12], we construct a homotopy $v(r, p): \Omega \times [0,1] \to R$ which satisfies

$$p(v, p) = (1-p)[(L(v) - L(u_0) + p + p[A(v) - f(r)] = 0$$

$p \in [0,1], r \in \Omega$ or

$$p(v, p) = L(v) - L(u_0) + pL(u_0) + pN(v) - f(v) = 0 ---- (iv)$$

Where $p \in [0,1]$ is an embedded parameter. u_0 Is the initial approximation of (i) which then satisfies

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$$p(v,0)=L(v)-L(u_0)=0-----------(v)$$

$$p(v,1)=A(v)-f(v)=0------------(vi)$$

We then assume the solution for (iv) as a power series in p as

$$v=v_0+pv_1+p^2v_2+..........................(vii)$$

Let $p=1$, then $u=\lim v=v_0+v_1+v_2+...as\ \ p\to 1$

III. Application to the Model

Consider the models

$$\frac{dS}{dt}=\pi-\delta S+\varepsilon H-\mu S \quad (1)$$

$$\frac{dH}{dt}=\delta S-\varepsilon H-(1-\varphi)kH-\mu H \quad (2)$$

$$\frac{dE}{dt}=(1-\varphi)kH-\phi E-\mu E \quad (3)$$

$$\frac{dI}{dt}=\phi E-\tau cI-\mu I \quad (4)$$

$$\frac{dI_2}{dt}=\tau cI-\mu I_2 \quad (5)$$

Subject to the following initial conditions

$S(0)>0,\ H(0)>0,\ E(0)>0,\ I(0)>0,\ I_2(0)>0$

We factor out the parameters to ease our analysis

$$\frac{dS}{dt}=\pi+\varepsilon H-(\delta+\mu)S \quad (6)$$

$$\frac{dH}{dt}=(\delta)S-(\varepsilon+(1-\varphi)k+\mu)H \quad (7)$$

$$\frac{dE}{dt}=((1-\varphi)k)H-(\phi+\mu)E \quad (8)$$

$$\frac{dI}{dt}=\varphi E-(\tau c+\mu)I \quad (9)$$

$$\frac{dI_2}{dt}=\tau cI-\mu I_2 \quad (10)$$

We let

$$m_1=\delta+\mu, m_2=\delta, m_3=\varepsilon+(1-\varphi)K+\mu, m_4=(1-\varphi)K$$

$$m_5=\phi+\mu, m_6=\tau c+\mu$$

Therefore our models will now become;

$$\frac{dS}{dt}=\pi+\varepsilon H-m_1S \quad (11)$$

$$\frac{dH}{dt}=m_2S-m_3H \quad (12)$$

$$\frac{dE}{dt}=m_4H-m_5E \quad (13)$$

$$\frac{dI}{dt}=\phi E-m_6I \quad (14)$$

$$\frac{dI_2}{dt}=\tau cI-\mu I_2 \quad (15)$$

We then assume solution to the models ranging from (11) to (15) as

$$S(t)=s_0+Ps_1+P^2s_2+... \quad (16)$$

$$H(t)=h_0+Ph_1+P^2h_2+... \quad (17)$$

$$E(t)=e_0+Pe_1+P^2e_2+... \quad (18)$$

$$I(t)=i_0+Pi_1+P^2i_2+... \quad (19)$$

$$I_2(t)=a_0+Pa_1+P^2a_2+... \quad (20)$$

We now apply HPM to (11) to (15)

Consider (15)

$$\frac{dS}{dt}=\pi+\varepsilon H-m_1S$$

The linear part is

$$\frac{dS}{dt}=0$$

And the nonlinear part is

$$\pi+\varepsilon H-m_1S=0$$

Applying the HPM, we then have

$$(1-P)\frac{dS}{dt}+p\left[\frac{dS}{dt}-(\pi+\varepsilon H-m_1S)\right]=0$$

This gives

$$\frac{dS}{dt}-P\frac{dS}{dt}+P\frac{dS}{dt}-P(\pi+\varepsilon H-m_1S)=0$$

this implies

$$\frac{dS}{dt}-P\pi-P\varepsilon H+Pm_1S=0 \quad (21)$$

Substituting (16) and (17) into (21), we have

$$(S_0'+PS_1'+P^2S_2'+...+)-P\pi-P\varepsilon(h_0+Ph_1+P^2h_2+...)+ Pm_1(s_0+Ps_1+P^2s_2+...)=0$$

This gives

$$S_0'+PS_1'+P^2S_2'+...+-(P\pi+P\varepsilon h_0+P\varepsilon Ph_1+\varepsilon P^3h_2)+...+ Pm_1s_0+Pm_1Ps_1+P^3m_1s_2+...=0$$

Collecting the coefficient of the power of P's, we have

$$P^0: S_0'=0 \quad (22)$$

$$P^1: S_1'-\pi-\varepsilon h_0+m_1s_0=0 \quad (23)$$

$$P^2: S_2'-\varepsilon h_1+m_1s_1=0 \quad (24)$$

From (12) we have

$$\frac{dH}{dt}=m_2s-m_3H$$

With the linear part as

$$\frac{dH}{dt}=0$$

And $m_2s-m_3H=0$ as the nonlinear part of (12)

Applying Homotopy to (12) we have

$$(1-P)\frac{dH}{dt}+p\left[\frac{dH}{dt}-(m_2s-m_3H)\right]=0$$

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$$\Rightarrow \frac{dH}{dt} + p\frac{dH}{dt} + P\frac{dH}{dt} - P(m_2 s - m_3 H) = 0$$

$$\Rightarrow \frac{dH}{dt} - P(m_2 s - m_3 H) \qquad (25)$$

$$\Rightarrow \frac{dH}{dt} - Pm_2 s + Pm_3 H = 0$$

Substituting (16) and (17) in (25) we have

$$h_0' + Ph_1' + P^2 h_2' + ... + -P(m_2(s_0 + Ps_1 + P^2 s_2 + ...)) + Pm_3(h_0 + Ph_1 + P^2 h_2 + ...) = 0$$

$$\Rightarrow h_0' + Ph_1' + P^2 h_2' + ... + -m_2 Ps_0 - m_2 Ps_1 - m_2 P^2 s_2 + ... + Pm_3 h_0 + m_3 P^2 h_1 + m_3 P^3 h_2 + ... = 0$$

Collecting the coefficient of P's we have

$$P^0 : h_0' = 0 \qquad (26)$$

$$P^1 : h_1' - m_2 s_0 + m_3 h_0 = 0 \qquad (27)$$

$$P^2 : h_2' - m_2 s_1 + m_3 h_1 = 0 \qquad (28)$$

Applying Homotopy perturbation method to (13) we have;

$$\frac{dE}{dt} = m_4 H - m_5 E = 0$$

With $\frac{dE}{dt} = 0$ as the linear part and

$m_4 H - m_5 E = 0$ as the nonlinear part, therefore

$$(1-P)\frac{dE}{dt} + p\left[\frac{dE}{dt} - (m_4 H - m_5 E)\right] = 0$$

$$(1-P)\frac{dE}{dt} + p\frac{dE}{dt} - Pm_4 H + Pm_5 E = 0$$

This gives

$$\frac{dE}{dt} - Pm_4 H + Pm_5 E = 0 \qquad (29)$$

Substituting (17) and (18) in (29), we have

$$e_0' + Pe_1' + P^2 e_2' + ... - Pm_4(h_0 + Ph_1 + P^2 h_2 + ...) + Pm_5(e_0 + Pe_1 + P^2 e_2 + ...) = 0$$

Collecting the powers of P's we have

$$P^0 : e_0' = 0 \qquad (30)$$

$$P^1 : e_1' - m_4 h_0 + m_5 e_0 = 0 \qquad (31)$$

$$P^2 : e_2' - m_4 h_1 + m_5 e_1 = 0 \qquad (32)$$

Applying the Homotopy perturbation method to (.14)

$$\frac{dI}{dt} = \phi E - m_6 I = 0$$

With the linear part as $\frac{dI}{dt} = 0$ and $\phi E - m_6 I$ as the nonlinear part; therefore

This gives

$$(1-P)\frac{dI}{dt} + p\left[\frac{dI}{dt} - (\phi E - m_6 I)\right] = 0$$

$$\Rightarrow \frac{dI}{dt} - P\phi E + Pm_6 I = 0 \qquad (33)$$

Substituting (18) and (19) in (.33) we have

$$i_0' + Pi_1' + P^2 i_2' + ... + -P\phi(e_0 + Pe_1 + P^2 e_2 + ...) + Pm_6(i_0 + Pi_1 + P^2 i_2 + ...) = 0$$

Collecting the powers of P's we have

$$P^0 : i_0' = 0 \qquad (34)$$

$$P^1 : i_1' - \phi e_0 + m_6 i_0 = 0 \qquad (35)$$

$$P^2 : i_2' - \phi e_1 + m_6 i_1 = 0 \qquad (36)$$

Applying the Homotopy perturbation method to (15)

$$\frac{dI_2}{dt} = \tau cI - \mu I_2$$

where $\frac{dI_2}{dt} = 0$ is the linear part and $\tau cI - \mu I_2$ as the other part

Therefore

$$(1-P)\frac{dI_2}{dt} + p\left[\frac{dI_2}{dt} - (\tau cI - \mu I_2)\right] = 0$$

Which gives

$$\frac{dI_2}{dt} - P\tau cI + P\mu I_2 = 0 \qquad (37)$$

Substituting (19) and (20) in (.37) we have

$$a_0' + Pa_1' + P^2 a_2' + ... + -P\tau c(i_0 + Pi_1 + P^2 i_2 + ...+) + P\mu(a_0 + Pa_1 + P^2 a_2 + ...+) = 0$$

Collecting the coefficient of the power of P's, we have

$$P^0 : a_0' = 0 \qquad (38)$$

$$P^1 : a_1' - \tau ci_0 + \mu a_0 = 0 \qquad (39)$$

$$P^2 : a_2' - \tau ci_1 + \mu a_1 = 0 \qquad (40)$$

From (.22), we have

$$S_0' = 0$$

$$\Rightarrow \frac{dS_0}{dt} = 0$$

Integrating we have

$$S_0 = C_1$$

Subject to the initial condition

$$S_0(0) = S_0$$

$$\Rightarrow C_1 = S_0$$

$$\therefore S_0 = S_0 \qquad (41)$$

From (26)

$$h_0' = 0$$

Integrating

$$\int h_0 = 0$$

$\Rightarrow h_0 = C_2$ where C_2 is the constant of integration

$$h_0(0) = H_0$$

We have

$\Rightarrow C_2 = H_0$

$h_0 = H_0$ (42)

And from (30)

$e_0' = 0$

$\Rightarrow \int e_0 = 0$

Integrating, we have

$e_0 = C_3$

from $e_0(0) = E_0$, *we have*

$e_0 = C_3 = E_0$

$\therefore e_0 = E_0$ (43)

From (34) we have

$i_0' = 0$

Integrating, $i_0 = C_4$

Applying the initial condition we have

$i_0(0) = I_0$

$C_4 = I_0$

$\Rightarrow i_0 = I_0$ (44)

And from (38) we have

$a_0' = 0$

Integrating, we have

$a_0 = C_5$

but $a_0(0) = I_2(0)$

$\Rightarrow C_5 = I_2(0)$

$\Rightarrow a_0 = I_2(0)$ (45)

From (23), we have

$S_1' = \pi - \varepsilon h_0 + m_1 s_0$ (46)

Substituting (41) and (42) in (46) we have

$S_1' = \pi + \varepsilon h_0 - m_1 s_0$

$$\Rightarrow \frac{dS_1}{dt} = \pi + \varepsilon h_0 - m_1 s_0$$

Integrating,

$\int dS_1 = (\pi + \varepsilon h_0 - m_1 s_0)\int dt$

$\Rightarrow S_1 = (\pi + \varepsilon h_0 - m_1 s_0)t + C_6$

Where C_6 is the constant of integration from $S_1(0) = 0$ we have that $C_6 = 0$ therefore

$S_1 = (\pi + \varepsilon h_0 - m_1 s)t$ (47)

From (.27), we have

$h_1' - m_2 s_0 + m_3 h_0 = 0$ (48)

Substituting (41) and (47) in (50) we have

$\Rightarrow h_1' = m_2 s_0 - m_3 H_0$

Integrating we have

$h_1 = (m_2 s_0 - m_3 H_0)t + C_7$

where C_7 is the constant of integration and $h_1(0) = 0 \Rightarrow C_7 = 0$

$h_1 = (m_2 s_0 - m_3 H_0)t$ (49)

From (4.5.31), we have

$e_1' = m_4 h_0 - m_5 e_0$ (50)

Substituting (42) and (43) in (52) we have

$e_1' = m_4 H_0 - m_5 E_0$

Integrating we have

$e_1 = (m_4 H_0 - m_5 E_0)t + C_8$

Where C_8 is the constant of integration and $e_1(0) = 0 \Rightarrow C_8 = 0$

$\Rightarrow e_1 = (m_4 H_0 - m_5 E_0)t$ (51)

From (4.5.35) we have

$i_1' = \phi e_0 - m_6 i_0$ (52)

Substituting (43) and (44) in (52) we have

$i_1' = (\phi E_0 - m_6 I_0)t$

Integrating we have

$i_1 = (\phi E_0 - m_6 I_0)t + C_9$

Where C_9 is the constant of integration

Applying the initial condition

$i_1(0) = 0 \qquad \Rightarrow C_9 = 0$

Therefore

$i_1 = (\phi E_0 - m_6 I_0)t$ (53)

And from (39)

$a_1' = \tau c i_0 - \mu a_0$

Substituting (44) and (45) in (56), we have

ss $a_1' = \tau c I_0 - \mu I_{2,0}$

Integrating, we have

$a_1 = (\tau c I_0 - \mu I_{2,0})t + C_{10}$

Where C_{10} is the constant of integration

Applying the initial conditions

$a_1(0) = 0$

$\Rightarrow C_{10} = 0$

Therefore

$a_1 = (\tau c I_0 - \mu I_{2(0)})t$ (54)

From (24)

$\Rightarrow S_2 = -(-\varepsilon h_1 + m_1 s_1)$ (55)

Substituting (.49) and (47) in (.58) we have

$S_2' = \varepsilon(m_2 S_0 - m_3 H_0)t - m_1(\pi + \varepsilon H_0 - m_1 S_0)t$

$= (\varepsilon m_2 S_0 - \varepsilon m_3 H_0)t - (m_1 \pi + m_1 \varepsilon H_0 - m_1^2 S_0)t$

$S_2' = ((\varepsilon m_3 H_0 - \varepsilon m_2 S_0) - m_1 \pi - m_1 \pi - m_1 \varepsilon H_0 + m_1^2 S_0))t$

$S_2' = (\varepsilon(m_3 - m_1)H_0 - (\varepsilon m_{2\_} m_1^2)S_0 - m_1 \pi)t$

Integrating we have

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$$S_2 = (\varepsilon(m_3 - m_1)H_0 - (\varepsilon m_2 - m_1^2)S_0 - m_1\pi)\frac{t^2}{2} + c_{11}$$

where C_{11} is the constant of integration

Applying the initial condition

$S_2(0) = 0 \Rightarrow C_{11} = 0$

Therefore

$$S_2 = (\varepsilon(m_3 - m_1)H_0 - (\varepsilon_2 - m_1^2)S_0 - m_1\pi)\frac{t^2}{2}$$

From (.16), we obtain the solution for (1) as;

$$S(t) = S_0 + PS_1 + P^2S_2 + ... +$$

$$\Rightarrow S(t) = S_0 + P(\pi + \varepsilon H_0 - m_1 S_t)t + P^2\left(\varepsilon(m_3 - m_1)H_0 - (\varepsilon m_2 - m_1\pi)S_0 - m_1\pi\right)\frac{t^2}{2} + ... +$$

Setting P = 0

$S(t) = S_0$

But setting P = 1, we have

$$S(t) = S_0 + (\pi + \varepsilon H_0 - m_1 S_0)t + \left(\varepsilon(m_3 - m_1)H_0 - (\varepsilon m_2 - m^2{}_1\pi)S_0 - m_1\pi\right)\frac{t^2}{2} + ... + \quad (56)$$

Now consider (28)

$$h_2' = m_2 s_1 - m_3 h_1 \quad (57)$$

Substituting (.47) and (49) in (57) we have

$$h_2' = m_2(\pi + \varepsilon H_0 - m_1 S_0)t - m_3(m_2 S_0 - m_3 H_0)t$$

$$= (m_2\pi + m_2\varepsilon H_0 - m_2 m_1 S_0 - m_3 m_2 S_0 + m_3{}^2 H_0)t$$

$$h_2' = (\pi m_2 - m_2(m_3 + m_1)S_0 + (m_2\varepsilon - m_3{}^2)H_0)t$$

$$= (m_2(\pi - (m_3 + m_1)S_0) + (m_2\varepsilon - m_3{}^2)H_0)t$$

Integrating, we have

$$h_2 = \left(m_2(\pi - (m_3 + m_1)S_0) - (m_2\varepsilon - m_3{}^2)H_0\right)\frac{t^2}{2} + C_{12}$$

Where C_{12} is the constant of integration

Applying the initial condition

$h_2(0) = 0 \qquad \Rightarrow C_{12} = 0$

Therefore (58)

$$h_2 = (m_2(\pi - (m_3 + m_1)S_0) + (m_2\varepsilon - m_3{}^2)H_0)\frac{t^2}{2}$$

From (4.5.17), we have

$$H = h_0 + Ph_1 + P^2 h_2 + ... +$$

Substituting (.42), (49) and (58) in (17) we obtain the solution for the model (12) as;

$$H(t) = H_0 + P(m_2 S_0 - m_3 H_0)t + P^2(m_2(\pi - (m_1 + m_3)S_0) + m_2(m_2\varepsilon - m^2{}_3)H_0)\frac{t^2}{2} + ... +$$

$$\lim_{P\to 0} H(t) = H_0$$

But

$$\lim_{P\to 1} H(t) \Rightarrow$$

$$H(t) = H_0 + (m_2 S_0 - m_3 H_0)t + (m_2(\pi - (m_1 + m_3)S_0) + (m_2\varepsilon - m^2{}_3)H_0)\frac{t^2}{2} + ... + \quad (59)$$

From (4.5.32) we have s

$$e_2' = m_4 h_1 - m_5 e_1 \quad (60)$$

Substituting (.49) and (51) in (.60), we have

$$e_2' = m_4(m_2 S_0 - m_3 H_0)t - m_5(m_4 H_0 - m_5 E_0)t$$

$$e_2' = (m_4 m_2 S_0 - m_4 m_3 H_0 - m_5 m_4 H_0 + m^2{}_5 E_0)t$$

$$e_2' = (m_4 m_2 S_0 - m_4(m_3 + m_5)H_0 + m^2{}_5 E_0)t$$

Integrating,

$$e_2 = (m_4 m_2 S_0 - m_4(m_3 + m_5)H_0 + m^2{}_5 E_0)\frac{t^2}{2} + C_{13}$$

Where C_{13} is the constant of integration

Applying the initial condition

$e_2(0) = 0 \qquad \Rightarrow C_{13} = 0$

Therefore

$$e_2 = (m_4 m_2 S_0 - m_4(m_3 + m_5)H_0 + m^2{}_5 E_0)\frac{t^2}{2} \quad (61)$$

Substituting (43),(51) and (61) in (18) we obtain

$$E(t) = E_0 + P(m_4 H - m_5 E_0)t + (m_4 m_2 S_0 - m_4(m_3 + m_5)H_0 + m^2{}_5 E_0)\frac{t^2}{2} + ...$$

Setting P = 0, we have

$E(t) = E_0$ *trivial solution*

And setting P = 1, we have

$$E(t) = E_0 + (m_4 H - m_5 E_0)t + (m_4 m_2 S_0 - m_4(m_3 + m_5)H_0 + m^2{}_5 E_0)\frac{t^2}{2} + ... + ... \quad (62)$$

From (4.4.36), we have

$$i_2' = \phi e_1 - m_6 i_1 \quad (63)$$

Substituting (4.5.51) and (4.5.53) in (4.5.63) we have

$$i_2' = \phi(m_4 H_0 - m_5 E_0)t - m_6(\phi E_0 - m_6 I_0)t$$

$$= (\phi m_4 H_0 - \phi m_5 E_0 - m_6\phi E_0 + m_6^2 I_0)t$$

$$= (\phi m_4 H - \phi(m_5 + m_6)E_0 + m_6^2 I_0)t$$

$$i_2' = (\phi(m_4 H_0 - \phi(m_5 + m_6)E_0) + m_6^2 I_0)t$$

Integrating, and applying the initial condition $i_2(0) = 0$ we have

$$i_2 = (\phi(m_4 H_0 - \phi(m_5 + m_6)E_0) + m_6^2 I_0)\frac{t^2}{2} + C_{14}$$

where C_{14} is the constant of integration

Substituting (44), (53) and (64) in (19)

We obtain

$$I(t) = I_0 + P(\phi E_0 - m_6 I_0)t + P^2(\phi(m_4 H_0 - (m_5 + m_6)E_0) + m^2{}_6 I_0)\frac{t^2}{2} + ...$$

Setting P = 0, we have

$I(t) = I_0$

And setting P = 1, we have

$$I(t) = I_0 + (\phi E_0 - m_6 I_0)t + (\phi(m_4 H_0 - (m_5 + m_6)E_0) + m^2{}_6 I_0)\frac{t^2}{2} + ... \quad (64)$$

And finally from (40) we have

$$a_2' = \tau c i_1 - \mu a_1 \quad (65)$$

Substituting (.53) and (54) in (.66)
We obtain

$$a_2^1 = \tau c(\varphi E_0 - m_6 I_0)t - \mu(\tau c I_0 - \mu I_{2,0})t$$

$$= (\tau c \varphi E_0 - \tau_c m_6 I_0 - \mu\tau c I_0 + \mu^2{}_7 I_{2,0})t$$

Integrating, we have

$$a_2 = (\tau c(\phi E_0 - (m_6 + \mu)I_0) + \mu^2 I_{2,0})\frac{t^2}{2} + C_{15}$$

Where C_{15} is the constant of integration, applying the initial conditions

$$a_2(0) = 0 \qquad \Rightarrow C_{15} = 0$$

Therefore

$$a_2 = (\tau c(\phi E_0 - (m_6 + \mu)I_0) + \mu^2 I_{2,0})\frac{t^2}{2} \quad (66)$$

Substituting (45), (54) and (67) in (20) we have

$$I_2(t) = I_{2,0} + P(\tau_c I_0 - \mu I_{2,0})t + P^2\left[\tau c(\phi E_0 - (m_6 + \mu I_0)I_0) + \mu^2 I_{2,0}\right]\frac{t^2}{2}$$

Setting P = 0, we have

$I_2(t) = I_{2,0}$ and

Setting P = 1, we have

$$I_2(t) = I_{2,0} + (\tau c I_0 - \mu I_{2,0})t + \left(\tau c(\phi E_0 - (m_6 + \mu I_0)I_0) + \mu^2 I_{2,0}\right)\frac{t^2}{2} + ... \quad (67)$$

Therefore equation (56), (59), (62), (65) and (67) are the solutions of our models.(1) to (5).

IV. Conclusion

In this paper, the homotopy perturbation method was applied on some systems of non-linear deterministic model on the control of HIV/AIDS pandemic using therapeutic dose in a heterogeneous population. The use of the method gives a very precise results analytically to the models showing that the method is very convenient and efficient, the numerical method leads to inaccurate results when the equation is highly dependend on time.

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