Thermo-elastic Analysis of a Rotating Solid Cylinder under Transient Thermal Stresses

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Abstract— *This paper presents thermoelastic analysis of a rotating solid cylinder under transient thermal stresses. An analytical method is presented to investigate transient thermal stresses in a rotating solid cylinder with constant angular velocity about its central axis. Finite element simulation was used to validate the analytical solution. Angular velocity changes, displacement, stress, and strain distributions in the transient state were investigated and displayed. The radial and circumferential stresses and radial strains in a rotating solid cylinder depend on both radius and angular velocity. The circumferential strain depends more on the angular velocity rather than the radius. The radial displacement almost only depends on the radius.*

Keywords— *CFD simulation, Hydrogen separation, Reforming products, Silica membrane.*

I. INTRODUCTION

Thermo-elastic analysis of non-uniform cylinders are important for efficient designing of mechanical, aerospace and civil structures. The phenomenon of elastic regime in the cylinder material is exhibited when thermal load exceeds the critical load to produce initial yielding condition in the cylinder. Due to the absence of comprehensive analytical solution of the problem, several efforts have been made to obtain effective methods of solution in the elasto region due to induced thermal stress. Solid circular cylinders are the most commonly used specimens in various standard tests in engineering application, such as the uniaxial compressive strength test, the triaxial compressive strength test, the Brazilian test, the double-punch test, the block punch index test and the point load strength test. In fact, the thermoelastic analysis of elastic solid circular cylinders is one of the most fundamental problems in theoretical thermoelasticity [1].

Recently, some studies have been carried out on mechanical and thermal analyses of various structures [2-16]. Khoei and Bahmani [17] presented a formulation for the pressure-dependent thermo-mechanical contact problem using the X-FEM method. A staggered approach based on the Newton–Raphson method was developed to show the effect of thermal contact in fracture modelling. The governing equations of a FGM Timoshenko beam resting on a non-linear strain foundation are derived and numerically solved by Sun et al. [18]. According to a power law function, they investigated thermal buckling and post buckling responses of the FGM beam with the assistance of shooting method. Hosseini [19] developed an asymmetric elastic-plastic-creep constitutive model for representation of thermo-mechanical response of cast irons under monotonic and cyclic loading conditions. The model was capable of effectively implementation to different Finite element (FE) packages.

Stasynk et al. studied the steady-state thermal stress of hollow cylinders considering the effect of thermal conductivity variation as a function of temperature. However, for large heat flow, the temperature and stresses difference between temperature dependent and independent thermal conductivity can be as much as 20% [20]. Naga presented the stress analysis and the optimization of both thick-walled permeable and impermeable cylinders under the combined effect of steady-state temperature and pressure gradient [21]. Zukhova and Pimshtein studied the one dimensional, steady-state thermal problem for a laminated cylinder consisting of concentric layers and subjected to internal and external loading. Their calculations showed that the radial compressive stress due to the internal pressure can permit external heating without layer separation [22]. Kindli et al. presented transient variation of thermal stresses within thick cylinders subjected to different operating conditions. They found that the value of the maximum effective stress at inner surface may be reduced to 50-60% [23]. Abd-alla et al. studied transient thermal stresses in rotation non-homogeneous cylindrically orthotropic composite tubes. They calculated numerical results of temperature, stress and displacement [24].

Malekzadeh et al. studied transient response of rotating laminated functionally graded cylindrical shells in thermal environment. concluded that the temperature dependence of material properties, material graded index, the convective heat transfer coefficient, the angular velocity, the boundary condition and the geometrical parameters have significant effects on the displacement and stress components of the rotating laminated FG cylindrical shell. However, in the author’s opinion, the exact analysis for a rotating solid cylinder under transient thermal has not been reported [25].

Derington [26] studied the principal stresses in a long elastic cylinder subjected to uniform internal or external pressure and steady state heat flow under a variety of loading conditions incorporating Tresea's yield criterion with no change of elastic constants with temperature. The effects of the gradation of strength and deformation of thick walled Functionally Graded (FG) tubes under internal pressure was investigated by Fukui and Yamanaka [27]. Obata and Noda [28] investigated the thermal stresses in a FGM hollow sphere and hollow circular cylinder. They also studied the effect of inner radius on the resulting stresses. Using incremental theory of plasticity method for thick walled cylindrical pressure vessels, Loghman and Wahab[29] obtained the plastic strain, plastic stress, and plastic zone progress for different loading conditions and thickness ratios. An exact solution for one-dimensional thermal stresses of FGM spheres [30] and cylinders [31] presented by Lutz and Zimmerman who considered variation of Young's modulus and the thermal expansion coefficient along the radius.

Shabana and Noda [32] obtained the elastoplastic thermal stresses in a rectangular plate subjected to different kinds of temperature conditions using finite element method. Tutuncu and Ozturk [33] presented the closed-form solutions for stresses and displacements in cylindrical and spherical vessels subjected to internal pressure alone, using the infinitesimal theory of elasticity.

In the present article, we have analysed a rotating solid cylinder under transient thermal stresses and derived thermoelastic equations for this cylinder and then heat equation was used into thermoelastic equations. Angular velocity changes, displacement, stress, and strain distributions in the transient state were investigated and displayed.

II. Problem Formulation

The problem to be considered is a rotating solid cylinder with radius a which is subjected to axisymmetric transient thermal loading $T(r,t)$. It is assumed that the solid cylinder rotates about its central axis with a constant angular velocity of ω.

For the problem under consideration, a response analysis of thermoelastic stresses is investigated within the framework of small deformation.

For an axisymmetric solid cylinder, the problem can be treated according to plane strain state. Consequently, all field variables are independent of circumferential direction θ, so there is only a radial displacement u_r. With u_r the radial and circumferential strains can be given by:

$$\varepsilon_r = \frac{\partial u_r}{\partial r}, \quad \varepsilon_\theta = \frac{u_r}{r} \tag{1}$$

In addition, thermoelastic constitutive relations take the following form:

$$\sigma_r = 2\mu\varepsilon_r + \lambda e - \beta\tau \tag{2}$$

$$\sigma_\theta = 2\mu\varepsilon_\theta + \lambda e - \beta\tau \tag{3}$$

$$e = \varepsilon_r + \varepsilon_\theta \tag{4}$$

$$\lambda = \frac{2\nu\mu}{1-2\nu}, \beta = \alpha(3\lambda + 2\upsilon), \mu = \frac{E}{2(1+\nu)}, \tau = T(r,t) - T_i \tag{5}$$

Where E is Young's modulus, ν Poisson ratio, T_i initial temperature, and α the coefficient of the thermal expansion, respectively.

Neglecting body force of the cylinder, the equilibrium equation for axisymmetric problems of a rotating solid cylinder is:

$$\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + \rho r\omega^2 = 0 \tag{6}$$

Here $\rho\omega^2$ is mass density; ω is the constant angular velocity and $\rho r\omega^2$ is the force per unit volume due to centrifugal force.

Substituting Eqs. (1), (2), (3), (4) and (5) into Eq. (6) gives the following equilibrium equation:

$$(2\mu + \lambda)\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}(2\mu + \lambda)\frac{\partial u_r}{\partial r} - (2\mu + \lambda)\frac{u_r}{r^2} - \beta\frac{\partial\tau}{\partial r} = -\rho r\omega^2 \tag{7}$$

With C_1 and C_2 as unknown constants, the complete solution of Eq. (7) is:

$$u_r = \frac{(1+\nu)}{(1-\nu)}\frac{\alpha}{r}\int_0^r \tau r\,dr + C_1 r + \frac{C_2}{r} - \frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\frac{\rho\omega^2 r^3}{8} \tag{8}$$

Substituting Eqs. (1) and (8) into Eqs. (2) and (3), the following equations can be obtained:

$$\sigma_r = \frac{-E(1-3\nu)}{(1-\nu)(1-2\nu)}\frac{\alpha}{r^2}\int_0^r \tau r\,dr + \frac{E}{(1+\nu)(1-2\nu)}C_1 - \frac{E}{1+\nu}\frac{C_2}{r^2} - \frac{3-2\nu}{1-\nu}\frac{\rho\omega^2 r^2}{8} \tag{9}$$

$$\sigma_\theta = \frac{E}{1-\nu}\frac{\alpha}{r^2}\int_0^r \tau r\,dr + \frac{E}{(1+\nu)}\frac{C_1}{(1-2\nu)} - \frac{E}{(1+\nu)(1-2\nu)}\frac{C_2}{r^2} - \frac{E\alpha}{1-\nu} - \left(\frac{1+2\nu}{1-\nu}\right)\frac{\rho\omega^2 r^2}{8} \tag{10}$$

Note that Eq. (8) is an integral equation with unknown constants C_1 and C_2, which can be determined using boundary conditions corresponding to center of cylinder and outer surface, respectively. In this case, mechanical stresses in the solid cylinder are only caused by external temperature change as well as centrifugal force. Under such circumstances, the corresponding mechanical boundary conditions can be stated below:

$$\sigma_r(0) = Finite, \sigma_r(a) = 0 \tag{11}$$

From Eq. (9), two unknown constants C_1 and C_2 can be determined. If inserted back into Eqs. (9) and (10) the following rearranged equations will be obtained:

$$\sigma_r = \frac{-E(1-3\nu)}{(1-\nu)(1-2\nu)}\frac{\alpha}{r^2}\int_0^r \tau r\,dr + \frac{E(1-3\nu)}{(1-\nu)(1-2\nu)}\frac{\alpha}{r^2}\int_0^a \tau r\,dr \tag{12}$$

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$$-\left(\frac{3-2\nu}{1-\nu}\right)\frac{\rho\omega^2 r^2}{8}$$

$$\sigma_\theta = \frac{\alpha}{r^2}\int_0^r \tau r\, dr + \frac{E\,(1-3\nu)}{(1-\nu)\,(1-2\nu)}\frac{\alpha}{r^2}\int_0^a \tau r\, dr - \frac{E\alpha}{1-\nu}\,\tau + \frac{2}{1-\nu}\frac{\rho\omega^2 r^2}{8} \tag{13}$$

Where $\tau(r,t) = T(r,t)-T_i$ is temperature changes. Heat equation can be expressed as (14):

$$T(r,t) = T_a - 2\,(T_a - T_i) \times \sum_{n=1}^{\infty}\left(\frac{\lambda h_a}{a}\frac{J_0(s_r)}{(\lambda^2 {s_n}^2 + {h_a}^2)J_0(s_n a)}\; e^{-k\,{s_n}^2\,t}\right) \tag{14}$$

$$\tau(r,t) = (T_a - T_i) \times \left\{1 - 2\sum_{n=1}^{\infty}\left(\frac{\lambda h_a}{a}\frac{J_0(s_r)}{(\lambda^2 {s_n}^2 + {h_a}^2)J_0(s_n r)}\; e^{-k\,{s_n}^2\,t}\right)\right\} \tag{15}$$

III. Numerical Results and Discussion

The analytical solution described in the preceding section for a rotating solid cylinder, with radius a = 0.5 m was considered with the Young's modulus E = 200 GPa, Poisson's ratio ν = 0.3, mass density ρ = 7860 kg/m^3, heat conduction coefficient k = 14.9 W/Km^2, heat convection coefficient h = 25 W/Km^2, thermal expansion coefficient α = 11.6e-6 1/°C, initial temperature T_i = 300 K and surface temperature T_a = 973 K. The finite element simulation was done for validation with the same inputs for analytical solution. Since the model was axisymmetric, only a quarter of the cylinder was considered. This simulation was done using ANSYS software. The analytical solution was performed by MATLAB software.

The distribution of radial stress for different values of angular velocity ω is shown in Fig. 1. As visible in r/a = 1, the radial stresses behave the same and converge. By increasing the angular velocity, radial stress increases. Moreover, radial stress has the maximum value at the center of the cylinder and at outermost part of the cylinder, it converges to zero.

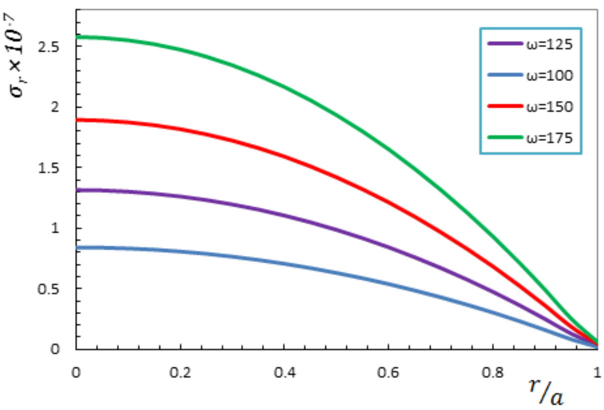

Fig. 1. Radial stress distributions in the solid cylinder for different angular velocities.

The distribution of circumferential stress for different values of ω is shown in Fig. 2. The circumferential stresses behave differently. By increasing the angular velocity, circumferential stress increases. Moreover, circumferential stress has the maximum value at the center of the cylinder and at outermost part of the cylinder, it reaches to its minimum value which is not zero unlike the radial stress.

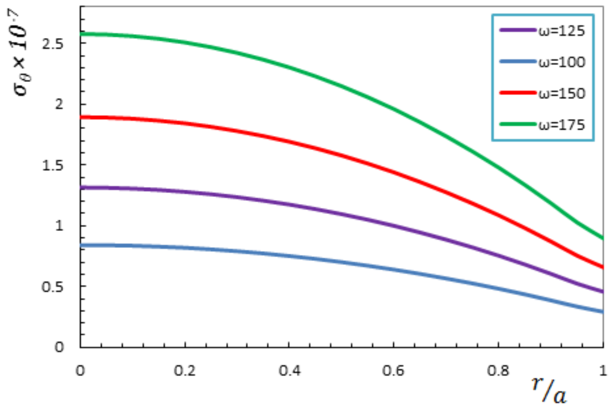

Fig. 2. Circumferential stress distributions in the solid cylinder for different angular velocities.

The distribution of radial strain for different values of ω is shown in Fig. 3. As can be seen in r/a = 0.9, the radial strains have the same value for any angular velocity. By increasing the angular velocity, radial strain increases. Moreover, radial strain has the maximum value at the center of the cylinder and at outermost part of the cylinder, it reaches to its minimum value.

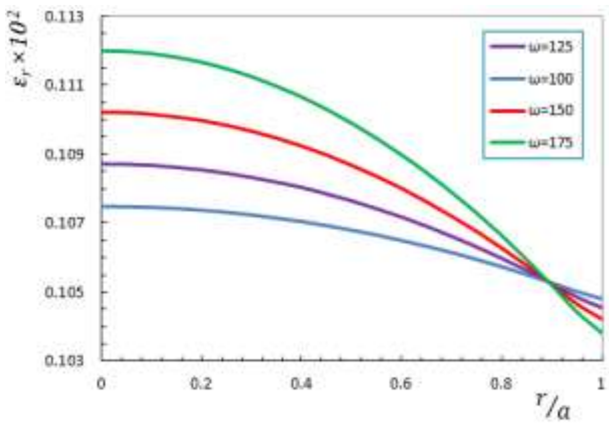

Fig. 3. Radial strain distributions in the solid cylinder for different angular velocities.

The distribution of circumferential strain for different values of ω is shown in Fig. 4. The circumferential strains increase by increasing the angular velocity but they are almost constant throughout different radius of the cylinder.

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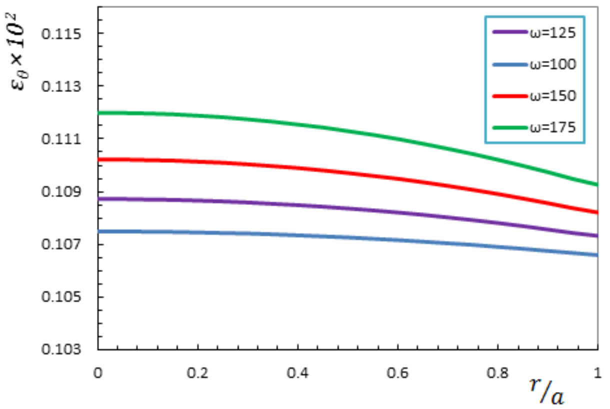

Fig. 4. Circumferential strain distributions in the solid cylinder for different angular velocities.

The distribution of radial displacement for different values of ω is shown in Fig. 5. It is observed that the radial displacements are linearly increasing by increasing the radius and the same for different values of the angular velocity.

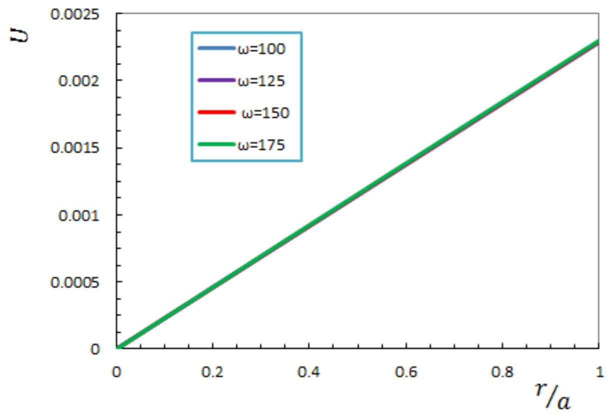

Fig. 5. Radial displacement distributions in the solid cylinder for different angular velocities.

Figs. 6-10, show the distribution of radial stress, circumferential stress, radial strain, circumferential strain and radial displacement for ω=100 r/s, respectively. In these figures, the analytical solution was compared with the results obtained using FEM. The results obtained in FEM study were compared with previously described analytical results and good agreements are found.

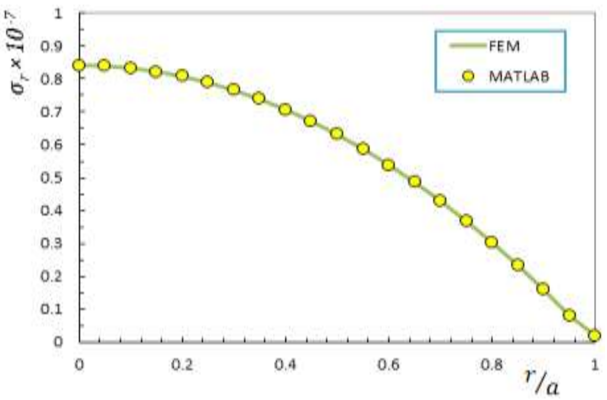

Fig. 6. Radial stress distributions in the solid cylinder for ω=100 r/s.

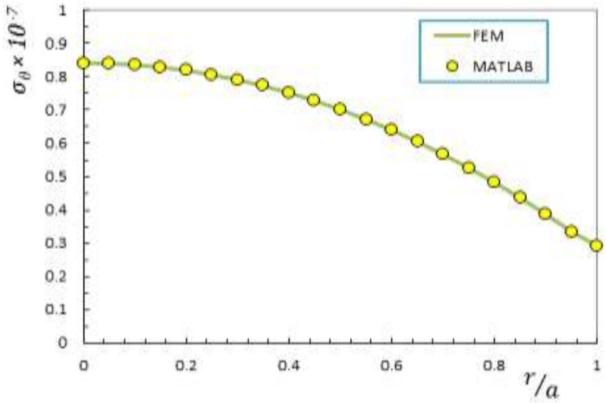

Fig. 7. Circumferential stress distributions in the solid cylinder for ω=100 r/s.

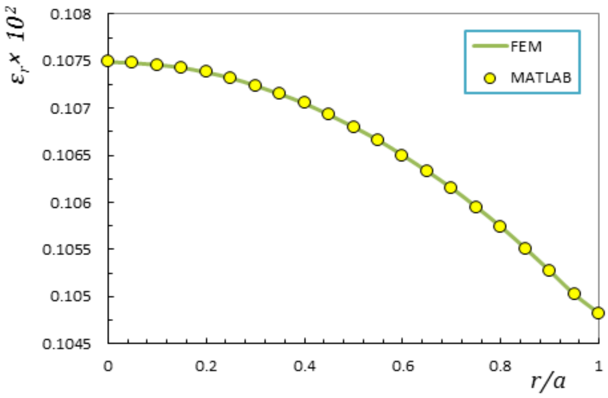

Fig. 8. Radial strain distributions in the solid cylinder for ω=100 r/s.

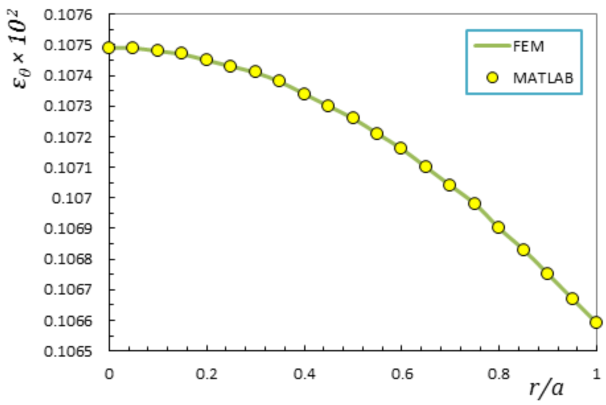

Fig. 9. Circumferential strain distributions in the solid cylinder for ω=100 r/s.

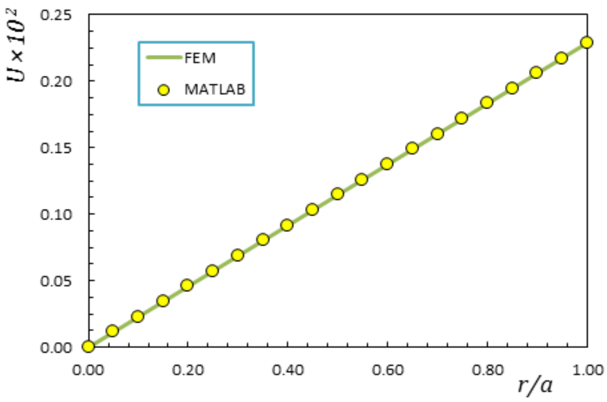

Fig. 10. Radial displacement distributions in the solid cylinder for ω=100 r/s.

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Figs. 11-15, show the color contour plot of radial stress, circumferential stress, radial strain, circumferential strain and radial displacement for ω=100 r/s, respectively.

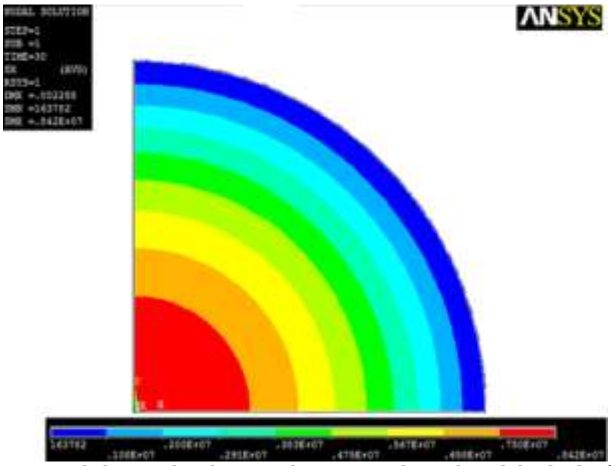

Fig. 11. Radial stress distribution color contour plot in the solid cylinder for ω=100 r/s.

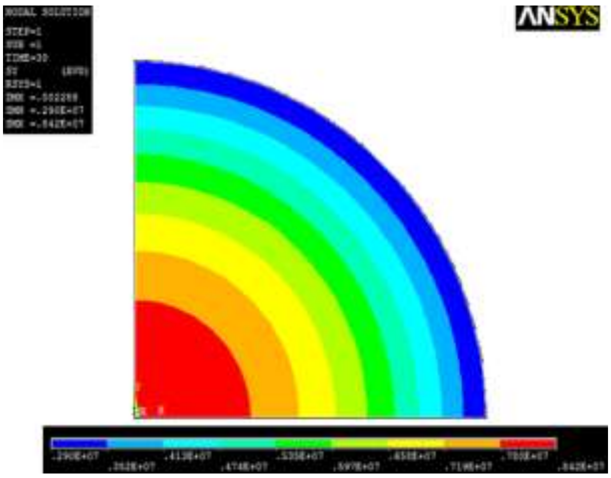

Fig. 12. Circumferential stress distribution color contour plot in the solid cylinder for ω=100 r/s.

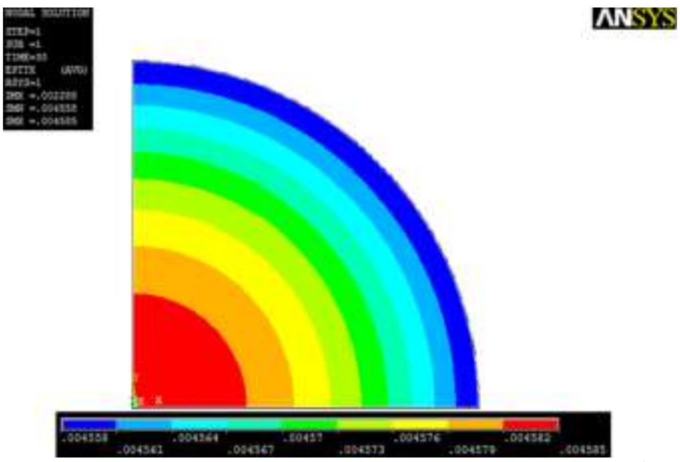

Fig. 13. Radial strain distribution color contour plot in the solid cylinder for ω=100 r/s.

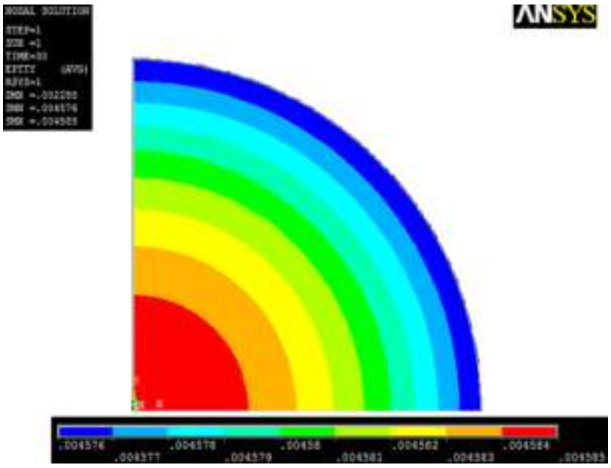

Fig. 14. Circumferential strain distribution color contour plot in the solid cylinder for ω=100 r/s.

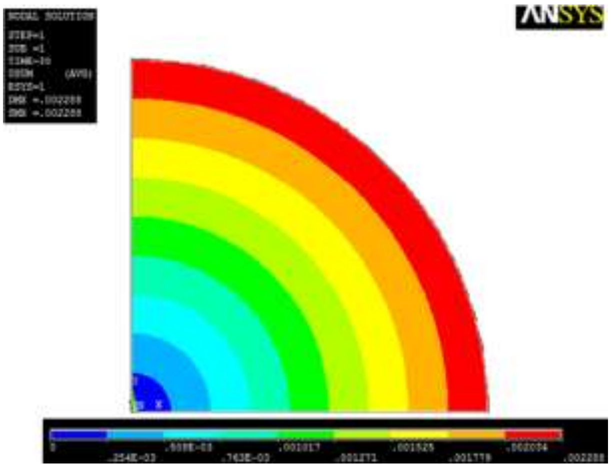

Fig. 15. Radial displacement distribution color contour plot in the solid cylinder for ω=100 r/s.

IV. Conclusion

The paper presents an analytical method to investigate transient thermal stresses in a rotating solid cylinder with constant angular velocity about its central axis. Radial and circumferential stresses, radial and circumferential strains and radial displacements equations were derived using the equilibrium equations.

The radial and circumferential stresses and radial strains in a rotating solid cylinder depend on both radius and angular velocity. The circumferential strain depends more on the angular velocity rather than the radius. The radial displacement almost only depends on the radius.

The greatest values of stress and strain are observed in the highest angular velocity ω = 175 r/s. As the value of ω increases, the radial, the circumferential stress, the radial strain and the circumferential strain increase. As the value of r increases, the radial, the circumferential stress, the radial strain and the circumferential strain decrease.

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