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Operation Research and Mathematical Modeling of Real-Time Signal Traffic Cycle Optimization in Ho Chi Minh City of Southern Viet Nam

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Abstract *– This paper presents a method of mathematical modeling for optimal determine the real time signal setting (RTSS) the cycle in order to reduce congested situation. Bases on the rate flow, the traffic density on the line, the intersection capacity and linking to near intersections as well, in order to compute the cycle more accurately. The result was assessed and compared to current traffic situation in some streets in Ho Chi Minh City of Southern Viet Nam. The results show that RTSS model can applying in real-life intersections, the clearing time and length of queue reduce over 90% compared with current signal cycle.*

Keywords *– Clearing time, length of queue, traffic density.*

I. Introduction

Clayton [1] was considered one of the first people who study delays at fixed-cycle signals. He built a model in which, both arrival flow and departures at the signal in fixed period of time and predicted that the queues are always exhausted at the end of a green phase, as long as the rate of arrival does not exceed the capacity of the intersection. The Clayton's model is not useful in high arrival rate situation.

Winsten's model [2] used a fixed pattern of departures (as Clayton), but with high arrival rate. The arrivals sharply increased, delay increased also. Winsten's model predicts delays consistently higher and closer than Clayton's, but still a little below, those found in practice. For light traffic, Winsten's binomial model gives a vehicle ratio for counts slightly less than one, approach to zero as the flow vehicle increases. In urban traffic system, in practice, the vehicle ratios just over one are researched for signals, but these usually increase with the volume of traffic.

Webster [3] and Newell [4] have built slightly better delay formulae by models with Poisson arrivals with variance vehicle ratios equal to equation (1).

Thamizh Arasan Venkatachalam and Dhivya Gnanavelu [5] gave a concept of time vehicle area density in time installed sensors, from built model presenting relation between speed and occupant area based on Indian traffic situation that is similar to Vietnam.

All models mentioned above just consider at only one intersection at a certain intervals. They has not yet linked to near intersections and not yet investigated to storage capacity of next traffic systems. In addition, these models assume that arrivals are completely cleared in one cycle that is not suitable for Ho Chi Minh City, Viet Nam current traffic.

Therefore, the paper will compute and determine optimal real-time signal cycle in order to reduce and clear queue length in shortest time based on storage capacity and released capacity of right intersection and near intersection. Besides, consider to queue length not exhausted in one cycle.

1. Computıng Sıgnal Traffic Cycle

Vehicle flow moving on the road has parameters as follows: Flow (q); average speed (v); traffic density (k).

+ Relationship between (q) and (k) [5]:

- When k = 0, q = 0 since no vehicle on the road; When k increases, q increases also; When k encreases over maximum point of flow (q_{max}), q descreases; If more and more vehicles on the road and reach to situration point, vehicles can not move and q = 0, k = k_{max}.

+ Relationship between (q) and speed (v) [5]:

- When q = 0 (no vehicle), v = 0;
- When q = q_{max} , v ∈ [0, v_{free}];

+ Relationship between (v) and density (k):

When k increases, v decreases; k decreases, v increases; k = 1, v = 0; When k near to 0, v approach to ∞:

$$\boldsymbol{v = \frac{1}{k} - 1} \tag{1}$$

+ Definitions:

- k: Vehicle density, is rate of vehicle occupated area per road area
- A_v: Traffic area in inspected intervals (m^2)
- A_r: Inspected road area in inspected intervals (m^2)
- a_{tb}: average occupated area of a vehicle (m^2)
- n_i: Total of vehical type i in inspected intervals
- N: Total of vehical all types in inspected intervals (vehicle)
- q: Number of arrivals in a hour (vehicle/h)
- N_a: Total of arrivals at a intersection in period of time t (vehicle)
- N_d: Total of departures at a intersection in period of time t (vehicle)
- t: inspected time (h)
- a_{vi}: Occupated area of vehicle type i (m^2)
- W: Width of road (m)
- l: Length of link between two intersections (m)
- C: Signal cycle (s)
- p: Departures exhaust intersection in seturation condition (vehicle).

Nguyen Chi Hung, "Operation Research and Mathematical Modeling of Real-Time Signal Traffic Cycle Optimization in Ho Chi Minh City of Southern Viet Nam," *International Research Journal of Advanced Engineering and Science*, Volume 3, Issue 2, pp. 197-201, 2018.

- s: Maximum total of vehicles exhaust in a hour (vehicle/h)
- r: Red time (s)
- g: Green time (s)

Let, $\boldsymbol{k} = \frac{A_v}{A_r}$ (2)

Where:

Traffic area: $\boldsymbol{A_v = \sum_i n_i a_{vi}}$ (3)

Road area: A_r = W.L (4)

$\boldsymbol{A_v = a_{tb} \sum_i n_i = N_a a_{tb}}$ (5)

$\boldsymbol{N_a = \sum_i n_i \Rightarrow A_v = N_a a_{tb} = kA_r}$ or $\boldsymbol{N_a = \frac{kA_r}{a_{tb}}}$,

$\Rightarrow \boldsymbol{a_{tb} = \frac{kA_r}{N}}$ (6)

Let, $\boldsymbol{k_1 = \frac{kW}{a_{tb}}}$ (7)

$\boldsymbol{q = \frac{N_a}{t} = k_1 V_{tb}}$ (8)

Departures exhausted at intersection in a cycle are expressed:

$\boldsymbol{p = \int_0^C sdt}$ or $\boldsymbol{s = \frac{dp}{dt}}$ (9)

If set the beginning of green time is zero value, the cycle is expressed as follows in Figure 1.

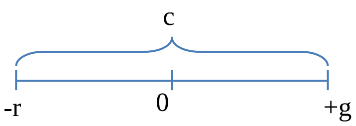

Fig. 1. Signal cycle.

Number of vehicals are computed in a cycle:

$\boldsymbol{p = \begin{cases} 0, -r \le t < 0 \\ sg, 0 < t \le g \end{cases}}$ → p = sg (10)

Arrivals in intervals of dt: $\boldsymbol{q = \frac{dN_a}{dt}}$ (11)

Total of arrivals in a cycle is:

$\boldsymbol{N_a = \int_{-r}^{g} qdt}$ → N_a = qc (12)

If sg < qc, queue length is not fully cleared in one cylcle. Therefore, the queue more and more extended after each cycle.

+ Computing Traffic Cycle Optimization.

The length of queue (L) always varys according to time and depends on flow rate on road. L will be increased in red time and descreased in green time. Consider in period of differntal time dt, the queue is created dl, where:

$\boldsymbol{dl = \frac{q}{k_1} dt \rightarrow L = \int_0^t \frac{q}{k_1} dt}$ (13)

If inspect in period of time t, arrivals (N_q) và departures (N_d) are determined:

$\boldsymbol{N_a = \int_0^t qdt = qt}$ (14)

Since depaetures exhaust uncontinuously at an intersection because of red time pause, it is expressed as follows:

$\boldsymbol{N_d = \frac{sg}{C} t = Qt}$ (15)

Where, Q capacity of intersection, In orther hand, it is number of vehical released per cycle (vehical),

where, $\boldsymbol{Q = \frac{sg}{C}}$ (16)

If arraivals beyond departures in intervals t, (qt – Qt) > 0 is a number of vehical waiting at intersection in period of time t. In order to clear fully in one cycle,

(qt – Qt) = QC and $\boldsymbol{t = \frac{Q}{q-Q} C}$ (17)

That means, in order to clear completely the queue that formed in period of time t at a phase in one cycle, it has to be increased green time to one more cycle. If not, the queue is more and more extended, lead to congestion. However, it makes the length of queue in other phase is increased also. If want the queue in other phase not to increase, cooperate with next intersections.

Consequently, it has to be inspected to capacity of intersections such as, length of link at intersections (l), number of vehicles exhausted intersection in one cycle (Q), these parameters are different at each one.

If it is considered in intervals t, from equation (17), time used to clear the queue formed in t is computed as follows:

$\boldsymbol{t_d = \left(\frac{q}{Q} - 1\right) t}$ (18)

t_d is necessary intervals for fully clearning a queue formed in period of time t. It's important to determine how long t_d is optimal. That depends on traffic condition at time.

If a point j that is far from the intersection a distance of l_j in a queue is inspected and t_j (delay time) is intervals from beginning green (0) untill vehicles at j moving, the average speed of vehicle (V) arriving to intersection is computed as follows :

$\boldsymbol{V = \frac{l_j}{t_j}}$ (19)

That means, vehicles at j have to take a period of time, t_j for moving at speed V to the intersection that is far from j a distance l_j. As a result, it takes $2t_j$ from beginning green (t = 0), so that vehicles at j exhaust the intersection.

From the point of view , if it's consedered in intervals t = g, length of queue departed (l_d) in one cycle is:

$\boldsymbol{l_d = \frac{1}{2} Vg}$ (20)

On the other hand, the length of queue departed (l_d) in t is:

$\boldsymbol{l_d = \frac{Vg}{2C} t = L_d t}$ (21)

Where: $\boldsymbol{L_d = \frac{Vg}{2C}}$ (22)

L_d: The length of queue departed in one cycle

Therefore, length of moving vehicle flow, $l_m = 2l_d$, and length of moving queue during one cycle, l_c = VC, length of moving queue during period of time t is l_a = Vt. It is supposed that flow rate (V) is stable during cycle.

If a queue at intersection is L, necessary time for fully clearing is computed:

$\boldsymbol{t_d = 2\frac{L}{V}}$ (23)

Hence,

L = l_a – l_d (Where, l_a is arrival queue) (24)

Reffer to equation (6):

$\boldsymbol{l_a = \frac{N_a . a_{tb}}{kW}, l_d = \frac{N_d . a_{tb}}{kW}}$ (25)

$N_a(t)$ = qt ∀ t

$$\boldsymbol{N_d(t) \begin{cases} nQ \ @ \ nC < t \le nC + r \\ (\text{red time}) \\ Qt \ @ \ nC + r < t \le (n+1)C \\ (\text{green time}), \\ \forall \ n = int(\frac{t}{C}) \end{cases}} \quad (26)$$

Reffer to equations (24), (25) and (26):

$$L = \begin{cases} (qt - nQ)\frac{a_{tb}}{k_r W} \ @\, nC < t \le nC + r \\ \quad (red\ time) \\ (qt - Qt)\frac{a_{tb}}{k_g W} \ @\, nC + r < t \le (n+1)C \\ \quad (green\ time) \\ \quad \forall\, n = int(\frac{t}{C}) \end{cases} \quad (27)$$

Where: k_r, k_g are red-time density and green-time density, repectively and $k_r > k_g$ (k_r = 1, not moving).

Reffer to equation (27), speed of forming the queue is:

$$\frac{dL}{dt} = \begin{cases} \frac{qa_{tb}}{k_d W}, \text{(red time)} \\ (q - Q)\frac{a_{tb}}{k_x W}, \text{(green time)} \end{cases} \quad (27)$$

There are two cases, q<s and q>s. In usual condition, q<s.

$k_c < \frac{q}{s} < 1$ where $k_c = \frac{g}{c} < 1$ (duty cycle) and

$qc - gs > 0$ (28)

It's a cause of forming queue. s almost stable during period of inspected intervals, s downward trend as arrival flow increasing in rush hour. Oppositely, q will be change according to time depending on number of vehicles at time.

Reffer to equation (6) and (27):

$$L = l\left(1 - \frac{sg}{qC^2}\right), \text{ k=1 at red time} \quad (29)$$

Where: l is length of link inspecting (m)

If L = l,

$$C_{min} = \sqrt{\frac{sg}{q}} \quad (30)$$

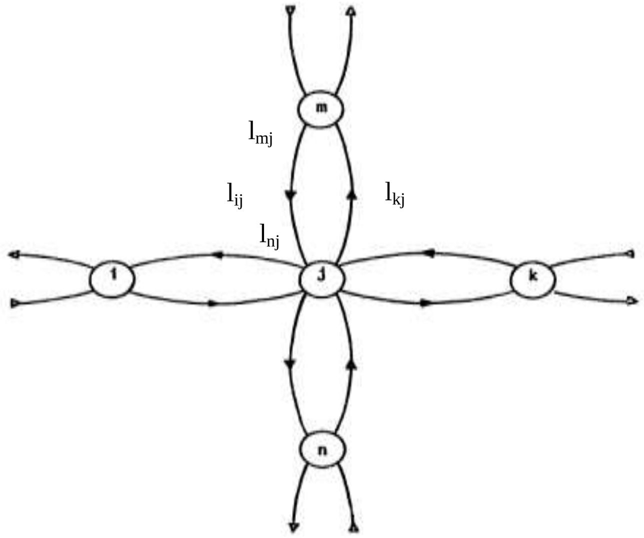

Fig. 2. Nodes and links in traffic network

Referring to Fig.2 [6] at node j, where:

l_{xj}: length of link from next node x to node j.

$x = [i, m, k, n]$; $l = [l_{ij}, l_{mj}, l_{kj}, l_{nj}]$

l_{aij}: length of arrival queue to j forward from i to j.

$l_a = [l_{aij}, l_{amj}, l_{akj}, l_{anj}]$

l_{djk}: length of departure queue away from j forward from j to k

$l_d = [l_{dij}, l_{dmj}, l_{dkj}, l_{dnj}]$

L_{ij}: length of queue at j forward from i to j.

Q_{ij}: Number of departuers node j forward from i to j in one cycle

q_{ij}: Number of arrivals to node j forward from i to j in one hour (vehicle/h)

τ_{ij} is period of time (s) moving from i to j

Hence, $L_{ij} < l_{ij}$,

$q_{ij}C_j = \alpha_{ij}Q_{ij}C_i$, (31)

In which, $\alpha_{ij} < 1$ is ratio of vehicles moving from i to j per total of vehicles released from i

C_j: signal cycle at node j (s)

$$L_{ij} = \frac{1}{k_1}\int_0^t (q_{ij} - Q_{jk})\,dt \quad (32)$$

Hence,

$g_{jk} = g_{ij} = r_{nj} = r_{mj}$ (33)

$L_{ij} = l_{dij} - l_{djk}$ (34)

If the signal cycle of node i is assigned reference point, cycles of near nodes will be difference versus i a value of ±β (time offset).

Where:

$t_j = t + \beta_{ij}$ (35)

β_{ij} time delay between two cycles i and j

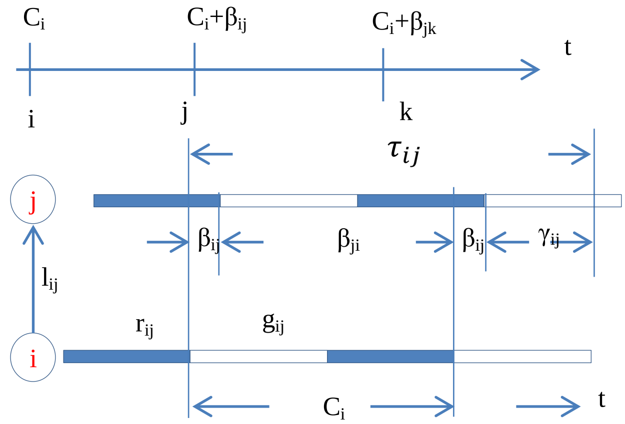

Fig. 3. Timeline graphic at near intersections

$\beta_{ij} + \beta_{ji} = C_i$ (36)

$$\tau_{ij} = \frac{l_{ij}}{V_{ij}} \quad (37)$$

Where:

V_{ij} is speed of vehicle moving from i to j (m/s).

Consequently, $l_{ij} = \beta_{ij}v_{ij} + L_{ij}$ (38)

Where: v_{ij} is speed of forming queue at node j.

Refer to equation (28):

$$v_{ij} = \frac{q_{ij}a_{tb}}{k_d W} = \frac{q_{ij}}{k_1} \quad (39)$$

Refer to equation (27), length of queue at j in intervals τ_{ij} is determined as follows:

$$L_{ij} = \frac{1}{k_1}\int_0^{\tau_{ij}} (q_{ij} - Q_{jk})\,dt \quad (40)$$

And Refer to equation (18), the necessary intervals for fully clearning a queue formed in period of time τ_{ij} is:

$$t_{dij} = \tau_{ij}\left(\frac{q_{ij}}{Q_{jk}} - 1\right) \quad (41)$$

Vehicles moving form i to j in τ is:

$N_{aij} = Q_{dij}.\,\tau_{ij} = q_{ij}.\,\tau_{ij}$ (42)

$\tau_{ij} = nC_j + \beta_{ij} + \gamma_{ij}$ (43)

Where: γ_{ij} is arrival time of vehicle to j each C_j.

Where, $-r_{jk} < \gamma_{ij} \le g_{jk}$ (44)

Refer to equations (16), (42) and (44):

Nguyen Chi Hung, "Operation Research and Mathematical Modeling of Real-Time Signal Traffic Cycle Optimization in Ho Chi Minh City of Southern Viet Nam," *International Research Journal of Advanced Engineering and Science*, Volume 3, Issue 2, pp. 197-201, 2018.

$$t_{dij} = \frac{n_{ij}q_{ij}}{s_{ij}g_{ij}} C_j^2 + \left[(\beta_{ij} + \gamma_{ij})\frac{q_{ij}}{s_{ij}g_{ij}} - n_{ij}\right] C_j - (\beta_{ij} + \gamma_{ij}) \quad (45)$$

$$\frac{dt_{dij}}{dC} = 0,\ C_j = \frac{s_{ij}g_{ij}}{2q_{ij}} - (\beta_{ij} + \gamma_{ij})\frac{1}{2n_{ij}} \quad (46)$$

Let $g_{ij} = \delta_{ij}C_j$, (47)

Where, δ_{ij} duty cycle ($\delta_{ij} < 1$),

Refer to equations (44), (47) and (48),

$$C_j = \frac{\tau_{ij}q_{ij}}{n_{ij}(\delta_{ij}s_{ij} - q_{ij})} \quad (48)$$

Since $C_j > 0$, therefore $\frac{s_{ij}}{q_{ij}} > \frac{1}{\delta_{ij}} > 1$ (49)

In general,

$$C_{max} = \frac{\tau q}{n(\delta s - q)} \quad (50)$$

As a result, equation (49) impels that, the signal cycle C_j at j is depended on length of link in front of it (l_{ij}) speed of vehicle flow (V_{ij}) showed by τ and n, on arrivals (q_{ij}) and departures (s_{ij}) at this node, providing that the condition (50) is still true.

Therefore, if q_{ij} and s_{ij} change, δ_{ij} change respectively so that it is not congested at j. It is considered at current intersections of traffic network in Ho Chi Minh City, Viet Nam and lead to congestion during rush hour when huge number of transportations in traffic, violate the condition equation (50) since the cycles are always stable. One more thing, the current signal cycle at intersections do not inspect l_{ij}, a factor of capacity of link. Equation (49) impels it.

There are always two phases for each intersection, following equation (51) there will be two value of C for two phases. Consequently, which C is shorter than other will be selected.

2. Practice

Practicing at intersections Nguyen Chi Thanh – Ly Thuong Kiet in District 10, Ho Chi Minh City, Viet Nam showed in Figure 4 and Figure 5 and collection status are showed in table I used for practicing in this study.

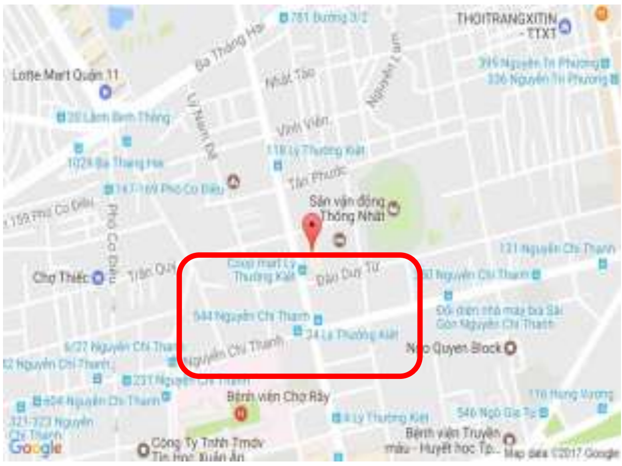

Fig. 4. Intersection Nguyen Chi Thanh – Ly Thuong Kiet on Google Map using for practice in this study.

The computed results based on real-life collected data of table I with n = 1 (because the distance between 2 intersection is less than 1 cycle) are showed in table II.

Fig. 5. Congestion of road traffic at intersection Nguyen Chi Thanh – Ly Thuong Kiet in District 10, Ho Chi Minh City, Viet Nam.

TABLE I. Collection data at Ly Thuong Kiet Street, Ho Chi Minh City, Viet Nam.

Time	N	q	s	V (m/s)
Morning				
07h40	159	109	104	5,14
07h45	118	81	77	4,71
07h50	169	84	80	4,42
08h00	126	64	60	4,24
08h09	156	89	85	3,92
At Noon				
11h40	140	70	66	5,98
11h45	108	70	67	4,37
11h55	136	80	78	5,63
12h07	101	63	62	4,78
Afternoon				
16h45	154,00	64	60	2,08
16h53	208,00	91	86	5,27
16h59	116,00	64	60	4,03
17h10	198,00	111	107	3,31
17h16	142,00	82	78	4,13
Cycle (s)	C = 74	Green: 29	Yellow: 3s	Red: 42s
Time offset (T off): β				2s
link (m)				160 m

TABLE II. Computed data at Ly Thuong Kiet Street, Ho Chi Minh City, Viet Nam.

Time	q (v/h)	s (v/h)	τ (s)	δ	C Computed
Morning	4154	10080	35,66	0.39	-661,99
At Noon	3442	8472	30,83	0.39	-769,31
Afternoon	4009	9708	42,49	0.39	-764,35

As a result, from real-life collected data such as, q, s, τ, δ, n, following equation (51), the value C is computed in table II. It implies that the signal cycle violates condition (50), lead to congestion. In order to adjust the cycle so that it suitable for real-time traffic situation, the value δ will be re-computed following equation (51). The result is expressed in table III.

TABLE III. Adjusted data at Ly Thuong Kiet Street, Ho Chi Minh City, Viet Nam.

Time	q (v/h)	s (v/h)	τ (s)	δ	C Computed
Morning	4154	10080	35,66	0,610	74,31
At Noon	3442	8472	30,83	0,580	72,09
Afternoon	4009	9708	42,49	0,650	74,03

II. Conclusion

In Vietnam, traffic congestion has been a major problem for many years. There are many solutions for reducing congestion. One of them is the use of Real-Time Signal Traffic Cycle Optimization. In this study we used mathematical modeling for reproducing and analyzing a broad variety of complex problems. The difficulty is that it can be too expensive or too dangerous to engage in studying such a problem. Traffic can be viewed as a complex system; therefore, researching and mathematical modeling is a suitable tool for analyzing traffic systems. The result was assessed and compared to current traffic situation in some streets in Ho Chi Minh City. As applying the model in real-life intersections, the clearing time and length of queue reduce over 90% compared with current signal cycle. In future studies, we will application of Internet of Thing (IoT), Mobile Communications and Applications, Creative Technologies, Smart Cities and so on should be considered.

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Biography

Nguyen Chi Hung was born in Long An Province Vietnam, on March 10, 1978. He received the B.Sc. and the M.Sc. degrees from the Department of Electrical and Electronic Engineering, University of Technology and Education (UTE), Ho Chi Minh City, Vietnam in 2000, 2006 and PhD. degree from the Department of Electrical Engineering, National Kaohsiung University of Applied Sciences (NKUAS), Kaohsiung, Taiwan in 2015. He is currently Head of Electrical Enginerring Division, Department of Electrical Electronic and Telecommunications (EET), Sai Gon University (SGU), Ho Chi Minh City, Viet Nam. His research interests are power system and quality, application of SFCL to power systems, protective coordination and Smart Cities.

Nguyen Chi Hung, "Operation Research and Mathematical Modeling of Real-Time Signal Traffic Cycle Optimization in Ho Chi Minh City of Southern Viet Nam," *International Research Journal of Advanced Engineering and Science*, Volume 3, Issue 2, pp. 197-201, 2018.