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Numerical Solution for Transient Heat Transfer in Longitudinal Fins

Navid Namdari, Morteza Abdi, Hamidreza Chaghomi, Farzad Rahmani
Mechanical Engineering Department, University of Toledo, OH, USA
Email address: navid.namdari@rockets.utoledo.edu

Abstract**—** *Longitudinal fins are extended surfaces to transfer the produced heat from the base to the environment. In the heat transfer models, the heat transfer coefficient and thermal conductivity are assumed as a linear or nonlinear function of the temperature. COMSOL software is a useful cross-platform finite element analysis which can be used for solving ordinary and partial linear or nonlinear heat transfer equations. The present paper aims to investigate transient heat transfer in longitudinal fins with different profiles including rectangular, parabolic, convex, exponential, sinusoidal, and cosine profile by modeling the fins in COMSOL. The results of transient heat transfer show that the nonrectangular profile provides a higher rate of heat transfer, which indicates its superiority over other types of fins.*

I. INTRODUCTION

Longitudinal fins are being widely used in aerospace, vehicle industries to increase the rate of heat transfer between a base and air by increasing the convection rate. convection rate and efficiency of a fin depends on the shape of the fin. Numerical and experimental studies have been done to determine the parameters, which affect the rate of heat transfer [1-11]. Researchers have implemented numerical software such ABAQUS, ANSYS, and COMSOL in order to solve the nonlinear equations in heat transfer, stress analysis, vibrations studies [12-27]. In the available heat transfer models, both heat transfer coefficient and thermal conductivity are dependent on local temperature and usually are assumed as a linear or nonlinear function of the temperature [28-49]. Nowadays developing advanced numerical techniques and computing tools enable researchers to use high performance software for obtaining a more accurate approximation of complex physical and mathematical problems with boundary conditions consistent to the reality. Particularly, COMSOL Multiphysics has provided a powerful finite element (FEM) partial differential equation (PDE) solver suitable to be applied for numerical solution to complicated problems in various areas of electrical, mechanical, and chemical engineering.

In the present study, transient heat transfer in longitudinal fins with different profiles including rectangular, parabolic, convex, exponential, sinusoidal and cosine profile is modeled in COMSOL. The results of transient heat transfer demonstrate that nonrectangular profiles have a higher rate of heat transfer which imparts the fin a higher performance.

1. Transient heat transfer in longitudinal fins

Different profiles of fins including rectangular, parabolic, convex profiles that have been modeled in COMSOL are illustrated in Figure 1.

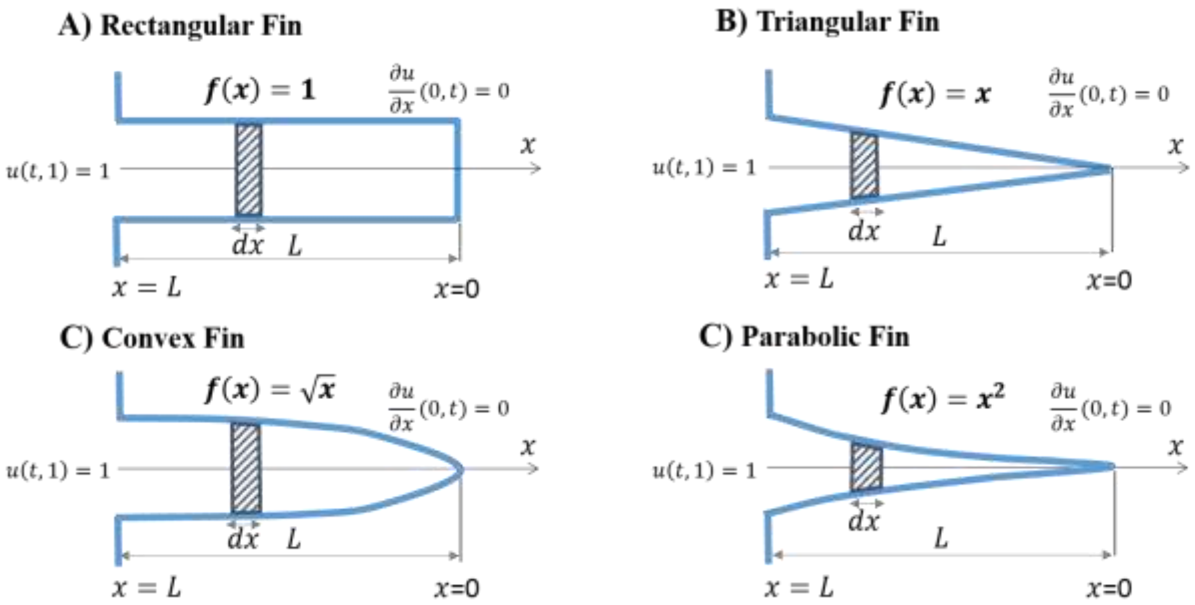

Fig. 1. Four different profiles of fins including rectangular, parabolic, convex, exponential, sinusoidal, and cosine profile.

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2. Nonlinear heat transfer equation in longitudinal fins

Considering the length of the longitudinal fins as one, nonlinear heat transfer equation in longitudinal fins can be written as:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}.\left[f(x).(1+BU).\frac{\partial U}{\partial x}\right]-M^2.U^{n+1} \qquad (1)$$

In this study, it is assumed that the fin temperature is zero at the onset of heat transfer. By considering the tip of the fin at a higher temperature and the base of the fin at a lower temperature, boundary conditions of heat transfer in a longitudinal fin are presented as follows:

$$U(0,x)=0$$
$$U(t,1)=1 \qquad (2)$$
$$\frac{U}{x}(0,t)=0$$

In 2016, Kadar et al. [26] considered a nonlinear thermal conductivity and presented a nonlinear equation for thermal conductivity as:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}.\left[f(x).\left[1+(n+2).\log(U)\right].\frac{\partial U}{\partial x}\right]-M^2.U^{n+1} \qquad (3)$$

In the last part of the present study Kadar PDE heat transfer equation for longitudinal fines is solved for different fin profiles by using COMSOL.

3. Results of nonlinear heat transfer in longitudinal fin

In this section by considering the corresponding function for profiles of the fins and considering the associated coefficients for B, n, and M, heat transfer equation in longitudinal fins has been extracted by solving each model in COMSOL. Figure 2 demonstrates the transient heat transfer of rectangular, parabolic, and convex profiles. The parabolic fin showed to have the lowest rate of heat transfer while the rectangular and convex profiles have the highest rate of heat transfer respectively.

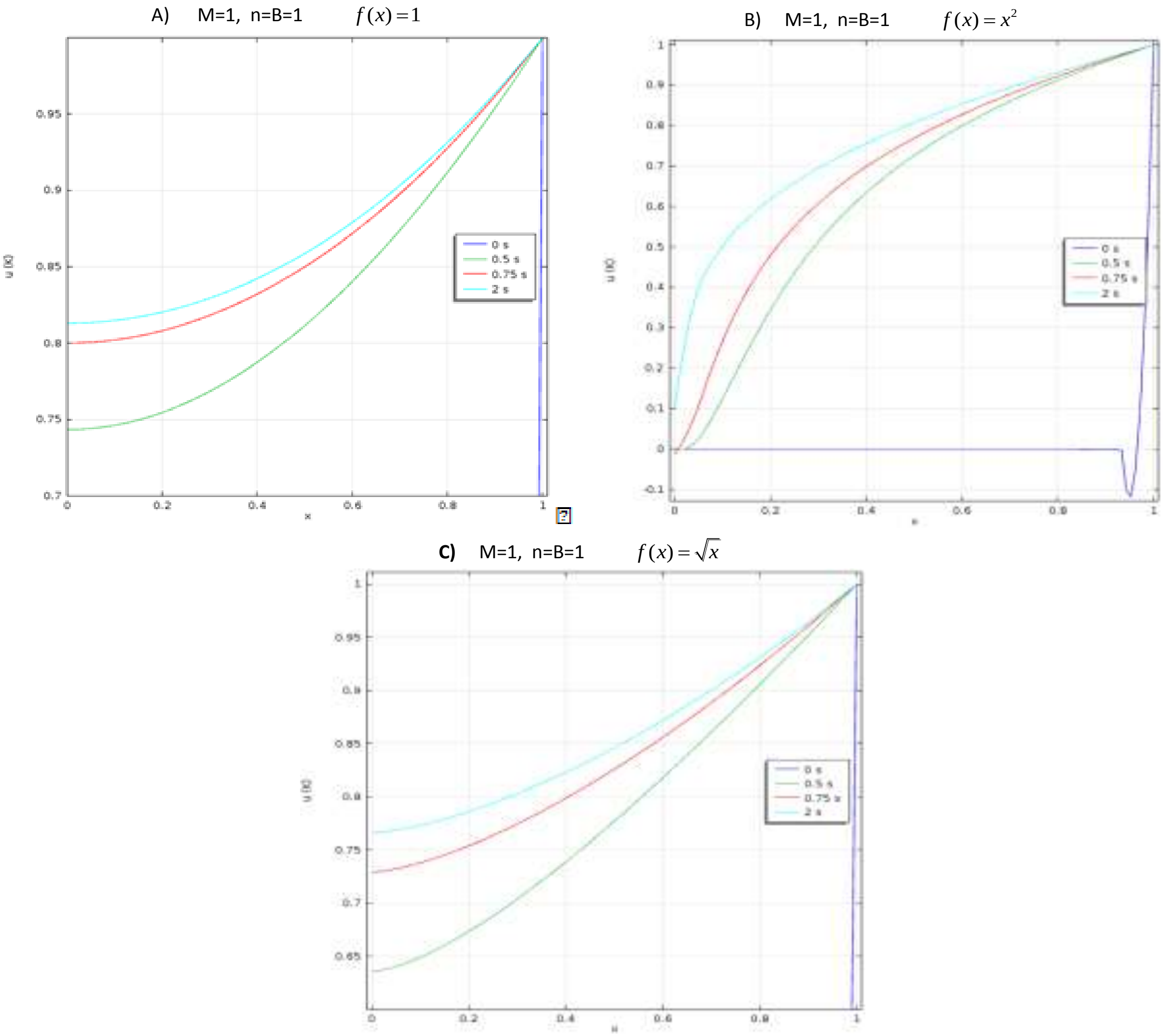

Fig. 2. The transient heat transfer of rectangular, parabolic, and convex profiles. (A: rectangular, B: Parabolic and C: convex profiles).

Figure 3 represents the transient heat transfer for sinusoidal, cosine, third order and fourth order profiles. The cosine fin shows to have higher rate of heat transfer with respect to its sinusoidal counterpart. Moreover, the results of simulations confirm incapability of cubic and fourth order fins in heat transfer because the temperature of the tip in transient solution remains standing at zero after 2 (s).

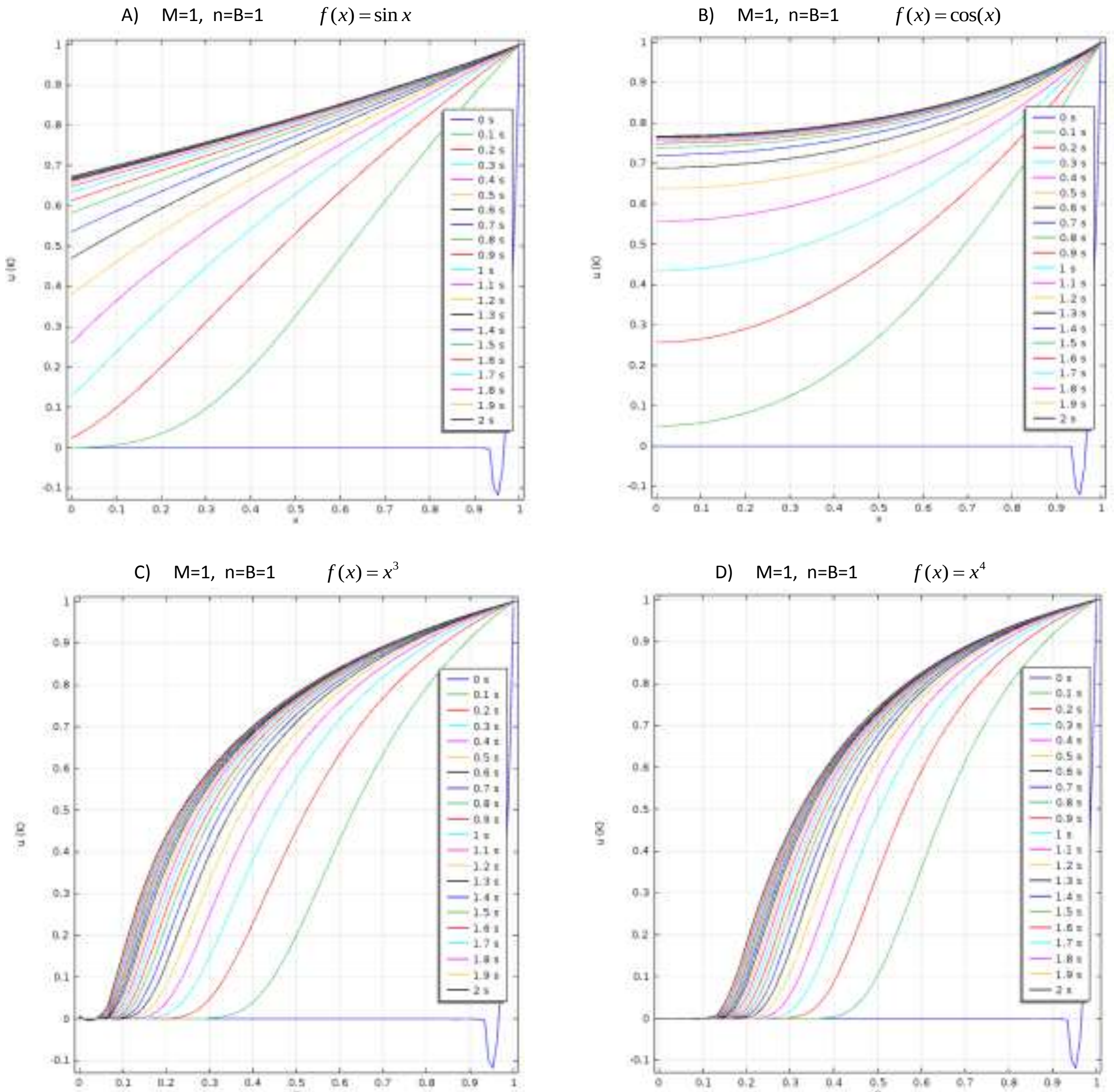

Fig. 3. The Transient heat transfer for sinusoidal, cosine, third order and fourth order profiles. (A: sinusoidal, B: Cosine, C: third order and D: forth order profiles).

Figure 4 illustrates the transient heat transfer of exponential profiles. As can be seen at t=0.1 (s) fin (B) has higher performance than fin (A) and this conclusion remains the same until the time reaches to t=2 (s).

Figure 5 shows the results of transient heat transfer for rectangular, triangular, parabolic and convex profiles using Equation (3) with selecting a nonlinear thermal conductivity.

As can be seen in the heat transfer figures (Figure 1 to 5), sinusoidal, cosine and exponential fins provide higher rates of heat transfer than rectangular fin that points out their higher functionality. Based on the heat transfer graphs (Figure 3), third order and fourth order fins do not produce an efficient heat transfer rate because at the area near to the tip of the fin temperature remains zero in the transient solution.

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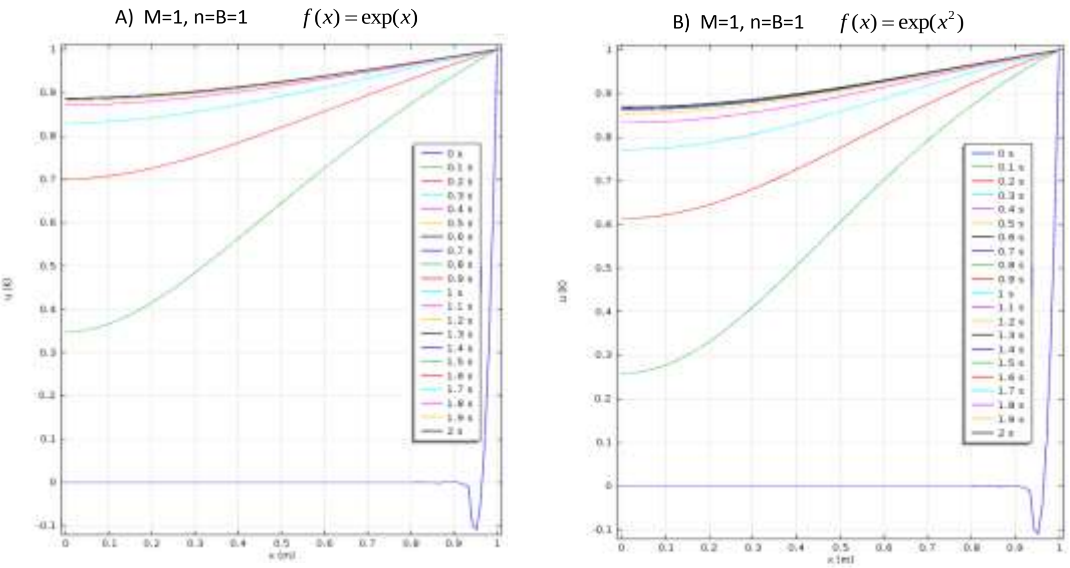

Figure 4. The transient heat transfer for exponential profiles.

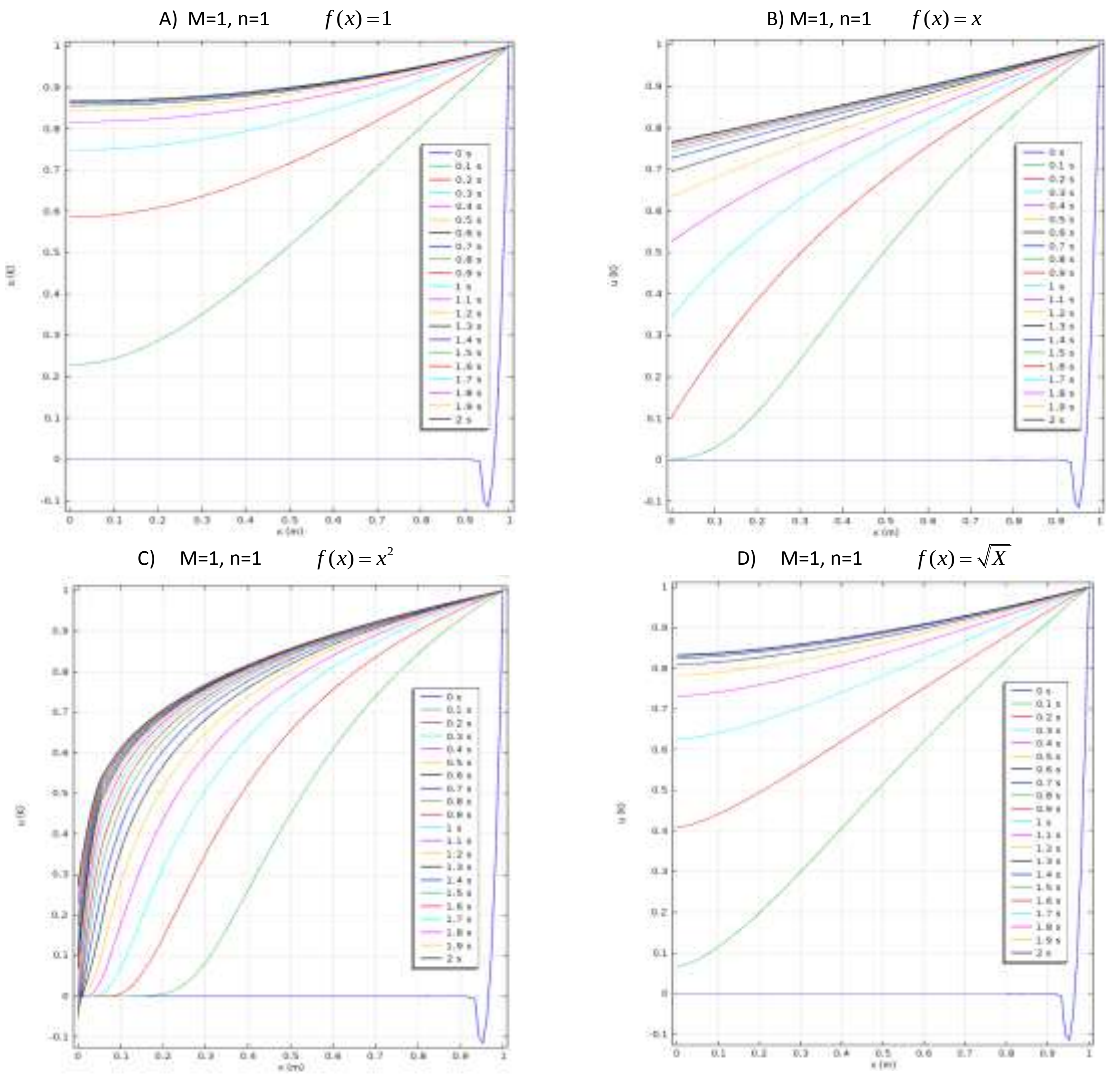

Figure 5. The transient heat transfer of rectangular, triangular, parabolic and convex profiles using Kadar et al. [26] PDE and considering a nonlinear thermal conductivity.

II. CONCLUSION

COMSOL is a useful cross-platform finite element analysis that can be used for solving ordinary and partial linear or nonlinear equations. In this paper, transient heat transfer in longitudinal fins for different profiles, including rectangular, parabolic, convex, exponential, and sinusoidal and cosine profile have been modeled. The results of the simulations showed sinusoidal, cosine and exponential fins have higher rates of heat transfer than rectangular fin that represents their higher functionality. This study also revealed that third order and fourth order fins do not present efficient heat transfer rates because in the transient solution of the fin, temperature remains at zero in the area near to the tip.

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