

Construction of Bayesian Single Sampling Plan by Attributes under the Conditions of Gamma Zero – Inflated Poisson Distribution

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Abstract—Sampling plans, systems and schemes are used to accept/reject the lots when inspecting a series of lots. A sampling system is a grouping of two or more sampling plans, with specified rules for switching between the plans and selecting the lots of finished products. Count a defecting unit in a production process in some situation that may have zeros. This situation very much suited distribution is Zero – inflated Poisson distribution than other process. This paper, construct single sampling plan by attribute in using Bayesian approach for assuming that the defective units are distributed to a gamma zero – inflated Poisson distribution. Employing an average of Gamma Zero – Inflated Poisson distribution and operating ratio of different producer and consumer risks have been analyses. Numerical examples are presented for Gamma Zero – Inflated Poisson distribution.

Keywords— Single sampling plan, sampling inspection by attributes, Gamma prior, Zero – Inflated Poisson distribution, operating characteristic function.

I. INTRODUCTION

The zero-inflated Poisson (ZIP) is an alternative process that can be considered here. This model allows for over dispersion assuming that there are two different types of individuals in the data: (1) Those who have a zero count with a probability of 1 (*Always-0 group*), and (2) those who have counts predicted by the standard Poisson (*Not always-0 group*). Observed zero could be from either group, and if the zero is from the Always-0 group, it indicates that the observation is free from the probability of having a positive outcome (Long 1997). The overall model is a mixture of the probabilities from the two groups, which allows for both the over dispersion and excess zeros that cannot be predicted by the standard Poisson model. Acceptance sampling is a major field/tool in Statistical Quality Control to inspect the quality of the product or raw material at various stages against the specified quality standards. Various sampling plans, systems and schemes were developed and applied in the industries based on the need of the shop floor situations. When 100% inspection is not possible there may be use acceptance sampling technique. Because when using sampling methods there arise more risks, but in acceptance sampling will reduce the risks of producer's as well as consumer's sides. Now a days 100% inspection is not possible for in all the industry. Buyer's thought will change day by day. So an industry wanted to maintain the quality standard becomes important and availability of that product in market is essential and price of the product must be considerable. These factors are may not be considered as

100% inspection, so there may be use acceptance sampling method. In acceptance sampling method classified by two major areas, one attributes and another variables sampling plans. This paper attempts to attribute single sampling plan in Bayesian approach. Many industries are manufactured items by continuous production process. An industry expects no defective items in the lot. The expectation to fulfill the probability distribution is Zero – inflated Poisson distribution. The production process may considered prior information so gamma prior also may be applied. The ZIP distribution can be viewed as a mixture of a distribution which degenerates at zero and a Poisson distribution.

ZIP distribution has been used in a wide range of disciplines such as agriculture, epidemiology, econometrics, public health, process control, medicine, manufacturing, etc. Some of the applications of ZIP distribution can be found in Bohning et al. (1999), Lambert (1992), Naya et al. (2008), Ridout et al. (1998), and Yang et al. (2011). Construction of control charts using ZIP distribution are discussed in Sim and Lim (2008) and Xie et al. (2001). Some theoretical aspects of ZIP distributions are mentioned in McLachlan and Peel (2000). Single sampling plans by attributes under the conditions of Zero – inflated Poisson distribution are determined by Loganathan and Shalini (2013), Suresh and Latha (2001), (2002) have given a procedure and tables for the selection of Bayesian SSP with gamma Poisson model. This paper attempts gamma zero - inflated Poisson distribution by attributing single sampling plan. Employing an average of gamma zero – Inflated Poisson distribution and operating ratio of different producer and consumer risks. Numerical examples are presented for Gamma Zero – Inflated Poisson distribution.

II. OPERATING PROCEDURE OF SSP

The operating procedure of a Bayesian SSP described as follows:

- 1) Draw a random sample of n units from a lot of N units.
- 2) Count the number of defective units, x in the sample
- 3) If $x \leq c$, accept the lot, otherwise, reject the lot.

Where, N , n , c are parameters of SSP, N is lots, n is sample, and c is acceptance constant.

III. OPERATING CHARACTERISTIC FUNCTION OF SSP WITH GAMMA- ZIP MODEL

The OC function of SSP is defined as

$$P_a(p) = P[X \leq c] \tag{1}$$

Where p is the probability of fraction defective
 The numbers of defects are zero for many samples there may consider Zero – inflated Poisson probability distribution. The probability mass function of the ZIP (ϕ, λ) distribution is given by Lambert (1992) and McLachlan and peel (2000)

$$P(X = x | \phi, \lambda) = \phi f(x) + (1 - \phi)P(X=x | \lambda) \tag{2}$$

where

$$f(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0, & \text{if } x \neq 0 \end{cases}$$

and

$$P(X = x / \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad \text{when } x = 0, 1, 2, \dots$$

The above probability mass function can also be expressed as

$$P(X = x | \phi, \lambda) = \begin{cases} \phi + (1 - \phi)e^{-\lambda} & \text{when } x = 0 \\ (1 - \phi) \frac{e^{-\lambda} \lambda^x}{x!}, & \text{when } x = 1, 2, \dots, 0 < \phi < 1, \lambda > 0 \end{cases}$$

In this distribution, ϕ may be termed as the mixing proportion. ϕ and λ are the parameters of the ZIP distribution. According to McLachlan and Peel (2000), a Zip distribution is a special kind of mixture distribution.

The OC function of the SSP under the conditions of ZIP (ϕ, λ) distribution can be defined as

$$P_a(p) = \sum_{x=0}^c P(X = x | \phi, \lambda) \tag{3}$$

$$P_a(p) = \phi + (1 - \phi)e^{-\lambda} + \sum_{x=1}^c (1 - \phi) \frac{e^{-\lambda} \lambda^x}{x!}$$

From the history of inspection it is known that p follows a Beta distribution which is for convenience approximated by a Gamma distribution (see Hald, 1981, p. 133) with density function $f(p)$.

$$f(p) = \frac{\beta^s}{\Gamma(s)} p^{(s-1)} e^{-\beta p} \tag{4}$$

Thus, the average probability of acceptance \bar{P} is approximately obtained by

$$\bar{P} = \int_1^c P_{(a)}(p) f(p) dp$$

$$\bar{P} = \phi + (1 - \phi)(1 - y)^s + \sum_{x=1}^c (1 - \phi) \binom{x+s-1}{s-1} y^x (1 - y)^s \tag{5}$$

Table I and II provides the average probability of acceptance against different values of s and acceptance number c as function of y .

The curve $\bar{P}(\mu)$ of the average probability of acceptance (APA), as function of μ with $n\mu = \frac{sy}{(1-y)}$ for different values of n, s and c can be taken instead of the OC curve for different values of n and c as done conventionally.

IV. DESIGNING BAYESIAN SINGLE SAMPLING PLANS

Sampling plans are constructed in such a way that protection to the producer as well as the consumer is ensured.

The optimum (n, c) could be determined satisfying the conditions

$$P_a(p_1) = 1 - \alpha \tag{6}$$

$$P_a(p_2) = \beta \tag{7}$$

where $p_1, \alpha, p_2,$ and β represent respectively, acceptable quality level, producer’s risk, rejectable quality level and consumer’s risk.

The \bar{P} values are computed for set of ($\phi, c, s,$ and $P_a(p)$), the different $P_a(p)$ values are 0.99, 0.95, 0.9, 0.5, 0.2, and 0.1. The fixed values of ϕ are 0.0001, 0.001, 0.01, 0.05, and 0.09 and S values are 1 (1) 9 for different c values are 1(1)9 are given in the Table I to III.

Hence, for specified (p_1, α, p_2, β) and ϕ a zero-acceptance number sampling plan can be determined from

$$n = \frac{s}{\mu} \left[\left(\frac{1 - \phi}{\bar{P} - \phi} \right)^{\frac{1}{s}} - 1 \right]$$

Satisfying $P_a(p) = 1 - \alpha$ and $P_a(p) = \beta$. Here, $p_1, \alpha, p_2,$ and β denote, respectively acceptable quality level, producer’s risk, rejectable quality level and consumer’s risk.

V. COMPARISON WITH CONVERSIONAL PLAN

From table I and II it is observed that for fixed value of $\phi, c,$ and \bar{P} as s value increases the $n\mu$ value approaches to the np value of the conversional plan. For large value of \bar{P} , $n\mu$ values increase with increase of s value, Where for $\bar{P} \leq 0.5,$ $n\mu$ values decrease with increase of s value and tends to the np values of conversional plan for large value of s .

Figure 1 shows the OC Curve for conversional Bayesian SSP for $n = 100, \phi = 0.001$ and $C=1$. It’s observed that Bayesian plan give more protection to the produced.

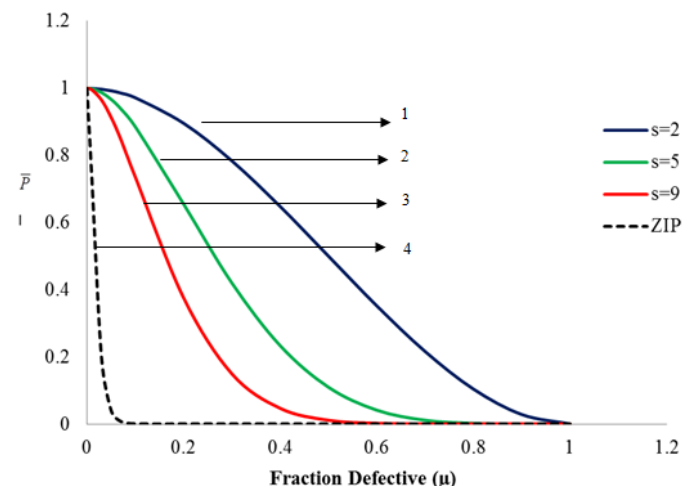


Fig. 1. OC Curve of Bayesian SSP with Gamma – ZIP Model and ZIP model, 1. Bayesian SSP with Gamma – ZIP Model($\phi = 0.001, C=1, s=2$), 2. Bayesian SSP with Gamma – ZIP Model($\phi = 0.001, C=1, s=5$), 3. Bayesian SSP with Gamma – ZIP Model($\phi = 0.001, C=1, s=9$), and 4. ZIP ($\phi = 0.001, \lambda$).

The operating ratio values calculated corresponding to ($\alpha=0.05, \beta=0.10$), ($\alpha=0.1, \beta=0.20$), $c=1(1)9$ and $s=1(1)9$ are listed in table III. $R1 = \alpha=0.05, \beta=0.10$ and $R2 = \alpha=0.1, \beta=0.20$.

For a given strength $(p_1, \alpha, p_2, \beta)$ and the value of ϕ , the plan parameters can be determined from these tables applying the following procedure:

- Step 1: Compute the operating ratio $R = p_2/p_1$.
- Step 2: Determine the acceptance number c from table corresponding to the values of ϕ , α , β , and to the value of R or it nearest.
- Step 3: Determine the unity values np_1 and np_2 from table I to III and calculate the sample size n as np_1/p_1 or np_2/p_2 , whichever is large (Schilling and Neubauer (2009)).

Illustration

Suppose that $\phi = 0.09$ and the plan specified by $p_1 = 0.005$, $\alpha = 0.05$, $p_2 = 0.15$ and $\beta = 0.10$. The value of the operating ratio

corresponding to these specifications is calculated as $R = \frac{p_2}{p_1} = \frac{0.15}{0.005} = 30$. The acceptance number can be found from Table corresponding to $\alpha = 0.05$, $\beta = 0.10$ and the nearest value 30 of R , as $c = 1$ and $s = 6$. The values corresponding to the values of ϕ , p_1 , α , p_2 and β are obtained from Table I as $np_1 = 0.356708$ and $np_2 = 11.85247$. Since $\frac{np_1}{p_1} = \frac{0.356708}{0.005} \approx 71$ and $\frac{np_2}{p_2} = \frac{11.85247}{0.15} \approx 79$, the number of products to be inspected is 79. Therefore, the sampling plan for the given specifications is (79, 1).

TABLE I. Bayesian SSPs under Gamma-Zero inflated Poisson for C=1.

ϕ	s	Pa(p)					
		0.99	0.95	0.9	0.5	0.2	0.1
0.0001	2	0.122188	0.312766	0.486886	2.000268	4.965491	8.200134
	3	0.135309	0.325244	0.49952	1.884978	4.186424	6.36598
	4	0.137847	0.33128	0.505905	1.829798	3.852226	5.624323
	5	0.138838	0.337242	0.512445	1.797922	3.659252	5.214353
	6	0.140959	0.338062	0.513168	1.776911	3.529309	4.961326
	7	0.144235	0.340996	0.51618	1.76468	3.455916	4.800601
	8	0.146376	0.344993	0.520804	1.751926	3.399108	4.667704
	9	0.148525	0.344495	0.519786	1.743443	3.349553	4.590836
	0.001	2	0.120816	0.314883	0.488624	2.002301	4.971271
3		0.135033	0.325466	0.499834	1.887035	4.196623	6.394443
4		0.137849	0.331456	0.506196	1.831688	3.860814	5.646741
5		0.138895	0.337419	0.512736	1.799716	3.666938	5.233669
6		0.139829	0.339547	0.514874	1.775986	3.544926	4.972853
7		0.144818	0.34116	0.516568	1.763497	3.456195	4.820559
8		0.146966	0.345158	0.518026	1.753669	3.405455	4.683058
9		0.146853	0.345291	0.520886	1.745236	3.356779	4.600838
0.01		2	0.123303	0.314701	0.48991	2.026329	5.125216
	3	0.136142	0.327174	0.502756	1.907947	4.302519	6.737653
	4	0.138601	0.333225	0.509133	1.850878	3.949696	5.88772
	5	0.139577	0.339202	0.515668	1.817924	3.746386	5.439771
	6	0.140504	0.341334	0.5178	1.793467	3.61861	5.188716
	7	0.145516	0.34295	0.519491	1.78061	3.523566	4.991583
	8	0.147673	0.34696	0.520947	1.770486	3.472802	4.846502
	9	0.147558	0.347091	0.523805	1.761794	3.421972	4.758684
	0.05	2	0.121649	0.325046	0.506511	2.145492	5.939954
3		0.137316	0.334361	0.515521	2.002794	4.87045	8.82999
4		0.139886	0.343492	0.525485	1.938108	4.417845	7.507113
5		0.142895	0.346722	0.52854	1.902189	4.16783	6.817185
6		0.135292	0.349166	0.530965	1.879073	4.002057	6.363328
7		0.141882	0.351064	0.532993	1.862814	3.89481	6.091586
8		0.145789	0.352716	0.532243	1.850386	3.816237	5.89397
9		0.147558	0.350697	0.535902	1.839997	3.757478	5.722639
0.09		2	0.132085	0.328831	0.51915	2.28265	7.212167
	3	0.143187	0.344004	0.531281	2.124481	5.720486	17.81308
	4	0.145118	0.354167	0.537519	2.048323	5.111409	14.03717
	5	0.146006	0.356708	0.543982	2.004582	4.770156	11.85247
	6	0.146951	0.358528	0.546049	1.972212	4.549256	10.7567
	7	0.152174	0.360149	0.547704	1.955349	4.424055	10.01012
	8	0.154414	0.364262	0.552339	1.941976	4.325485	9.494808
	9	0.154279	0.364374	0.552128	1.930457	4.242042	9.111523

TABLE II. Bayesian SSPs under Gamma-Zero inflated Poisson for C=4.

ϕ	s	Pa(p)					
		0.99	0.95	0.9	0.5	0.2	0.1
0.0001	2	0.010071	0.051819	0.107577	0.889265	4.607208	9.559558
	3	0.010407	0.051733	0.107239	0.820963	4.143625	8.364277
	4	0.010061	0.05157	0.106528	0.788854	4.006436	7.893525
	5	0.01006	0.051535	0.106385	0.769186	3.926679	7.659043
	6	0.010742	0.051505	0.106255	0.757092	3.907544	7.517462
	7	0.010057	0.051481	0.106151	0.746996	3.912319	7.421056
	8	0.010228	0.052321	0.106068	0.742323	3.9291	7.350209
	9	0.01024	0.052349	0.106046	0.738288	3.986799	7.295411
	0.001	2	0.002275	0.011633	0.023566	0.164064	0.885955
3		0.010075	0.051734	0.107108	0.820045	4.164254	8.329202
4		0.010071	0.051619	0.106637	0.790025	4.028909	7.936059
5		0.010069	0.051583	0.106487	0.77036	3.950751	7.698493
6		0.010752	0.051553	0.106357	0.758225	3.93346	7.553785
7		0.010067	0.051529	0.106253	0.748094	3.939478	7.455095
8		0.010238	0.05237	0.106169	0.743406	3.957217	7.382506
9		0.010249	0.052397	0.106147	0.739357	4.015409	7.326335
0.01		2	0.002278	0.011644	0.023588	0.166689	0.947788
	3	0.010167	0.052217	0.108136	0.833808	4.410439	8.903018
	4	0.010163	0.052103	0.107671	0.802732	4.261195	8.405713
	5	0.010162	0.052067	0.107519	0.782331	4.19691	8.115674
	6	0.01085	0.052036	0.107386	0.76976	4.191164	7.936174
	7	0.010159	0.052011	0.10728	0.759271	4.20605	7.812484
	8	0.010331	0.05286	0.107195	0.754429	4.229983	7.720968
	9	0.010343	0.052887	0.107171	0.750241	4.289655	7.649946
	0.05	2	0.002298	0.011871	0.024871	0.178183	1.193445
3		0.010598	0.054484	0.11305	0.901689	5.602545	12.7064
4		0.010594	0.05437	0.112523	0.865257	5.401852	11.38004
5		0.010592	0.054331	0.112357	0.841	5.348817	10.79431
6		0.01131	0.054297	0.112212	0.826152	5.370456	10.34173
7		0.010589	0.05427	0.112096	0.813797	5.372462	10.02293
8		0.010769	0.055153	0.112003	0.808161	5.379682	9.788969
9		0.01078	0.05518	0.11197	0.803253	5.389165	9.609457
0.09		2	0.002396	0.012386	0.026038	0.191878	1.456162
	3	0.011067	0.056967	0.118413	0.983456	7.165441	26.47432
	4	0.011062	0.056843	0.117834	0.940048	6.864694	22.02535
	5	0.011061	0.0568	0.117652	0.910648	6.727694	19.26866
	6	0.01181	0.056763	0.117493	0.892784	6.61919	17.57789
	7	0.011057	0.056734	0.117365	0.877969	6.584079	16.75406
	8	0.011245	0.057655	0.117263	0.871312	6.556093	15.93867
	9	0.011257	0.057681	0.117221	0.865461	6.535195	15.26868

TABLE III. The operating ratio values for different c, s and ϕ values.

ϕ	S	C=1		C=4	
		R1	R2	R1	R2
0.0001	2	26.22	10.20	184.48	42.83
	3	19.57	8.38	161.68	38.64
	4	16.98	7.61	153.06	37.61
	5	15.46	7.14	148.62	36.91
	6	14.68	6.88	145.96	36.78
	7	14.08	6.70	144.15	36.86
	8	13.53	6.53	140.48	37.04
	9	13.33	6.44	139.36	37.60
	0.001	2	26.17	10.17	139.36
3		19.65	8.40	161.00	38.88
4		17.04	7.63	153.74	37.78
5		15.51	7.15	149.24	37.10
6		14.65	6.89	146.52	36.98
7		14.13	6.69	144.68	37.08
8		13.57	6.57	140.97	37.27
9		13.32	6.44	139.82	37.83
0.01		2	27.69	10.46	145.90
	3	20.59	8.56	170.50	40.79
	4	17.67	7.76	161.33	39.58
	5	16.04	7.27	155.87	39.03
	6	15.20	6.99	152.51	39.03

0.05	7	14.55	6.78	150.21	39.21
	8	13.97	6.67	146.07	39.46
	9	13.71	6.53	144.65	40.03
	2	37.92	11.73	179.04	47.99
	3	26.41	9.45	233.21	49.56
	4	21.86	8.41	209.31	48.01
	5	19.66	7.89	198.68	47.61
	6	18.22	7.54	190.47	47.86
	7	17.35	7.31	184.69	47.93
0.09	8	16.71	7.17	177.49	48.03
	9	16.32	7.01	174.15	48.13
	2	91.36	13.89	269.62	55.92
	3	51.78	10.77	464.73	60.51
	4	39.63	9.51	387.48	58.26
	5	33.23	8.77	339.24	57.18
	6	30.00	8.33	309.67	56.34
	7	27.79	8.08	295.31	56.10
	8	26.07	7.83	276.45	55.91
9	25.01	7.68	264.71	55.75	

VI. CONCLUSION

Bayesian acceptance sampling is the best technique which deals with the procedure in which decision to accept or reject

lots or process based on their examination of past history or knowledge. Count data with extra zeros are common in many medical applications. The zero-inflated Poisson (ZIP) model is useful to analyse such data. For hierarchical or correlated count data where the observations are either clustered or represent repeated outcomes from individual subjects. However, the ZIP parameter estimation can be severely biased if the non-zero counts are over dispersed in relation to the Poisson distribution. The ZIP distribution has been shown to be useful for modelling outcomes of manufacturing process producing numerous defect-free products. When there are several types of defects, the multivariate ZIP model can be useful to detect specific process equipment problems and to reduce multiple types of defects simultaneously. In a production process being well-monitored, occurrence of non-defects would be more frequent and a ZIP distribution is the appropriate probability distribution to the number of defects per unit. The Gamma ZIP distribution reduces both producer's risk as well as consumer's risk. When inspection errors are taken into account, similar procedure can be followed to determine the plan parameters corresponding to the apparent lot fraction nonconforming.

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