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Implementation of Kalman-Bucy Filter for Continuous Time State Estimation in Simulink

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Abstract—*This paper has presented implementation of Kalman-Bucy filter for continuous time state estimation in Simulink. Many practical cases involving engineering science and embedded system requires filtering application. A Kalman-Bucy filter implemented in Simulink is used to estimate the output temperature of a typical heating system whose transfer function is considered in this paper. The result obtained showed that the designed filter performed effectively by rejecting the noise and tracking the input.*

Keywords— *Embedded system, Filtering, heating system, Kalman-Bucy, Simulink.*

I. INTRODUCTION

In many practical cases, involving engineering and embedded system, filtering application is often times desirable. One of such filter that has find wide acceptance and application in theory and in practice is the Kalman filter (KF). As a very powerful filter that has been used in various aspects, it can be used to perform the estimation of: the past, present, and even future states, and even in situation when the exact nature of modeled system is not known. In practice, it has helped to perform many tasks that would not have been possible without it. Its immediate areas of applications have been in the control of intricate dynamic systems like: in continuous manufacturing processes, aircraft, ships, or spacecraft. The Kalman filter is tool for obtaining estimates that are reliable [3].

The Kalman filter (KF) is a discrete time filter. In some cases a continuous time filter may be desired to estimate unmeasured states of a linear continuous time process. Then it will be required to design a continuous time filter that will replace the KF so as to be able to estimate unmeasured states of a linear continuous process. The Kaman-Bucy filter is such continuous time form of the KF. These filtering techniques to a large extended have impacted on control theory, signal processing and time series analysis [1].

In the study of the Kalman-Bucy filter for integrable L´evy process with infinite second momemt, [1] applied the Kalman-Bucy filter to a situation where a finite dimensional L´evy processs drive both the system and observation processes. The result obtained showed that the observation noise components which have infinite variance contributed nothing to the filtering equations. In [2], an improved approach to Kalman-Bucy filtering using the identification algorithm was presented. It stated that accounting for the time delaying a system could render the KBF ineffective. Hence, the paper presented an identification algorithm that improved the KBF in the presence of time delay. A simple method for estimation of parameters in first order systems is presented in [4]. It used a step response technique to estimate the parameters of a first order system with time delay. In order to perform an estimate of the state of a continuous-time process, [5] presented a new nonlinear filter for continuous time measurement which has a more general stochastic measurement. In the study of estimation of the centre of gravity of a manoeuvring aircraft using Kalman filters and the ADMIRE aircraft model, [6] used a nonlinear aircraft model to estimate the centre of gravity during the manoeuvring. In order to effectively deal with nonlinearities in the ADMIRE aircraft model, the Kalman-Bucy filter coefficients for pitching moment were modified as the aircraft manoeuvred. A Kalman-Bucy filter is presented in [8] for multivariable ship motion control system. Shaolin et al [9] presented an outlier- tolerant Kalman filter of state vectors in linear stochastic system.

In this paper, the focus is on the implementation of Kalman- Bucy filter for a continuous time signal in Simulink. The Kalman-Bucy filter is used to estimate unmeasured states and output of a temperature system [10].

A. Problem Statement

It is required to properly estimate the unmeasured states and output temperature of a heat system. In order to carry out this estimate, a Kalman-Bucy filter is chosen because the system is a continuous time process. Hence, it is assumed in this context that the temperature system is a linear continuous-time process whose unmeasured states and output will be estimated using only the process input and measurement noise.

B. The Kalman-Bucy Filter

If the inputs and measured outputs are given such that assumptions are made on the process and output noise, the purpose of the Kalman-Bucy filter is to estimate unmeasured states (considering them to be observable) and the actual process outputs. Figure 1 shows the estimated states $\hat{z}$, and $\hat{y}$ are the estimated measured outputs.

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Fig. 1. Block diagram of input-output relation of the Kalman-Bucy filter.

The Kalman-Bucy filter requires a differential Ricaati equation to be integrate through time unlike the Kalman-filter which uses a predictor-corrector algorithm to update the state estimates [7].

The mathematical equations representing the filter update are stated as:

$$G = PC^T V^{-1} \quad (1)$$

$$\dot{\hat{z}} = A\hat{z}(t) + Bu((t)) + G(y(t) - C\hat{z}(t)) \quad (2)$$

$$\dot{P} = AP + PA^T - PC^T V^{-1} CP + Q \quad (3)$$

In the above equations, G is Kalman-Bucy filter observer gain matrix which makes the observer sensitive to sensor noise, P is an estimate of the covariance of the measurement error and satisfies the Riccati equation, C^T is the transpose of the measurement matrix C, V is a weighting matrix of measurement (sensor) noise, Q is the a weighting matrix of process (state) noise, A is the system matrix, and B is the input matrix. For the filter implementation, both $\dot{\hat{z}}$ and $\dot{P}$ must be integrated through time.

II. Methodology

A. Modelling of Temperature System

It is required to estimate the state and measured output temperature of a heat system whose dynamic equation is represented in form of a transfer function as [10]:

$$G(s) = \frac{0.8}{3s+1} \quad (4)$$

B. State Representation

In order to present the system state space, it is assumed that the system is a linear continuous –time process with input and measurement noise as shown in figure 2.

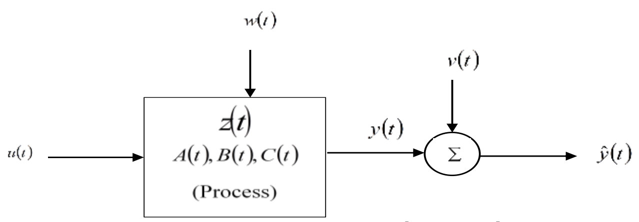

Fig. 2. Linear continuous-time process with input and output noise

$$\dot{z} = Az + Bu + w \quad (5)$$

$$y = Cz \quad (6)$$

$$\hat{y} = y + v \quad (7)$$

where u is the vector of inputs, z is the actual states vector, y is the actual process outputs vector, $\hat{y}$ is a vector of the actual process outputs, w and v are state and output noise respectively. In this context, the state or process and output noise are assumed to be zero Gaussian with covariance Q and R respectively.

The state space representation of Eq. (4) is as follows:

$$\dot{z} = -\frac{1}{3}z + \frac{1}{3}u \quad (8)$$

$$y = 0.8z \quad (9)$$

III. System Configuration

Figure 3 is the block diagram for implementing the Kalman-Bucy filter in Matlab/Simulink. FSI and FSO means FilterStatein and FilterStateout respectively. The Simulink block model uses an Embedded Matlab function named Kalman-Bucy Filter as shown in figure 3. The filter is used to estimate the unmeasured states of a continuous process (temperature of a heat system) and the output of the process using only the process noise and a noisy measurement.

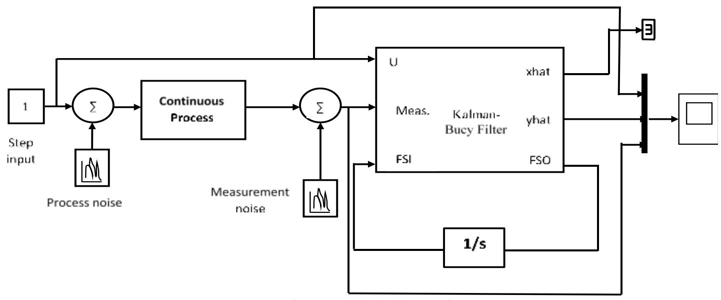

Fig. 3. Simulink block model for Kalman-Bucy filter implementation.

IV. Simulation Results and Discussion

A. Results Overview

The results obtained from the simulations carried-out in the Simulink environment of the Matlab software are presented in figure 4, 5, 6, and 7.

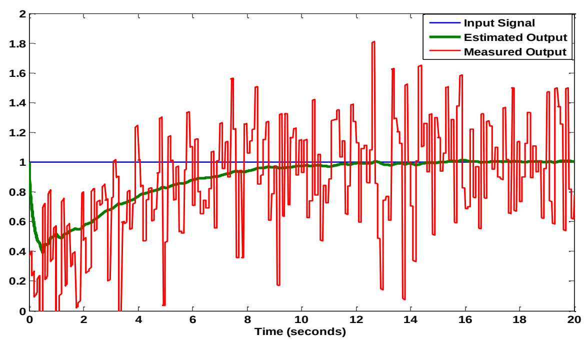

Fig. 4. Step response (Variance = 0.01, 0.1).

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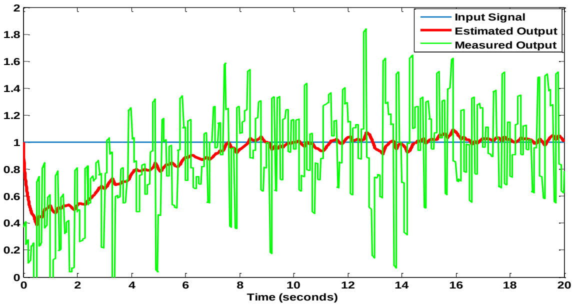

Fig. 5. Step response (Variance = 0.1, 0.1).

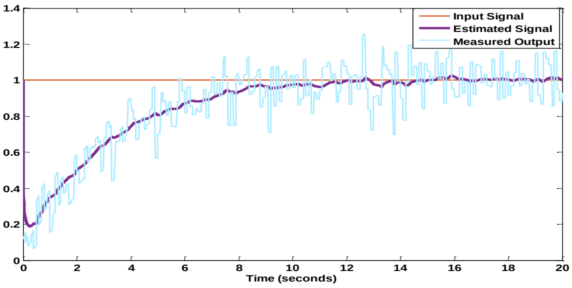

Fig. 6. Step response (Variance = 0.01, 0.01).

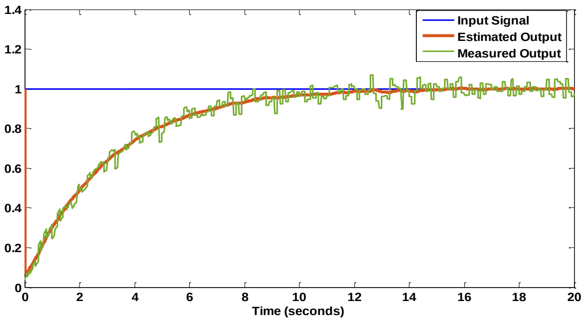

Fig. 7. Step response (Variance = 0.001, 0.001).

B. Discussion

The plots above were obtained by subjecting the considered continuous time process to two sources of zero-mean Gaussian noise using the random signal of the Matlab/Simulink block. In figure 4, the input signal is corrupted with noise whose variance is 0.01 while the output signal is corrupted with noise whose variance is 0.1. In figure 5, the input signal is corrupted with noise whose variance is 0.1 while the output signal is corrupted with noise whose variance is 0.1. In figure 6, the input signal is corrupted with noise whose variance is 0.01 while the output signal is corrupted with noise whose variance is 0.01. In figure 7, the input signal is corrupted with noise whose variance is 0.001 while the output signal is corrupted with noise whose variance is 0.001. It can be seen from the simulation results that the performance of the Kalman-Bucy filter in terms of rejecting the noise and estimating an output signal that tracks the step input signal accurately improves within the selected variance.

V. Conclusion

The paper has presented implementation of Kalman-Bucy filter for continuous time state estimation in Simulink. In this paper, it should be noted that both the Kalman- Bucy filter actually calculated the estimated state and the output but only the plots of the estimated output signal are presented. It can be seen that the implemented Kalman-Bucy filter performs very well by rejecting the noise signal and estimating a signal that tracks the referenced step input. Conclusively, the accurate tracking of the referenced step input by the estimating signal is achieved by selecting appropriate state and measurement noise variance.

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