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# Heat and Mass Transfer Effects on MHD Free Convection Flow over an Inclined and Exponentially Accelerated Plate Embedded in Porous Medium with Heat Source

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***Abstract***— *The effects of Heat and mass transfer on MHD free convection flow over an inclined and exponentially accelerated plate embedded in porous medium with heat source has been discussed. An exact solution of the governing equations for fully developed flow is obtained in closed form. Detailed computations of the influence of the Grash of number, modified Grash of number, Hartmann number, slip parameter, porosity parameter and chemical reaction parameter are discussed with the help of tables and graphs*.

***Keywords***— *Inclined Plane, exponentially accelerated, porous medium, Heat Source, MHD, and mass transfer*.

## I. INTRODUCTION

Free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems, etc. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Studies pertaining to coupled heat and mass transfer due to free convection has got wide applications in different realms, such as, mechanical, geothermal, chemical sciences, etc. Many industrial, technological and physical setups such as nuclear reactors, food processing, polymer production, etc. considers the effect of heat and mass transfer. In nature, there exist flows which are caused not only by the temperature differences but also by concentration differences. These mass transfer differences do affect the rate of heat transfer. In industries, many transport process exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect of thermal and mass diffusion. In many practical situations such as condensation, evaporation and chemical reactions the heat transfer process is always accompanied by mass transfer process. Recently, Magetohydrodynamics is attracting the attention of the many authors due to its applications in geophysics; it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. At high temperatures in some engineering devices like in MHD pumps, in MHD bearings, etc. gases can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic effects and alter heat transfer and friction characteristics.

Israel-Cookey et al. [5] discussed Influence of viscous dissipation on unsteady MHD free convection flow past an infinite heated vertical plate in porous medium with time-dependent suction. Kim [4] discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Senapati et al. [6] have studied magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in the presence of chemical reaction. Senapati et al. [7] also discussed the chemical effects on mass and heat transfer on MHD free convection flow of fluids in vertical plates and in between parallel plates in poiseuille flow. Jha et al. [1] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy et al. [2] have studied mass transfer effect on exponentially accelerated isothermal vertical plate. Senapati et al. [3], have analyzed the effect of heat and mass transfer on MHD free convection flow past in oscillating vertical plate with variable temperature embedded in porous medium. Unsteady hydromagnetic convective flow past an infinite vertical porous medium has been discussed by Das et al. [8]. Mangathai et al. [9] studied the effects of heat and mass transfer on MHD free convection flow over an inclined plate embedded in a porous medium,

In this paper, closed form of analytic solutions for the Heat and mass transfer effects on MHD free convection flow over an inclined and exponentially accelerated plate embedded in porous medium with heat source has been developed.

## II. MATHEMATICAL FORMULATION

Let us consider the unsteady flow of an electrically conducting incompressible viscous fluid past an infinite inclined plate exponentially accelerated with variable heat and mass transfer and constant suction. The x’ axis is taken along the plate which makes an angle α with the vertical and y’ axis is perpendicular to the plate. In y’ direction, a uniform magnetic field $B_0$ is applied transversally to the plate. Initially both plate and fluid are at rest with constant Temperature $T_\infty^{'}$ and constant mass concentration $C_\infty^{'}$. At the time $t^{'} > 0$, the plate is given a sudden jerk making exponential movements



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having velocity $u = U_0 e^{At'}$. The temperature and mass concentration of the plate are raised linearly w.r.t. time t' with same parameter A = $\frac{U_0^2}{\nu}$. Since the plate is in x'y' plane, so all the physical variables are of the functions of $y'$ and $t'$ only. Considering all above assumptions and usual Boussineq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right)cos\alpha + g\beta_c\left(C' - C'_\infty\right)cos\alpha - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - S'\left(T' - T'_\infty\right) \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - R'\left(T' - T'_\infty\right) \tag{3}$$

With the following boundary conditions

$$\begin{aligned} &t' \le 0: \quad u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad for\, all\, y' > 0. \\ &t' > 0: \begin{cases} u' = U_0 e^{At'}, T' = T'_\infty + \left(T'_w - T'_\infty\right)At', C' = C'_\infty + \left(T'_w - T'_\infty\right)At' \; at y' = 0 \\ u' = 0,\ T' = T'_\infty,\ C' = C'_\infty \; for\, all\, y' \to \infty \end{cases} \end{aligned} \tag{4}$$

where $u'$ is axial velocity, $\nu = \frac{\mu}{\rho}$, K' is the permeability of porous medium, g is the acceleration due to gravity, $\beta$ is co efficient of thermal expansion, $\beta_c$ is the co-efficient mass concentration expansion, k is thermal diffusion and D is mass diffusion.

Let us introduce the following non-dimensional quantities

$$\left.\begin{aligned} &u = \frac{u'}{U_0}, t = \frac{t'U_0^2}{\nu}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gr = \frac{g\beta\nu\left(T'_w - T'_\infty\right)}{U_0^3}, Gm = \frac{g\beta_c\nu\left(C'_w - C'_\infty\right)}{U_0^3} \\ &Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ K = \frac{U_0^2 K'}{\nu^2},\ M = \frac{\sigma B_0^2 \nu}{\rho U_0^2},\ R = \frac{R'\nu}{U_0^2},\ S = \frac{S'\nu}{U_0^2} \end{aligned}\right\} \tag{5}$$

The equations (1) to (3) with boundary condition (4) reduce to

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\,\theta\, cos\alpha + Gm\,C\, cos\alpha - \left(M + \frac{1}{K}\right)u \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} - S\theta \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - RC \tag{8}$$

with the boundary conditions

$$\begin{aligned} &t \le 0:\ u = 0,\ \theta = 0,\ C = 0 \quad for\, all\, y > 0 \\ &t > 0: \begin{cases} u = e^t,\ \theta = t, \quad C = t \;\; at\, y = 0 \\ u \to 0,\ \ \theta \to 0,\ C \to 0 \;\; as\, y \to \infty \end{cases} \end{aligned} \tag{9}$$

## III. Method of Solution

Let us consider the velocity, temperature and mass concentration of the flow field as

$$u = u_0 e^{\omega t},\ \theta = \theta_0 e^{\omega t}\ and\ C = C_0 e^{\omega t} \tag{10}$$

Substituting relation (10) in equations (6) to (8) with boundary conditions (9), we get

$$\frac{d^2 u_0}{dy^2} - \left(\omega + M + \frac{1}{K}\right)u_0 = -\left(Gr\theta_0 + GmC_0\right)Cos\alpha \tag{11}$$

$$\frac{d^2 \theta_0}{dy^2} - \left(\omega + S\right)\theta_0 = 0 \tag{12}$$

$$\frac{d^2 C_0}{dy^2} - \left(\omega + R\right)C_0 = 0 \tag{13}$$

with the following boundary conditions

$$t > 0: \begin{cases} u_0 = e^{(1-\omega)t},\ \theta_0 = te^{-\omega t},\ C_0 = te^{-\omega t} \;\; at\, y = 0 \\ u_0 \to 0,\ \ \theta_0 \to 0,\ \ C_0 \to 0 \;\; as\, y \to \infty \end{cases} \tag{14}$$

By solving equations (11) to (13) with boundary conditions (14) using equation (10), we get

$$\left.\begin{aligned} &u = \left(a_1 e^{-K_3 y} + a_{12} e^{-K_1 y} + a_3 e^{-K_2 y}\right)e^{\omega t} \\ &\theta = \left(te^{-K_1 y}\right)e^{\omega t} \\ &C = \left(te^{-K_2 y}\right)e^{\omega t} \end{aligned}\right\} \tag{15}$$

Now, it is important to calculate Skin friction, Nusselt Number and Sherwood Number using equation (10).

The skin friction near the plate from velocity field is

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\left(K_3 a_1 + K_1 a_2 + K_2 a_3\right)e^{\omega t} \tag{16}$$

The Rate of heat transfer / Nusselt number near the plate from temperature field is

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \left(K_1 t\right)e^{\omega t} \tag{17}$$

The Rate of mass transfer / Sherwood Number near the plate from mass concentration field is

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = \left(K_2 t\right)e^{\omega t} \tag{18}$$

Where

$$a_1 = e^{(1-\omega)t} + \left(\frac{GrtCos\alpha}{K_1^2 - K_3^2} + \frac{GmtCos\alpha}{K_2^2 - K_3^2}\right),$$

$$a_2 = -\frac{GrtCos\alpha}{K_1^2 - K_3^2},\ a_3 = -\frac{GmtCos\alpha}{K_2^2 - K_3^2},$$

$$K_1 = \sqrt{(S+\omega)Pr}, K_2 = \sqrt{(R+\omega)Sc}, K_3 = \sqrt{\left(M + \frac{1}{K} + \omega\right)}$$

## IV. Graphical Results and Discussion

In this paper, the Effect of Heat and Mass transfer effects on MHD free convection flow over an inclined and exponentially accelerated plate embedded in porous medium with heat source has been discussed. The effect of the parameters Gr, Gm, R, S, Sc, Pr, M, K, α, t and ω on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w.r.t y.



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Shearing Stress, Nusselt number and Sherwood Number at walls are obtained in the tables for different parameters.

*Velocity Profiles*

The velocity profiles are depicted in Figures 1-5. Figure 1 shows the effect of the parameters R and S on velocity at any point of the fluid, when Gr=2, Gm=2, M=0.5, K=2, Sc=0.23, Pr=0.71, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t = 0.5. It is noticed that the velocity decreases with the increase of Chemical reaction parameter (R) and Source parameter (S).

Figure 2 shows the effect of the parameters Sc and Pr on velocity at any point of the fluid when Gr=2, Gm=2, M=0.5, K=2, R=2, S=2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t=0.5. It is noticed that the velocity decreases with the increase of Schmidt number (Sc) and Prandtl number (Pr).

Figure 3 shows the effect of the parameters Gr and Gm on velocity at any point of the fluid, when Sc=0.23, Pr=0.71, M=0.5, K=2, R=2, S=2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t=0.5. It is noticed that the velocity increases with the increase of Grashoff number (Gr) and modified Grashoff number (Gm).

Figure 4 shows the effect of the parameters M and K on velocity at any point of the fluid when Sc=0.23, Pr=0.71, Gr=2, Gm=2, R=2, S=2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t=0.5. It is noticed that the velocity decreases with the increase of Magnetic parameter (M), whereas increases with increase of permeability parameter of porous medium (K).

Figure 5 shows the effect of the parameters t, ω and α on velocity at any point of the fluid, when Sc=0.23, Pr=0.71, M=0.5, K=2, R=2, S=2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t=0.5. It is noticed that the velocity increases with the increase of Oscillating frequency (ω), whereas decreases with the increase of time (t) and Inclination angle (α).

*Temperature Profile*

Figure 6 shows the effect of the parameters S, Pr, t and ω on Temperature profile, in the absence of other parameters. It is noticed that the temperature rises with the increase of time (t) and oscillating frequency (ω), whereas temperature falls in the increase of (S) and Prandtl number (Pr).

*Mass Concentration Profile*

Figure 7 shows the effect of the parameters Sc and R on mass concentration profile at any point of the fluid when ω=1, t=0.5 in the absence of other parameters. It is noticed that the mass concentration decreases with the increase of Schmidt number (Sc) and Chemical reaction parameter (R).

Table I shows the effects of different parameters on Skin friction at plate. It is noticed that skin friction increases with the increase of Chemical reaction parameter (R), Source parameter(S), Prandtl number (Pr), Oscillating frequency (ω) and Magnetic parameter (M), whereas decreases with the increase of Grashoff number (Gr), modified Grashoff number (Gm), Schmidt number (Sc), time (t) and Inclination angle (α).

Table II shows the effects of S, Pr, t and ω on Nusselt number at the plate. It is noticed that Nusselt number increases with the increase of Source parameter(S), Prandtl number (Pr), Oscillating frequency (ω) and time (t).

Table III shows the effects of Sc, R, t and ω on Sherwood Number at the plate. It is noticed that Sherwood Number increases with the increase of Schmidt number (Sc), frequency of oscillation (ω) and Chemical reaction parameter (R) near the plate.

TABLE I. Effect of different parameters on skin friction.

| R | S | Pr | M | K | Gr | Gm | Sc | t | α | ω | Skin Friction at plate |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 0.71 | 0.5 | 2 | 2 | 2 | 0.23 | 0.5 | $\pi/6$ | 1.5 | 1.1987 |
| 3 | | | | | | | | | | | 1.2332 |
| 4 | | | | | | | | | | | 1.2606 |
| 2 | 3 | | | | | | | | | | 1.2349 |
| | 4 | | | | | | | | | | 1.2627 |
| | 2 | 2 | | | | | | | | | 1.3261 |
| | | 7 | | | | | | | | | 1.4575 |
| | | 0.71 | 1 | | | | | | | | 1.5452 |
| | | | 2 | | | | | | | | 2.1272 |
| | | | 0.5 | 3 | | | | | | | 1.0713 |
| | | | | 4 | | | | | | | 1.0049 |
| | | | | 2 | 4 | | | | | | 0.7019 |
| | | | | | 6 | | | | | | 0.2050 |
| | | | | | 2 | 4 | | | | | 0.5627 |
| | | | | | | 5 | | | | | 0.2447 |
| | | | | | | 2 | 0.6 | | | | 1.5824 |
| | | | | | | | 0.78 | | | | 1.3498 |
| | | | | | | | | 0.7 | | | 0.9166 |
| | | | | | | | | 1 | | | 0.1085 |
| | | | | | | | | 0.5 | $\pi/4$ | | 1.9066 |
| | | | | | | | | | $\pi/3$ | | 1.6776 |
| | | | | | | | | | $\pi/6$ | 1.5 | 1.2865 |
| | | | | | | | | | | 2 | 1.2921 |

TABLE II. Effect of different parameters on rate of heat transfer / Nusselt number.

| S | Pr | ω | t | Nusselt Number(Nu) |
|---|---|---|---|---|
| 2 | 0.71 | 1 | 0.5 | 1.2031 |
| 3 | | | | 1.3892 |
| 4 | | | | 1.5532 |
| 2 | 2 | | | 2.0193 |
| | 7 | | | 3.7777 |
| | 0.71 | 1.5 | | 1.6686 |
| | | 2 | | 2.2905 |
| | | 1 | 1 | 3.9672 |
| | | 1 | 1.5 | 9.8112 |

TABLE III. Effect of different parameters on rate of mass transfer / Sherwood number.

| Sc | R | ω | t | Sherwood Number (Sh) |
|---|---|---|---|---|
| 0.23 | 2 | 1 | 0.5 | 0.6848 |
| 0.3 | | | | 0.7821 |
| 0.6 | | | | 1.1060 |
| 0.78 | | | | 1.2610 |
| | 3 | | | 0.7907 |
| | 4 | | | 0.8840 |
| | | 1.5 | | 0.9497 |
| | | 2 | | 12.2756 |
| | | | 1 | 2.2560 |
| | | | 1.5 | 5.5542 |



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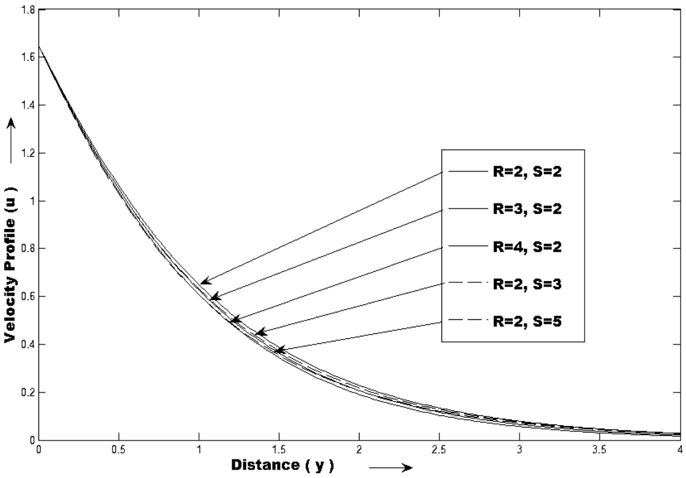


Fig. 1. Effect of R and S on velocity profile, when Gr = 2, Gm = 2, M = 0.5, K = 2, Sc = 0.23, Pr = 0.71, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t = 0.5

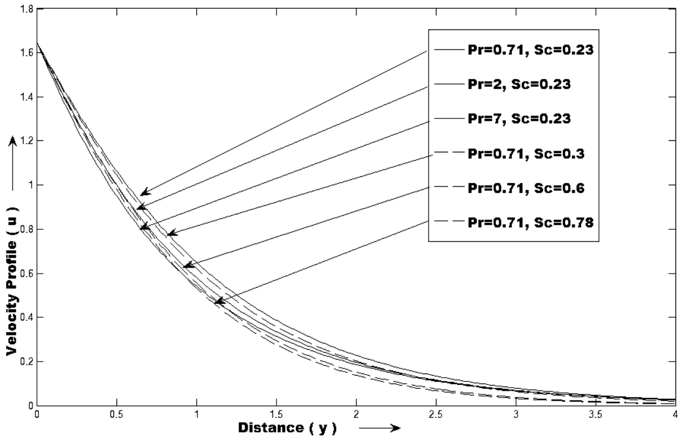


Fig. 2. Effect of Pr and Sc on velocity profile, when Gr = 2, Gm = 2, M = 0.5, K = 2, R = 2, S = 2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t = 0.5.

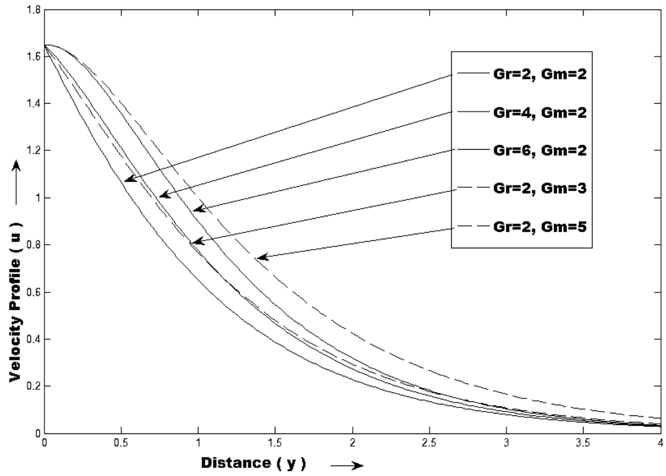


Fig. 3. Effect of Gr and Gm on velocity profile, when Sc = 0.23, Pr = 0.71, M = 0.5, K = 2, R = 2, S = 2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t = 0.5.

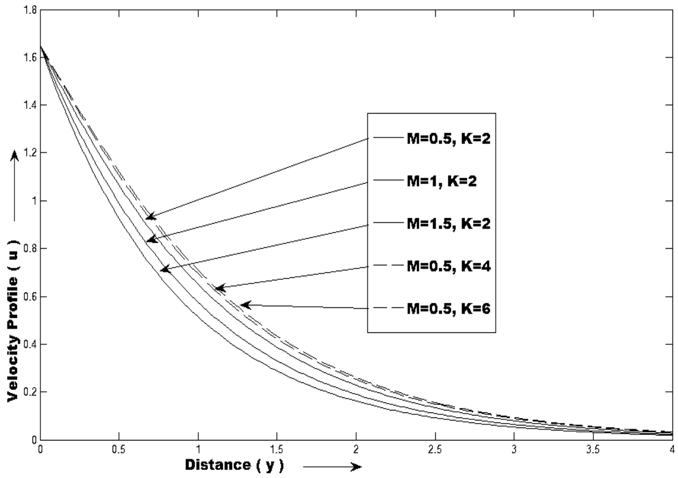


Fig. 4. Effect of M and K on velocity profile, when Sc = 0.23, Pr= 0.71, Gr = 2, Gm = 2, R = 2, S = 2, $\alpha = \frac{\pi}{6}$, $\omega = 1$ and t = 0.5.

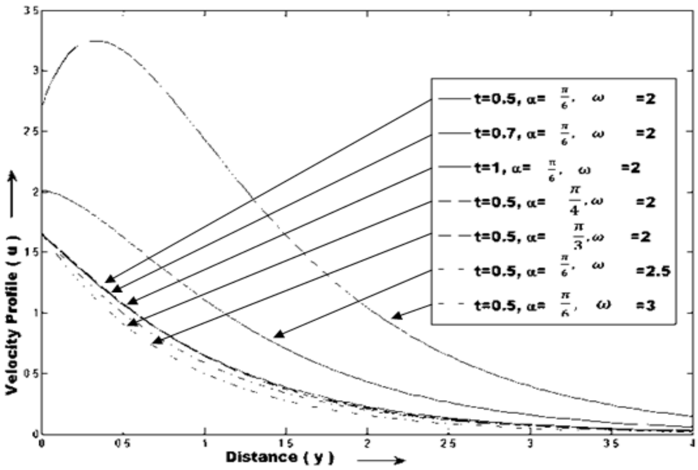


Fig. 5. Effect of α , t and ω on velocity profile, when Sc = 0.23, Pr = 0.71, M = 0.5, K = 2, R = 2, S = 2, $Gr=2$ and Gm = 2.

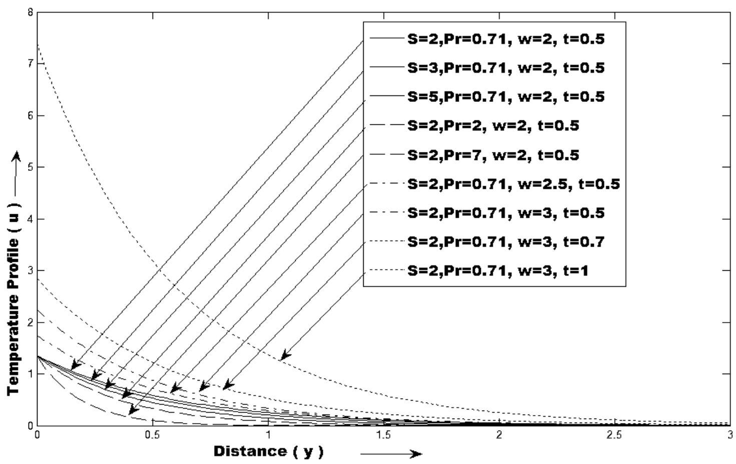


Fig. 6. Effect of S, $Pr$ , t and ω on Temperature profile, in the absence of other parameters.

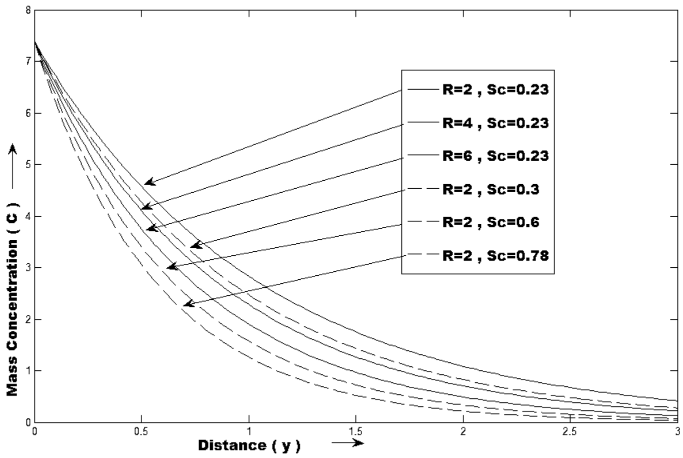


Fig. 7. Effect of R and $Sc$ on Mass Concentration, when t = 0.5, $\omega = 2$.

## V. CONCLUSIONS

In this study, the following conclusions are set out:

1. It is observed that the velocity increases with decreasing of all parameters except Grashoff number (Gr), modified Grashoff number (Gm) and permeability parameter of porous medium (K).
2. It is observed that the temperature rises with increase of time (t) and Oscillating frequency $(\omega)$, whereas falls with the increase of (S ) and prandtl number (Pr).
3. It is observed that the mass concentration decrease with increase of Schmidt number (Sc) and Chemical reaction parameter (R).
4. Skin friction at plate increases with increase of Chemical reaction parameter (R), Source parameter (S), Prandtl number (Pr), Oscillating frequency $(\omega)$ and Magnetic parameter (M) and decreases for other parameters. Also, Nusselt number increases with increase of Source parameter (S) and Prandtl number (Pr), where as Sherwood Number increases in the increase of Schmidt number (Sc), frequency of oscillation (ω) and Chemical reaction parameter (R).

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