

# Wavelet Bayes Adaptive Image Denoising

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**Abstract**— The class of natural images that we encounter in our daily life is only a small subset of the set of all possible images. This subset is called an image manifold. The Adaptive Digital Image Processing applications are becoming increasingly important and they all start with a mathematical representation of the image. In Bayesian restoration methods, the image manifold is encoded in the form of prior knowledge that express the probabilities that specified combinations of pixel intensities can be experiential in an image. Because image spaces are high-dimensional, one often isolates the manifolds by decomposing images into their components and by fitting probabilistic models on it. The construction of a Bayesian network involves prior knowledge of the probability relationships between the variables of interest. Learning approaches are widely used to construct Bayesian networks that best represent the joint probabilities of training data. In practice, an optimization process based on a heuristic search technique is used to find the best structure over the space of all possible networks. However, the approach is computationally intractable because it must explore several combinations of dependent variables to derive an optimal Bayesian network. The difficulty is resolved in this paper by representing the data in wavelet domains and restricting the space of possible networks by using certain techniques, such as the “maximal weighted spanning tree”. With the use of biorthogonal wavelets, the perceptual quality of the reconstructed image has been improved. Three wavelet properties - sparsity, cluster, and motion can be oppressed to reduce the computational complexity of learning a Bayesian network.

**Keywords**— Adaptive Bayesian network; biorthogonal wavelet transform; image denoising.

## I. INTRODUCTION

During the last decades, multi resolution image representations, like wavelets, have received much attention for the construction of Bayesian networks, due to their sparseness which manifests in highly non-Gaussian statistics for wavelet coefficients. Marginal histograms of wavelet coefficients are typically leptokurtic and have heavy tails [8], [9]. In literature, many wavelet-based image denoising methods have arisen exploiting this property, and are often based on simple and elegant shrinkage rules. In addition, joint histograms of wavelet coefficients have been studied in taking advantage of correlations between wavelet coefficients either across space, scale or orientation, additional improvement in denoising performance is obtained. The Gaussian Scale Mixture (GSM) model, in which clusters of coefficients are modelled as the artifact of a Gaussian random vector and a positive scaling variable, has been shown to produce outcome that are appreciably better than marginal models [10] sharing the significant features still present in the degraded image, but with the artifacts censored. Image denoising is an important image processing assignment, both as a process itself, and as a module in other processes. There exists several ways to

denoise an image or a set of records. The main properties of an excellent image denoising model are that it will eliminate noise while preserving edges. Generally linear models have been used. One common technique is to use a Gaussian filter, or homogeneously solving the heat-equation with the noisy image as input-data, i.e. a linear, 2nd order PDE-reproduction. For some purposes this kind of denoising is sufficient. One large advantage of linear noise removal models is the speed. But a drawback of the linear models is that they do not preserve edges in an excellent way. Edges, which are recognized as discontinuities in the image, are dirty out. Nonlinear models on the other hand can handle edges in a much better way than linear models. TV filter is very good at preserving edges, but smooth unstable regions in the input image are transformed into piecewise constant regions in the output image. Using the TV-filter as a denoiser leads to solve a 2nd order nonlinear PDE. Since smooth regions are transformed into piecewise constant regions when using the TV-filter, it is desirable to generate a model for which smoothly changeable regions are transformed into smoothly unreliable regions, and yet the edges are preserved. This can be done for example by solving a 4th order PDE instead of the 2nd order PDE from the TV-filter. Result show that the 4th order filter produces better results in smooth regions, and unmoving preserves edges in a very excellent way. Image denoising algorithms may be the oldest in image processing. Various methods, in spite of implementation, share the similar basic plan noise reduction through image blurring. Blurring can be done nearby, as in the Gaussian smoothing model or in anisotropic filtering; by calculus of variations; or in the frequency domain, such as Weiner filters. But a universal best approach has yet to be found.

Novel adaptive and patch-based approach [13] is proposed for image denoising and representation. The method is based on a point wise selection of small image patches of fixed size in the variable neighborhood of each pixel. My involvement is to associate with each pixel the weighted sum of data points within an adaptive neighborhood, in a manner that it balances the exactness of approximation and the stochastic error, at each spatial location. This method is general and can be applied under the assumption that there exist repetitive patterns in a local neighborhood of a point. By introducing spatial adaptively, I expand the work earlier described by Bauds et al. which can be measured as an addition of bilateral filtering to image patches. Finally, a nearly parameter-free algorithm for image denoising is recommended. The scheme is applied to both artificially despoiled (white Gaussian noise) and real images and the performance is extremely close to, and in some cases yet surpasses, that of the already published denoising schemes. A novel adaptive and exemplar-based

approach is proposed for image restoration and representation. The method is based on a point wise selection of small image patches of fixed size in the variable neighbourhood of each pixel. The core idea is to associate with each pixel the weighted sum of data points within an adaptive neighbourhood. This method is general and can be applied under the assumption that the image is a locally and fairly stationary process. This paper is a spotlight on the problem of the adaptive neighbourhood selection in a manner that it balances the accuracy of approximation and the stochastic error, at each spatial location. Thus, the new proposed point wise estimator mechanically adapts to the degree of underlying smoothness which is unidentified with minimal a priori assumptions on the function to be recovered [14].

Consider the image denoising difficulty, where zero-mean white and homogeneous Gaussian additive noise is to be detached from a given image. The steps taken is based on sparse and redundant representations over trained dictionaries. Using the K-SVD algorithm, a dictionary that describes the image content effectively is achieved. Two training options are measured: using the corrupted image itself, or training on an amount of high-quality image database. Since the K-SVD is limited in management small image patches, I expand its deployment to arbitrary image sizes by defining a global image prior that forces sparsity over patches in every location in the image. I here illustrate how such Bayesian treatment leads to a simple and effective denoising algorithm. This lead to a state-of-the-art denoising presentation, equivalent and sometimes surpassing recently published leading alternative denoising methods. The planned method is based on local operations and involves sparse decompositions of each image block under one fixed over-complete dictionary, and a simple average calculation. The content of the dictionary is of main importance for the denoising method. I have shown that a dictionary trained for natural real images, as well as an adaptive glossary trained on patches of the noisy image itself, both present very well [16].

## II. ADAPTIVE WAVELET BAYESIAN NETWORK

In this approach, the denoising problem is basically a prior probability modeling and estimation that exploits a hidden Bayesian system [1], constructed from wavelet coefficients, to model the previous probability of the original image. Then, we use the belief propagation (BP) method [1], which estimates a coefficient based on all the coefficients of an image, as the maximum-a-posterior (MAP) estimator to develop the denoised wavelet coefficients. It is also explained that if the network is a spanning tree, the standard BP algorithm can execute MAP estimation competently. The experiment results demonstrate that, in conditions of the peak-signal-to-noise-ratio and perceptual quality, the projected approach out performs state-of-the-art algorithms on various images, particularly in the textured regions, with various amounts of White Gaussian noise [20].

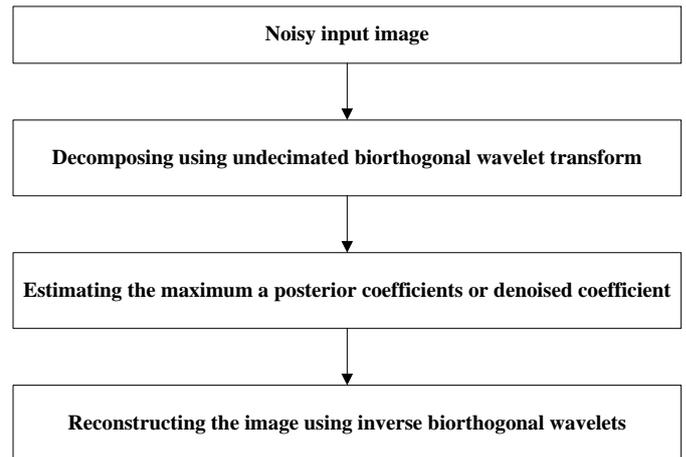


Fig. 1. Basic flow chart.

A Bayesian network, denoted as  $B = (V, E, P)$ , comprises a set of random variables and their conditional dependencies represented by a directed acyclic graph in which the nodes represent the elements in  $V$ [1]. Each edge element in  $E$  takes the form of a directed arc  $x \rightarrow y$ , where  $x$  and  $y \in V$ . The likelihood  $p(y | x) \in P$  of an edge  $x \rightarrow y \in E$  is the conditional probability of observing  $y$  given that  $x$  exists. The Bayesian networks constructed in wavelet domains is called Wavelet Bayesian Networks (WBNs). The primary objective of this paper is to construct a WBN from the undecimated Discrete Wavelet Transform (DWT) of a single image.

Initially, wavelet decomposition of an image  $F$  yields three images of wavelet coefficients with horizontal, vertical, and diagonal orientations respectively, and one approximate image of  $F$ . Then, at the next scale, the approximated image is further decomposed to obtain three images of the wavelet coefficients and one coarser approximate image of  $F$ . Let  $w_{h,j} F(u,v)$ ,  $w_{v,j} F(u,v)$ , and  $w_{d,j} F(u,v)$  denote, respectively, the horizontal, vertical, and diagonal images of the wavelet coefficients at scale  $2^j$ ; and let  $A_j F(u, v)$  represent the approximated image at the same scale. If the undecimated DWT is decomposed  $J$  times, we will have wavelet coefficients  $w_{h,j} F$ ,  $w_{v,j} F$ , and  $w_{d,j} F$  with  $j = 1 \dots J$  and the coarsest approximate image. To construct a WBN, first group sub-bands with the same orientation together to obtain a horizontal-group, a vertical-group, and a diagonal-group of wavelet coefficients. Then, construct a Bayesian network  $B$  for each group. Let  $B_h = (V_h, E_h, P_h)$ ,  $B_v = (V_v, E_v, P_v)$ , and  $B_d = (V_d, E_d, P_d)$  denote the Bayesian networks constructed from the horizontal-group, vertical-group, and diagonal-group of wavelet coefficients respectively. The WBN,  $B = (V, E, P)$  is derived from  $B_h, B_v$ , and  $B_d$  by  $V = V_h \cup V_v \cup V_d$  (4)  $E = E_h \cup E_v \cup E_d$  and (5)  $P = P_h \cup P_v \cup P_d$ . (6) Next, explain how to construct the Bayesian network  $B_u(V_u, E_u, P_u)$  that corresponds to the  $u$ -orientation, where  $u \in \{h, v, d\}$ [2].

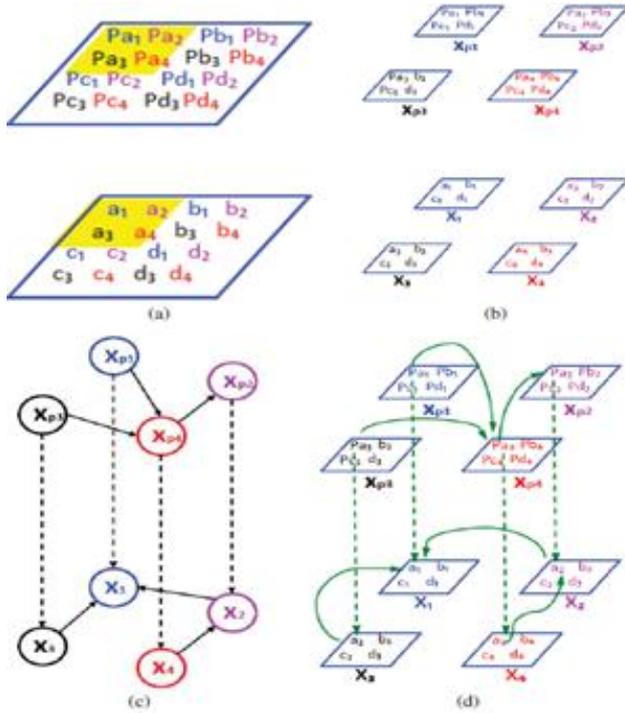


Fig. 2. Constructing a multilayer hidden network. (a) Two subbands, with the coarser subband on top. (b) Procedure creates two wavelet patches, each of which is associated with a subgraph. Subband coefficients are assigned to the nodes, as specified in Fig. 1(c). (c) Nodes in the two-layer network are linked by intra-scale (solid) arcs and inter-scale (dashed) arcs. (d) To derive the prior probability.

The normal distribution is immensely useful because of the central limit theorem, which states that, under mild conditions, the mean of many random variables independently drawn from the same distribution is distributed approximately normally, irrespective of the form of the original distribution. Physical quantities that are expected to be the sum of many independent processes (such as measurement errors) often have a distribution very close to the normal. Moreover, many results and methods (such as propagation of uncertainty and least squares parameter fitting) can be derived analytically in explicit form when the relevant variables are normally distributed.

### III. ADAPTIVE DATA FITTING WITH NORMAL PROBABILITY DISTRIBUTION

Some special properties define why the normal distribution (or Gaussian) is used for fitting the random data input.

- The value of the probability density function approaches zero as input random variable approaches positive and negative infinity.
- The probability density function is centered at the mean, and the maximum value of the function occurs at when random variable become equal to mean
- The probability density function for the normal distribution is symmetric about the mean.

The posterior probabilities of each of the different components in the Gaussian mixture distribution defined by object for each observation in the data. Figure 3 shows the posterior map with respect to the data network.

TABLE I. Data fitting with normal probability distribution.

Input	Mean & variance	Approximation	Horizontal	Vertical	Diagonal
 Lena	Mean	259.8	-1.46	0.37	0.60
	Variance	25.21	26.4	22.1	17.9

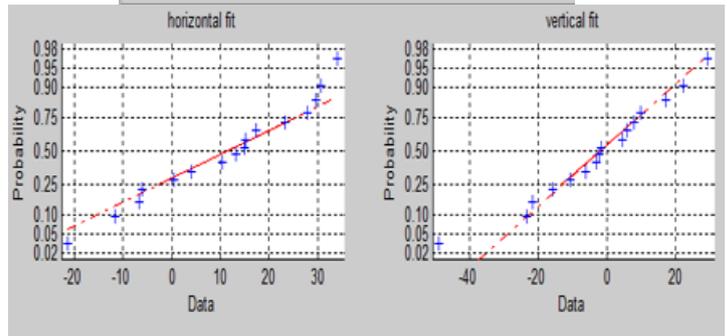
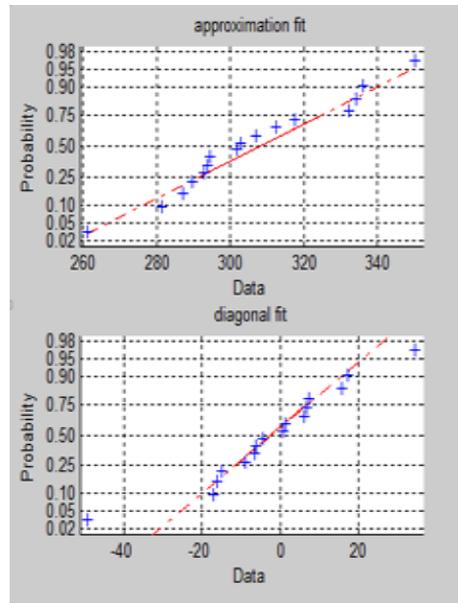


Fig. 3. Approximation, diagonal, horizontal and vertical data fitting.

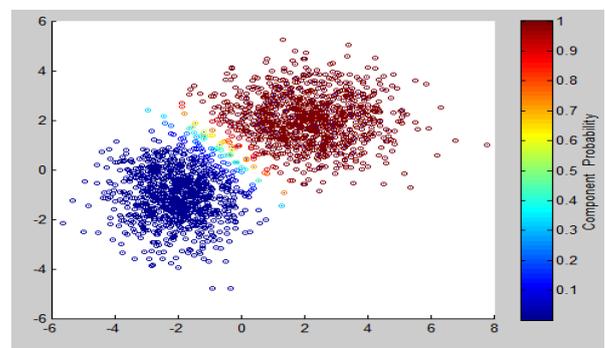


Fig. 4. Posterior probability map.

### IV. DENOISING ALGORITHM AND EXPERIMENTAL RESULT

This section present the proposed denoising algorithm, its implementation, and its performance comparison with that of other methods.

The proposed algorithm is summarized. The steps are as follows: Step (1) calculates the undecimated DWT of the input image; Steps (2) to (5) construct the WBN  $B[1]$ ; and Steps (6) to (8) create the WBN  $Bn$  [1] used for denoising purposes; In Step (9), the wavelet coefficient are estimated from  $Bn$  by applying the adaptive max-product algorithm to the factor graph  $Fn$  for each realization of  $Bn$ . Here I use Bior4.4 filters to process the undecimated DWT. Because Bior4.4 filters are close to orthogonal wavelet filters, the noise variance of subbands at all scales can be set at  $\sigma^2n$ , which is the image noise variance. The variance  $\sigma^2n$  is used in the Wiener filtering in Step (2) as well as in deriving the MAP estimation of the wavelet coefficients in Step (9). The frequency count in Step (4) indicates the number of wavelet coefficients in a quantization bin. The size of a subband's quantization bin is set at 14 of the standard deviation, measured from the wavelet coefficients in the subband.



Fig. 6. Denoised output.

TABLE II. Comparison of low and high edge images with different variance.

Image	Variance	PSNR(dB)		SSIM	
		MRF	OUR	MRF	OUR
Lenna 512x512	0.01	25.555	32.275	0.695	0.903
	0.02	25.007	29.803	0.695	0.868
	0.05	22.517	26.799	0.704	0.779
	0.09	19.234	24.300	0.681	0.725
Baboon 256x256	0.01	19.156	21.550	0.579	0.613
	0.02	16.029	17.858	0.369	0.433
	0.05	11.849	13.716	0.255	0.266
	0.09	10.803	11.718	0.179	0.189

From table II it is understood that the noise variance change is gradually affect the proposed method i.e. the proposed method preserve edges more compared with the existing method [1].

### V. IMPLEMENTATION OF CURRENT ALGORITHM FOR RGB IMAGE

The current algorithm can be extend for denoising of color images. Figure 5 and figure 6 shows the execution result of the current algorithm in color images, Table III shows the different color images with PSNR and SSIM evaluation results.

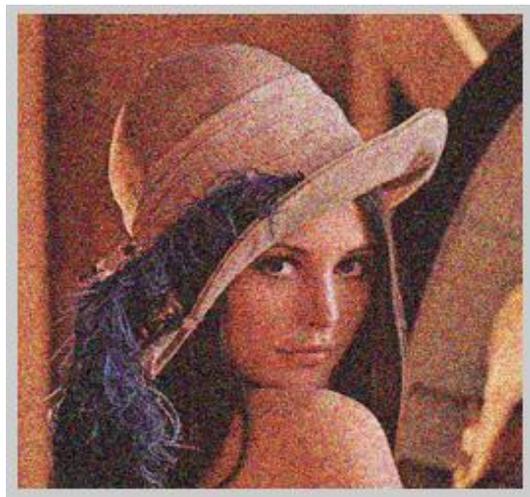


Fig. 5. Noisy input.

TABLE III. Evaluation of PSNR and SSIM for color images.

Image	PSNR(dB)	SSIM
Lenna	31.5875	0.896
Brick house	27.3958	0.7334
Leaf	26.7192	0.7024

### VI. CONCLUSION

The Bayesian formula indicates that the denoising problem is essentially a prior probability modeling and estimation task. The constructive data-adaptive procedure that derives a hidden graph structure from the wavelet coefficients. The graph is then used to model the prior probability of the original image for denoising purposes.

The existing Wavelet Bayesian Network Image Denoising [1] performs the denoising algorithm effectively. With the use of spanning tree approach on wavelet domain make the MAP estimation easily. The posterior probability estimation preserve the edges significantly compared with the MRF [2] approach.

The modified Wavelet Bayes Adaptive Image Denoising algorithm use the Gaussian spanning tree approach, it preserve the edges effectively and also can use the current algorithm for color image denoising.

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