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$P2-$Like and P^*-Generalized $\mathcal{B}R-$ Recurrent Space

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Abstract— *In this paper, we introduced the generalized $\mathcal{B}R-$ recurrent Finsler space, i.e. characterized by the following condition*

$$\mathcal{B}_m R^i_{jkh} = \lambda_m R^i_{jkh} + \mu_m(\delta^i_j g_{kh} - \delta^i_k g_{jh}), \quad R^i_{jkh} \neq 0,$$

where $\mathcal{B}_m$ is Berwald's covariant differential operator with respect to x^m, λ_m and μ_m are known as recurrence vectors, which is a $P2-$like space and P^*- space [satisfies their conditions], we called them a $P2-$like generalized $\mathcal{B}R-$recurrent space and a P^*- generalized $\mathcal{B}R-$ recurrent space, respectively. The purpose of the present paper to develop the above spaces by using the properties of a $P2-$like space and a P^*- space. Also to obtain different theorems for some tensors satisfy in above spaces. Various identities are established in our spaces.

Keywords: *a $P2-$like generalized $\mathcal{B}R-$recurrent space and a P^*- generalized $\mathcal{B}R-$ recurrent space*

I. INTRODUCTION

R. Verma [13] obtained some results when R^h- recurrent and $C-$ concircularly spaces are $P2-$like spaces. S.Dikshit [14] obtained certain identities in a $P2-$like R^h-birecurrent space. F.Y.A. Qasem [6] obtained certain identities in a $P2-$like R^h- generalized and $P2-$like R^h- special generalized birecurrent spaces of the first and the second kind. F.Y.A. Qasem and A.A.A. Muhib [7] obtained certain identities in a $P2-$like R^h-trirecurrent space. A.A.A. Muhib [1] established different identities concerning $P2-$like R^h- generalized and $P2-$like R^h- special generalized trirecurrent spaces. A.A.M. Saleem [2] discussed $P2-$like C^h- generalized and $P2-$like C^h- special generalized birecurrent spaces and obtained the necessary and sufficient condition of some tensors to be generalized birecurrent and special generalized birecurrent, also obtained some identities in such spaces. A.M.A. Al – qashbari [3] introduced and discussed $P2-$like generalized H^h, R^h and K^h-recurrent spaces. A.M.A. Hanballah [4] introduced and studied $P2-$like K^h- generalized and special generalized birecurrent spaces.

R. Verma [13] obtained some results when R^h- recurrent and $C-$ concircularly spaces are P^*-Finsler spaces. C.K. Mishra and G. Lodhi [5] discussed C^h- recurrent Finsler space of second order and obtained different theorems regarding this space when it is P^*-Finsler space. A.A.M. Saleem [2] obtained different theorems in C^h- generalized birecurrent and C^h-special generalized birecurrent spaces when they are P^*-Finsler space. A.M.A. Al – qashbari [3] introduced and discussed P^*-generalized H^h, R^h and K^h- recurrent spaces. A.M.A. Hanballah [4] introducted and studied P^*-K^h- generalized and special generalized birecurrent spaces.

Let F_n be an n – dimensional Finsler space equipped with the metric function F(x,y) satisfying the request conditions [10] .

The vector y_i is defined by

(1.1) $\quad y_i = g_{ij}(x,y)y^j.$

The two sets of quantities g_{ij} and its associative g^{ij}, which are components of a metric tensor connected by

(1.2) $$g_{ij}g^{ik} = \delta^k_j = \begin{cases} 1 & if \;\; j=k, \\ 0 & if \;\; j\neq k. \end{cases}$$

In view of (1.1) and (1.2), we have

(1.3) a) $\delta^i_k y_i = y_k$, b) $\delta^i_i = n$ and c) $\delta^i_j g_{ir} = g_{jr}$.

The tensor C_{ijk} is defined by

$$C_{ijk} = \tfrac{1}{2}\dot\partial_k g_{ij}$$

which is positively homogeneous of degree -1 in y^i and symmetric in all its indices and called *(h)hv-torsion tensor* [12] and its associative C^i_{jk} is positively homogeneous of degree -1 in y^i and symmetric in its lower indices and called *(v)hv- torsion tensor*. According to Euler's theorem on homogeneous functions, these tensors satisfy the following:

(1.4) a) $C_{ijk}y^i = C_{kij}y^i = C_{jki}y^i = 0$, b) $C^i_{jk}y^k = 0 = C^i_{kj}y^k$,
c) $C^i_{ir} = C_r = C^i_{ri}$, d) $y_i C^i_{kh} = 0$
e) $g_{ij}C^i_{kh} = C_{jkh}$ and f) $C^i_{jr}\delta^r_k = C^i_{jk}$

Berwald covariant derivative $\mathcal{B}_k T^i_j$ of an arbitrary tensor field T^i_j with respect to x^k is given by

$$\mathcal{B}_k T^i_j := \partial_k T^i_j - (\dot\partial_r T^i_j)G^r_k + T^r_j G^i_{rk} - T^i_r G^r_{jk}$$

The processes of Berwald's covariant differentiation and the partial differentiation, for an arbitrary tensor field T^i_j , commute according to

$$(\dot\partial_k \mathcal{B}_h - \mathcal{B}_k\dot\partial_h)T^i_j = T^r_j G^i_{khr} - T^i_r G^r_{khj}$$

Berwald's covariant derivative of the vectors y^i and y_i vanish identically, i.e.

(1.5) a) $\mathcal{B}_k y^i = 0$ and b) $\mathcal{B}_k y_i = 0$.

Berwald's covariant derivative of the metric tensor g_{ij} doesn't vanish and given by

(1.6) $\quad \mathcal{B}_k g_{ij} = -2C_{ijk|h}y^h = -2y^h\mathcal{B}_h C_{ijk}$.

The h - curvature tensor (*Cartan's third curvature tensor*) is defined by

$$R^i_{jkh} = \partial_h\Gamma^{*i}_{jk} + (\partial_l\Gamma^{*i}_{jk})G^l_h + C^i_{jm}(\partial_k G^m_h - G^m_{kl}G^l_h) + \Gamma^{*i}_{mk}\Gamma^{*m}_{jh} - k/h^* .$$

The curvature tensor R^i_{jkh} and the h(v)- torsion tensor H^i_{kh} are connected by

(1.7) $R^i_{jkh}y^j = H^i_{kh} = K^i_{jkh}y^j$.

The tensor $P^{\,i}_{jkh}$ is positively homogeneous of degree zero in $y^{\,i}$ and satisfies

(1.8) $P^i_{jkh}y^j = (\partial_j \Gamma^{*i}_{hk})y^j = \Gamma^{*i}_{hjk}y^j = P^i_{kh} = C^i_{kh|r}y^r$

and

(1.9) $P^i_{jkh}y^k = 0 = P^i_{jkh}y^h$,

where

(1.10) $\dot{\partial}_h \Gamma^{*i}_{jk}y^j = 0.$

This curvature tensor is positively homogeneous of degree zero in $y^{\,i}$ and skew-symmetric in its last two lower indices .

The torsion tensor P^i_{kh} satisfies the following relations

(1.11) a) $P^i_{kh}y^k = 0$, b) $P^i_{kh}y_i = P_{kh}$ and c) $g_{ir}P^i_{kh} = P_{rkh}$

Ricci tensor P_{jk} and the curvature vector P_k of the curvature tensor P^i_{jkh} are given by

(1.12) a) $P^i_{jki} = P_{jk}$ and b) $P^i_{ki} = P_k$.

In view of (1.4b) and since the vector y^j is h – covariant constant , i.e. $y^j{}_{|k} = 0$, we have

(1.13) $C^i_{jh|k}y^j = 0$

* k/h means the subtraction from the former term by interchanging the indices k and h.

2. $P2$ –Like Generalized $\mathcal{BR}$ –Recurrent Space

Let us consider a $P2$ –Like space which is characterized by the condition ([11], [12]

(2.1) $P^i_{jkh} = \phi_j C^i_{kh} - \phi^i C_{jkh}$,

where ϕ_j is a non- zero covariant vector field and ϕ^i is a non-zero contravariant vector field .

Definition 2.1. The generalized $\mathcal{B}R$ –recurrent space which is $P2$ –Like space [satisfies the condition (2.1)], will be called a *$P2$ –Like generalized $\mathcal{B}R$ –recurrent space* and will denote it briefly by a *$P2$ –Like $G(\mathcal{B}R) - RF_n$*.

Remark 2.1. It will be sufficient to call the tensor which satisfies the condition of $P2$ –Like $G(\mathcal{B}R) - RF_n$ as *generalized $\mathcal{B}$ –recurrent tensor* (briefly $G\mathcal{B} - R$) .

Let us consider a $P2$ –Like $G(\mathcal{B}R) - RF_n$.

Taking the covariant derivative for the condition (2.1) with respect to x^m in the sense of Berwald, we get

(2.2) $\mathcal{B}_m P^i_{jkh} = (\mathcal{B}_m\phi_j)C^i_{kh} + \phi_j(\mathcal{B}_m C^i_{kh}) - (\mathcal{B}_m\phi^i)\,C_{jkh} - \phi^i(\mathcal{B}_m C_{jkh})$.

Suppose that the $v(hv)$ – torsion tensor C^i_{kh} and the $h(hv)$ – torsion tensor C_{jkh} satisfy the conditions

(2.3) a) $\mathcal{B}_m C^i_{kh} = \lambda_m C^i_{kh} + \mu_m(\delta^i_k y_h - \delta^i_h y_k)$

and

b) $\mathcal{B}_m C_{jkh} = \lambda_m C_{jkh} + \mu_m(g_{kj}y_h - g_{hj}y_k)$,

respectively .

Substituting the conditions (2.3a), (2.3b) and (2.1) in (2.2), we get

(2.4) $\mathcal{B}_m P^i_{jkh} = \lambda_m P^i_{jkh} + \mu_m\phi_j(\delta^i_k y_h - \delta^i_h y_k) + \mu_m\phi^i(g_{kj}y_h - g_{hj}y_k) + (\mathcal{B}_m\phi_j)C^i_{kh} - (\mathcal{B}_m\phi^i)C_{jkh}$.

This shows that

(2.5) $\mathcal{B}_m P^i_{jkh} = \lambda_m P^i_{jkh} + \mu_m\phi_j(\delta^i_k y_h - \delta^i_h y_k)$

if and only if

(2.6) $(\mathcal{B}_m\phi_j)C^i_{kh} - (\mathcal{B}_m\phi^i)C_{jkh} + \mu_m\phi^i(g_{kj}y_h - g_{hj}y_k) = 0$.

Thus, we conclude

Theorem 2.1. *In $P2$ –Like $G(\mathcal{B}R) - RF_n$, the covariant derivative of first order for Cartan's second curvature tensor P^i_{jkh} in the sense of Berwald is given by the condition (2.5), if and only if (2.6) holds good [provided the conditions (2.3a) and (2.3b) hold].*

Transvecting (2.4) by y^j , using (1.8), (1.5a), (1.1) and (1.4a), we get

(2.7) $\mathcal{B}_m P^i_{kh} = \lambda_m P^i_{kh} + \gamma_m(\delta^i_k y_h - \delta^i_h y_k) + (\mathcal{B}_m\phi_j)C^i_{kh}y^j$,

where $y^j\phi_j = \phi$ and $\gamma_m = \phi\mu_m$.

This shows that

(2.8) $\mathcal{B}_m P^i_{kh} = \lambda_m P^i_{kh} + \gamma_m(\delta^i_k y_h - \delta^i_h y_k)$

if and only if

(2.9) $(\mathcal{B}_m\phi_j)C^i_{kh}y^j = 0$.

Thus, we conclude

Theorem 2.2. *In $P2$ –Like $G(\beta R) - RF_n$, the covariant derivative of first order for the torsion tensor P^i_{kh} in the sense of Berwald is given by the condition (2.8), if and only if (2.9) holds . [provided the conditions (2.3a) and (2.3b) hold].*

Contracting the indices i and h in (2.4), using (1.12a), (1.3a), (1.3b) and (1.4c), we get

(2.10) $\mathcal{B}_m P_{jk} = \lambda_m P_{jk} + (1-n)\mu_m\phi_j y_k + \mu_m\phi^i(g_{kj}y_i - g_{ij}y_k) + (\mathcal{B}_m\phi_j)C_k - (\mathcal{B}_m\phi^i)C_{jki}$.

This shows that

(2.11) $\mathcal{B}_m P_{jk} = \lambda_m P_{jk} + (1-n)\mu_m\phi_j y_k$

if and only if

(2.12) $\mu_m\phi^i(g_{kj}y_i - g_{ij}y_k) + (\mathcal{B}_m\phi_j)C_k - (\mathcal{B}_m\phi^i)C_{jki} = 0$.

Thus, we conclude

Theorem 2.3. *In $P2$ –Like $G(\mathcal{B}R) - RF_n$, the P – Ricci tensor P_{jk} is non vanishing if and only if (2.12) holds good [provided the conditions (2.3a) and (2.3b) hold].*

Contracting the indices i and h in (2.7), using (1.12b), (1.3a), (1.3b) and (1.4c), we get

(2.13) $\mathcal{B}_m P_k = \lambda_m P_k + (1-n)\gamma_m y_k + (\mathcal{B}_m\phi_j)C_k y^j$.

where $y^j\phi_j = \phi$ and $\gamma_m = \phi\mu_m$.

This shows that

(2.14) $\mathcal{B}_m P_k = \lambda_m P_k + (1-n)\gamma_m y_k$

if and only if

(2.15) $(\mathcal{B}_m\phi_j)\,C_k y^j = 0.$

Thus, we conclude

Theorem 2.4. *In $P2$ –Like $G(\mathcal{B}R) - RF_n$, the curvature vector P_k is non-vanishing if and only if (2.15) holds .[provided the conditions (2.3a) and (2.3b) hold].*

we know that [10]

(2.16) $R^{\,i}_{\,jkh|s} + R^{\,i}_{\,jsk|h} + R^{\,i}_{\,jhs|k} + y^{\,m}(R^{\,r}_{\,mhs}P^{\,i}_{\,jkr} + R^{\,r}_{\,mkh}P^{\,i}_{\,jsr} + R^{\,r}_{\,msk}P^{\,i}_{\,jhr}) = 0$.

Taking the covariant derivative for (2.16) with respect to x^m in the sense of Berwald, we get

(2.17) $\mathcal{B}_m(R^{\,i}_{\,jkh|s} + R^{\,i}_{\,jsk|h} + R^{\,i}_{\,jhs|k}) + \mathcal{B}_m[\,y^{\,m}(R^{\,r}_{\,mhs}P^{\,i}_{\,jkr} + R^{\,r}_{\,mkh}P^{\,i}_{\,jsr} + R^{\,r}_{\,msk}P^{\,i}_{\,jhr})] = 0$.

Using (1.7) in (2.17), we get

(2.18) $\mathcal{B}_m(R^{\ i}_{jkh|s} + R^{\ i}_{jsk|h} + R^{\ i}_{jhs|k}) + \mathcal{B}_m(H^{\ r}_{hs}P^{\ i}_{jkr} + H^{\ r}_{kh}P^{\ i}_{jsr} + H^{\ r}_{sk}P^{\ i}_{jhr}) = 0$

or

(2.19) $\mathcal{B}_m\big(R^{\ i}_{jkh|s} + R^{\ i}_{jsk|h} + R^{\ i}_{jhs|k}\big) + P^{\ i}_{jkr}(\mathcal{B}_m H^{\ r}_{hs}) + H^{\ r}_{hs}\big(\mathcal{B}_m P^{\ i}_{jkr}\big) + P^{\ i}_{jsr}(\mathcal{B}_m H^{\ r}_{kh}) + H^{\ r}_{kh}\big(\mathcal{B}_m P^{\ i}_{jsr}\big) + H^{\ r}_{sk}\big(\mathcal{B}_m P^{\ i}_{jhr}\big) + P^{\ i}_{jhr}(\mathcal{B}_m H^{\ r}_{sk}) = 0$.

Using (2.1) in (2.19), we get

(2.20) $\mathcal{B}_m(R^{\ i}_{jkh|s} + R^{\ i}_{jsk|h} + R^{\ i}_{jhs|k}) + (\phi_j C^i_{kr} - \phi^i C_{jkr})(\mathcal{B}_m H^{\ r}_{hs}) + H^{\ r}_{hs}\mathcal{B}_m(\phi_j C^i_{kr} - \phi^i C_{jkr}) + (\phi_j C^i_{sr} - \phi^i C_{jsr})(\mathcal{B}_m H^{\ r}_{kh}) + H^{\ r}_{kh}\mathcal{B}_m(\phi_j C^i_{sr} - \phi^i C_{jsr}) + (\phi_j C^i_{hr} - \phi^i C_{jhr})(\mathcal{B}_m H^{\ r}_{sk}) + H^{\ r}_{sk}\mathcal{B}_m(\phi_j C^i_{hr} - \phi^i C_{jhr}) = 0$

or

(2.21) $\mathcal{B}_m(R^{\ i}_{jkh|s} + R^{\ i}_{jsk|h} + R^{\ i}_{jhs|k}) + \phi_j\big[C^i_{kr}(\mathcal{B}_m H^{\ r}_{hs}) + C^i_{sr}(\mathcal{B}_m H^{\ r}_{kh}) + C^i_{hr}(\mathcal{B}_m H^{\ r}_{sk})\big] - \phi^i\big[C_{jkr}(\mathcal{B}_m H^{\ r}_{hs}) + C_{jsr}(\mathcal{B}_m H^{\ r}_{kh}) + C_{jhr}(\mathcal{B}_m H^{\ r}_{sk})\big] + H^{\ r}_{hs}\mathcal{B}_m\big(\phi_j C^i_{kr}\big) + H^{\ r}_{kh}\mathcal{B}_m\big(\phi_j C^i_{sr}\big) + H^{\ r}_{sk}\mathcal{B}_m\big(\phi_j C^i_{hr}\big) - \big[H^{\ r}_{hs}\mathcal{B}_m(\phi^i C_{jkr}) + H^{\ r}_{kh}\mathcal{B}_m(\phi^i C_{jsr}) + H^{\ r}_{sk}\mathcal{B}_m(\phi^i C_{jhr})\big] = 0.$

Transvecting (2.21) by y^j, using (1.7), (1.5a) and (1.4a), we get

(2.22) $\mathcal{B}_m(H^{\ i}_{kh|s} + H^{\ i}_{sk|h} + H^{\ i}_{hs|k}) + \phi\big[C^i_{kr}(\mathcal{B}_m H^{\ r}_{hs}) + C^i_{sr}(\mathcal{B}_m H^{\ r}_{kh}) + C^i_{hr}(\mathcal{B}_m H^{\ r}_{sk})\big] + \phi(H^{\ r}_{hs}\mathcal{B}_m C^i_{kr} + H^{\ r}_{kh}\mathcal{B}_m C^i_{sr} + H^{\ r}_{sk}\mathcal{B}_m C^i_{hr}) = 0$,

where $\phi = \phi_j y^j$.

Thus, we conclude

Theorem 2.5. *In $P2$ –Like $G(\mathcal{B}R) - RF_n$, we have the identity (2.22) [provided the conditions (2.3a) and (2.3b) hold].*

3. P^* – Generalized $\mathcal{B}R$ – Recurrent Space

A P^* – Finsler space is characterized by the condition ([8], [9])

(3.1) $P^i_{kh} = \phi\, C^i_{kh}$,

where

(3.2) $P^i_{jkh} y^j = P^i_{kh} = C^i_{kh|s} y^s$.

Definition 3.1. The generalized $\mathcal{B}R$ – recurrent space which is a P^* – space [satisfies the condition (3.1)], will be called a P^* – *generalized $\mathcal{B}R$ – recurrent space* and will denote it briefly by $P^* - G(\mathcal{B}R) - RF_n$.

Remark 3.1. It will be sufficient to call the tensor which satisfies the condition of $P^* - G(\mathcal{B}R) - RF_n$ as *generalized $\mathcal{B}$ –recurrent tensor* (briefly $G\mathcal{B} - R$).

Remark 3.2. All results in a $P2$ –Like $G(\mathcal{B}R) - RF_n$ which obtained in the previous section satisfy in an $P^* - G(\mathcal{B}R) - RF_n$.

Let us consider an $P^* - G(\mathcal{B}R) - RF_n$.

Taking the covariant derivative for the condition (3.1) with respect to x^m in the sense of Berwald, we get

(3.3) $\mathcal{B}_m P^i_{kh} = (\mathcal{B}_m\phi)C^i_{kh} + \phi(\mathcal{B}_m C^i_{kh})$.

Using the conditions (2.3a) and (3.1) in (3.3), we get

(3.4) $\mathcal{B}_m P^i_{kh} = \lambda_m P^i_{kh} + \gamma_m(\delta^i_k y_h - \delta^i_h y_k) + (\mathcal{B}_m\phi)C^i_{kh}$,

where $\gamma_m = \phi\mu_m$.

This shows that

(3.5) $\mathcal{B}_m P^i_{kh} = \lambda_m P^i_{kh} + \gamma_m(\delta^i_k y_h - \delta^i_h y_k)$

if and only if

(3.6) $(\mathcal{B}_m\phi)C^i_{kh} = 0$.

Thus, we conclude

Theorem 3.1. *In $P^* - G(\mathcal{B}R) - RF_n$, the covariant derivative of first order for the $v(hv)$ – torsion tensor P^i_{kh} in the sense of Berwald is given by the condition (3.5) if and only if (3.6) holds [provided the conditions (2.3a) holds].*

Contracting the indices i and h in (3.4), using (1.12b), (1.3a), (1.3b) and (1.4c), we get

(3.7) $\mathcal{B}_m P_k = \lambda_m P_k + \gamma_m(1-n)y_k + (\mathcal{B}_m\phi)C_k$.

This shows that

(3.8) $\mathcal{B}_m P_k = \lambda_m P_k + \gamma_m(1-n)y_k$

if and only if

(3.9) $(\mathcal{B}_m\phi)C_k = 0$.

Thus, we conclude

Theorem 3.2. *In $P^* - G(\mathcal{B}R) - RF_n$, the curvature vector P_k is non-vanishing if and only if (3.9) holds [provided the conditions (2.3a) holds].*

Transvecting (3.4) by y_i , using (1.11b), (1.5b), (1.3a) and (1.4d), we get

(3.10) $\mathcal{B}_m P_{kh} = \lambda_m P_{kh}$.

Thus, we conclude

Theorem 3.3. *In $P^* - G(\mathcal{B}R) - RF_n$, the P – Ricci tensor P_{kh} behaves as recurrent [provided the condition (2.3a) holds].*

Transvecting (3.4) by g_{ir} , using (1.11c), (1.6), (1.3c) and (1.4e), we get

(3.11) $\mathcal{B}_m P_{rkh} = \lambda_m P_{rkh} + \gamma_m(g_{kr}y_h - g_{hr}y_k) + (\mathcal{B}_m\phi)C_{rkh} - 2y^h P^i_{kh}\mathcal{B}_h C_{irm}$.

This shows that

(3.12) $\mathcal{B}_m P_{rkh} = \lambda_m P_{rkh} + \gamma_m(g_{kr}y_h - g_{hr}y_k)$

if and only if

(3.13) $(\mathcal{B}_m\phi)\, C_{rkh} = 2y^h P^i_{kh}\mathcal{B}_h C_{irm}$.

Thus, we conclude

Theorem 3.4. *In $P^* - G(\mathcal{B}R) - RF_n$,the covariant derivative of first order for the associative tensor P_{rkh} in the sense of Berwald is Given by the condition (3.12), if and only if (3.13) holds [provided the conditions (2.3a) holds].*

We know that [10]

(3.14) $P^{\ i}_{jkh} = \dot\partial_h \Gamma^{*i}_{jk} + C^{\ i}_{jr}P^r_{kh} - C^{\ i}_{jh|k}$.

Taking the covariant derivative for (3.14) with respect to x^min the sense of Berwald, we get

$$\mathcal{B}_m P^{\ i}_{jkh} = \mathcal{B}_m\,(\dot\partial_h \Gamma^{*i}_{jk} + C^{\ i}_{jr}P^r_{kh} - C^{\ i}_{jh|k})$$

or

(3.15) $\mathcal{B}_m P^{\ i}_{jkh} = \mathcal{B}_m\,\dot\partial_h \Gamma^{*i}_{jk} + P^r_{kh}(\mathcal{B}_m C^{\ i}_{jr}) + C^i_{jr}(\mathcal{B}_m P^r_{kh}) - \mathcal{B}_m\, C^{\ i}_{jh|k}$.

Using the conditions (2.5), (3.5) and using (1.4f) in (3.15), we get

(3.16) $\mathcal{B}_m(\dot\partial_h \Gamma^{*i}_{jk} - C^{\ i}_{jh|k}) + P^r_{kh}(\mathcal{B}_m C^{\ i}_{jr}) + \lambda_m C^i_{jr}P^r_{kh} + \gamma_m(C^i_{jk}y_h - C^i_{jh}y_k) - \lambda_m P^i_{jkh} - \phi_j\mu_m(\delta^i_k y_h - \delta^i_h y_k) = 0$.

where $\gamma_m = \phi\,\mu_m$

Using the condition (3.1) in (3.16), we get

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(3.17) $\mathcal{B}_m(\dot{\partial}_h \Gamma^{*i}_{jk} - C^{\ i}_{jh|k}) + P^r_{kh}(\mathcal{B}_m C^{\ i}_{jr}) + \lambda_m C^i_{jr} P^r_{kh} + \mu_m(P^i_{jk} y_h - P^i_{jh} y_k) - \lambda_m P^i_{jkh} - \phi_j \mu_m(\delta^i_k y_h - \delta^i_h y_k) = 0$.

Thus, we conclude

Theorem 3.5. *In $P^* - G(\mathcal{B}R) - RF_n$, we have the identity (3.17) if and only if (2.6) and (3.6) hold [provided the conditions (2.3a) holds].*

Transvecting (3.17) by y^j, using (1.5a),(1.10), (1.13), (1.4b), (1.11a) and (1.8), we get

(3.18) $\lambda_m P^i_{kh} + \phi\, \mu_m(\delta^i_k y_h - \delta^i_h y_k) = 0$.

where $\phi_j y^j = \phi$

Thus, we conclude

Theorem 3.6. *In $P^* - G(\mathcal{B}R) - RF_n$, we have the identity (3.18) if and only if (2.6) and (3.6) hold [provided the conditions (2.3a) holds].*

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