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Research Identifying Shear Modulus of Soil under Cyclic Loading at Medium and Small Strain Level

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Abstract— *The main purpose of this paper is to investigate the stiffness of soil under cyclic loading at small and medium strain level. The influence of several important factor on shear modulus of soil such as: the strain level, the plasticity index and the main effective stress are discussed in detail in this paper. Behaviour of different kind of soils is also compared based on available laboratory through some experimental results as well as numerous results from FEM. Besides, an experimental method to identify the shear modulus of soils through Cross-hole seismic Survey is also gaven in this paper.*

Keywords— *Shear modulus, degradation factor, soil, shear strain, void ratio.*

I. Introduction

In many hands-on cases the ground response under seismic loading is evaluated by using the well-known linear equivalent method in which compatible values of shear modulus and damping ratio are chosen according to the shear strain level in soil deposit. In this simplified approach the developed pore water pressure and the residual soil displacements cannot be calculated. However, In this research, author only concentrate on consider linear equivalent method for soils under cyclic loading at small and medium strain level ($10^{-6} \leq \varepsilon \leq 10^{-2}$) also do not involve to large strain level ($\varepsilon > 10^{-2}$). From that, it could be considered acceptable in a practical point of view.

To use of this method of analysis need to reliable strain-dependent shear modulus and damping ratio curves.

In this paper only the factors affect to the shear modulus of soils are treated and discussed in detail. The text is essentially divided in two parts. In the first part, the stiffness of soil at small strain level ($\varepsilon \approx 10^{-6}$) is analyzed. In the second part, the investigation proceeds with the study of the shear modulus degradation at higher strain level until 10^{-2}.

II. Initial Shear Modulus

In many years recently, many studies were carried out to investigate behaviour of soil at small strain level. The initial shear modulus G_0 (for $\varepsilon \approx 10^{-6}$) is a very important parameter not only for seismic ground response analysis but also for a variety of geotechnical applications.

A number of considerable results about empirical relationships have been proposed for estimating initial shear modulus for different kinds of soils: Hardlin and Black (1969), Iwasaki and Taksuoka (1977), Marcusson and Wahls (1972), Kokusho (1972), Kokusho and Esashi (1981), Nishio et al., (1985), Biarez et al., (1999), J.A Santos and A. Gomes Correia (2000). All of these relationships are based on two experimental evidences: the shear wave velocity (V_s) is linear function of void ratio (e) and depends on the mean effective stress (p') with a power of n/2, as proposed originally by Hardin and Richart (1963).

$$V_s = C.(B-e).\, p'^{n/2} \tag{1}$$

where B, C and n are the constant coefficients depend on different type of soils

The relationship between the total mass density (ρ) and the void ratio of soil can be obtained as follow:

$$\rho = \frac{\rho_s}{1+e} \tag{2}$$

In which, ρ_s, is the mass density of solid particles.

Based on the basic of wave propagation it is well-known that the value of G_0 can be obtained from the shear wave velocity:

$$G_0 = \rho.v_s^2 \tag{3}$$

From combining the results of the previous equations (1), (2) and (3), it will give the typical formula as follow:

$$G_0 = \frac{\rho_s}{1+e}.C^2.(B-e)^2.p'^n = A.\frac{(B-e)^2}{1+e}.p'^n = A.F(e).p'^n \tag{4}$$

More recently, another empirical void ratio function of the form $F(e) = e^{-x}$ proposed by other researchers such as in [1], [2], [6]. This empirical function is based on experimental data and can be explained by using the simple theory of Hertizan related to perfect spheres. According to the study of (Biarez and Hicher, 1994) a different arrangement of a group of identical spheres would be characterised by a different void ratio and a coefficient G(e) which express the arrangement of the spheres .

By considering an idealized continuous medium of spheres with identical sizes, "reference [2] "showed that the initial shear modulus can be expressed as a function of G(e).

$$G_0 \propto G(e)^{-2/3}.p'^{1/3} \tag{5}$$

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It can be deduced the void ratio function F(e) is proportional to $G(e)^{-\frac{2}{3}}$ and can be approximated by an exponential function as shown in figure 1:

$$F(e) \propto G(e)^{-2/3} \cong e^{-0.7} \tag{6}$$

According to (Cascante and Santamarina, 1996), the exponent n can be an indicator of the type of contacts. For

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example, n=1/3 for contacts between spheres, whereas n=1/2 for cone-to-plane contacts.

For real soil, which is a random package of particles of different sizes and shapes it was found that the exponent n can be taken equal to 1/2 for clays and also in a simplified way for sands [6].

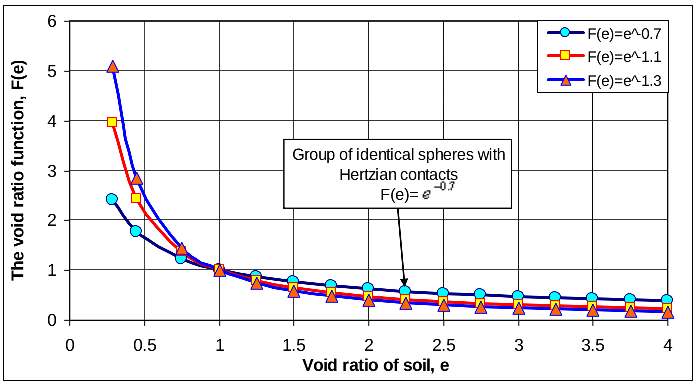

Fig. 1. Curves of relationship between void ratio, e and funtion F(e) with medium of spheres.

Based on available laboratory data mentioned before and including some experimental results obtained by the authors (resonant column tests), "reference [2]" proposed two unified curves that represent the lower and upper bound values of G_0:

$$G_0 = 4000.e^{-1/3}.p'^{0.5}; \quad \text{For the lower bound} \qquad (7)$$

$$G_0 = 8000.e^{-1.1}.p'^{0.5}; \quad \text{For the upper bound} \qquad (8)$$

These curves and the experimental data are represented in figure 2 for a given value of p'=1000kPa. The void ratio functions are also of the form $F(e) = e^{-x}$ and are plotted in figure 1.

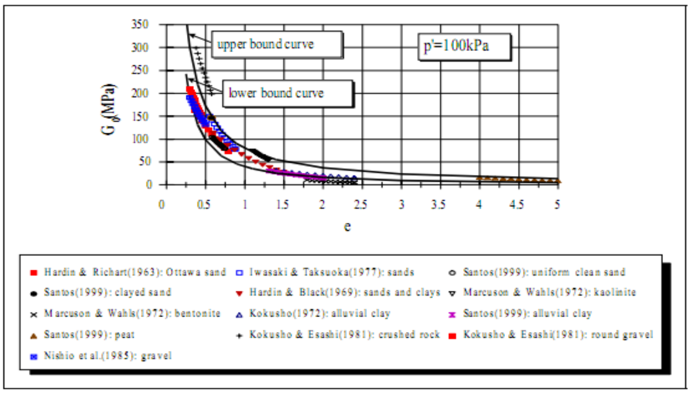

Fig. 2. Relationship between initial shear modulus G_0 as a funtion of void ratio e [2].

It is remarkable to observe that, in general, the experimental data lie between the two bound curves. Some discrepancies were observed only particular case for a crushed rock material. It is also important to emphasise that the data plotted in figure 2 represent many experimental results obtained by different authors using several types of testing techniques such as seismic test, resonant column test, cyclic torsion shear test, cyclic tri-axial test with local measurements. So, the proposed curves seem to be a consistent tool that can be applied for sands and clays as a guide for practical purposes. For gravel, the proposed bound curves may not be recommended because the exponent n shows also some dependency on the coefficient of uniformity [1]. Similar conclusions can be taken for any other value of p'.

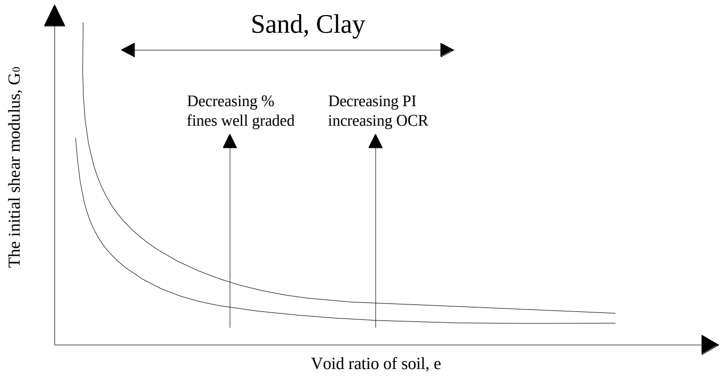

Fig. 3.Influence of several factors to relationship curve G_0 - e.

The influence of some important factors on the initial shear modulus can be deduced from the data reproduced in figure 2 and represented schematically in figure 3 includes percentage of fines, material grading, plasticity index (IP) and stress history (overconsolidation ratio, OCR). At such small shear strain level, the effects of loading frequency (f) and the number of cycle (N) are not significant and can be neglected.

III. SHEAR MODULUS DEGRADATION FACTOR

In recent years, due to the improvements in laboratory testing such as local and overall deformation measurement and stress-path control tests, many reliable experimental data has accumulated allowing a considerable advance in the knowledge of the stress strain behavior of soil.

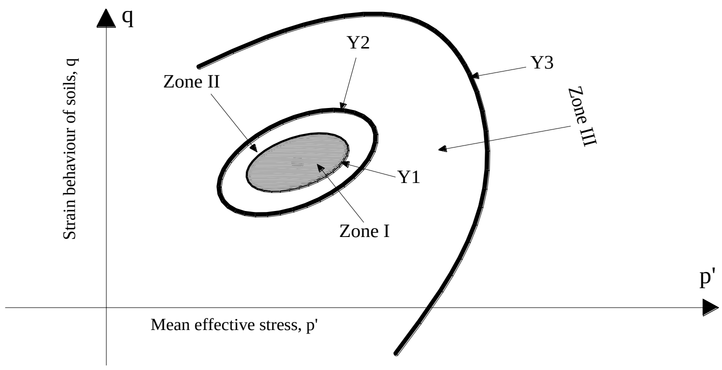

Fig. 4. Influence of several factors to relationship curve G_0 - e.

It has been demonstrated that the behaviour of soils can be described using the concept of kinematic regions in stress space. In the simplify scheme proposed by (Jardine, 1992) the current effective stress point is surrounded by two-sub yield surfaces Y_1 and Y_2. Inside the surface Y_2 (Zone II) the response is non-linear but fully recoverable (elastic). It appears that inside Zone II a small region (Zone I-inside surface Y_1) can exist in which the response is linear elastic. The surface Y_2 defines the threshold at which drained or undrained cyclic loading starts to affect the soil significantly. When the Y_2 surface is reached significant plastic straining begins to occur until the Y_3 yield surface.

Based on a similar idea [4] explained some fundamental aspects of cyclic response of saturated soils. In figure 5, which are represented the strain-dependent shear modulus degradation factor (G/G_0) and damping ratio (ξ) of soils that author suggested the definition of two level of cyclic shear strain: the linear threshold shear strain, γ_t^e [5, 9] and the volumetric threshold shear strain, γ_t^v. The latter is the most important and represent the limit beyond which the soil structure starts to change irreversibly: permanent volume change will take place in drained conditions, whereas pore water pressure will build up in undrained conditions.

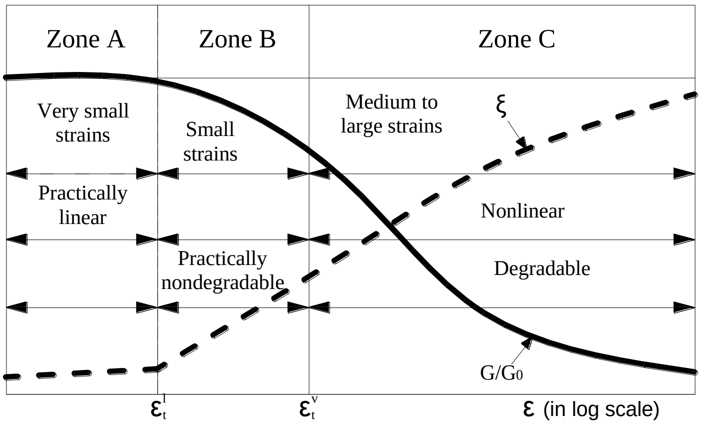

Fig. 5. Typical G/G0-ε and ξ - ε curves proposed by Vucetic [2].

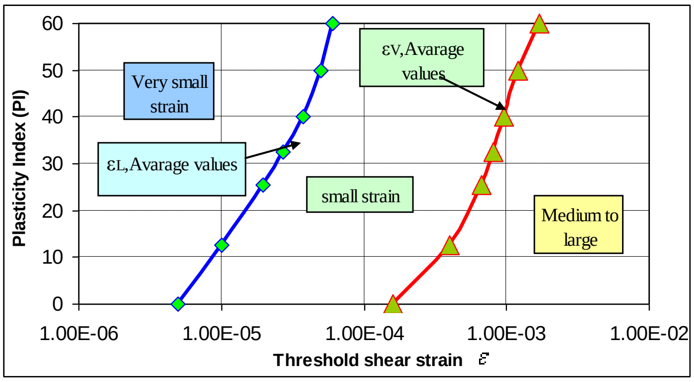

Fig. 6. Relationship between threshold shear strain ε and plasticity index PI follow Vucetic, 1994 [2].

The agreement of the two models can be clearly seen in table I.

TABLE I. Typical soil behaviour for different zones.

[Jardine, 1992]	[Vucetic, 1994]	Soil behaviour
Zone I	Zone A	Linear elastic
Zone II	Zone B	Non-linear elastic
Zone III	Zone C	Elastoplastic

"Reference [4]" suggested that γ_t^e depends on soil microstructure and can be seen possibly correlated to the soil 'IP'. "Reference [4], [5]" proposed G/G_0 degradation curves and approximate range of γ_t^v according to the plasticity index of soil (figure 6 and 7).

Indeed, the stiffness degradation curve must be affected by other factors such as: stress history, mean effective stress, number of cycles, loading frequency, etc. For example, figure 8 shown clearly that for sands the influence of the main effective stress can be quite important at low confining pressure conditions:

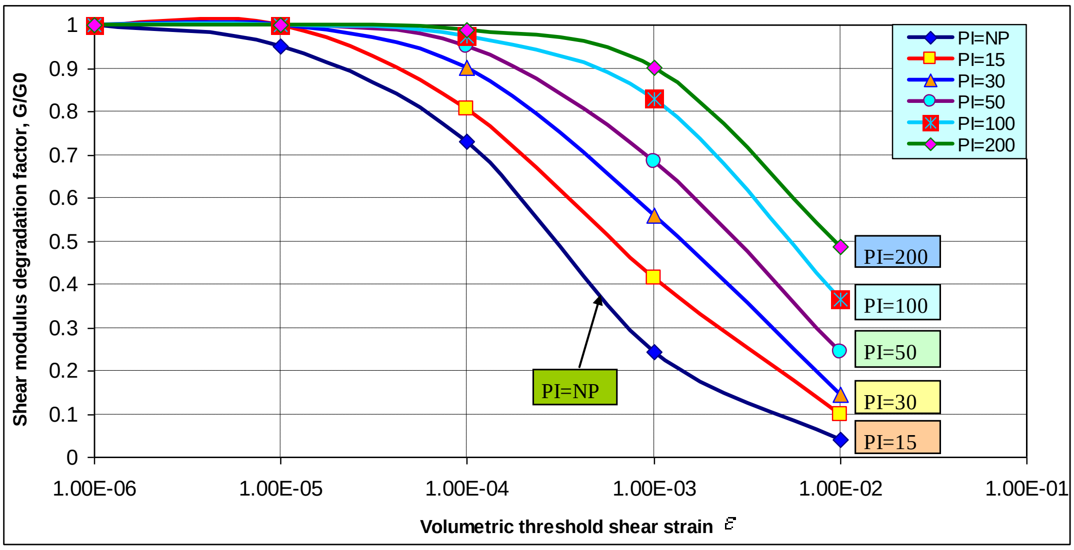

Fig.. 7. Relationship between stiffness degradation factor G/G_0 with threshold shear strain ε and IP

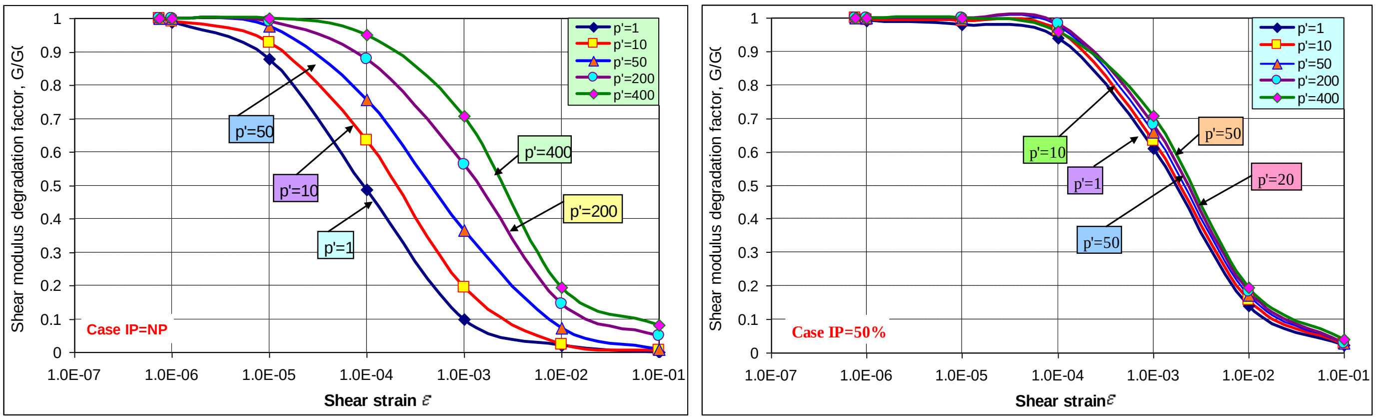

Fig. 8. Relationship between stiffness degradation factor G/G_0 with curves ε -p' (case IP=50% and case IP=NP).

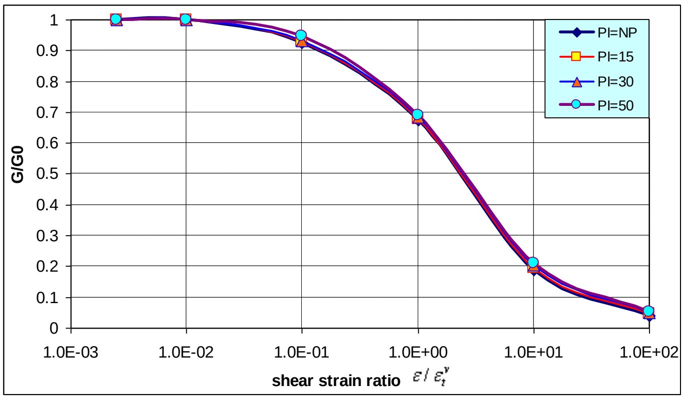

Fig. 9. Results of curves G/G_0-$\varepsilon/\varepsilon^v$ follow values IP.

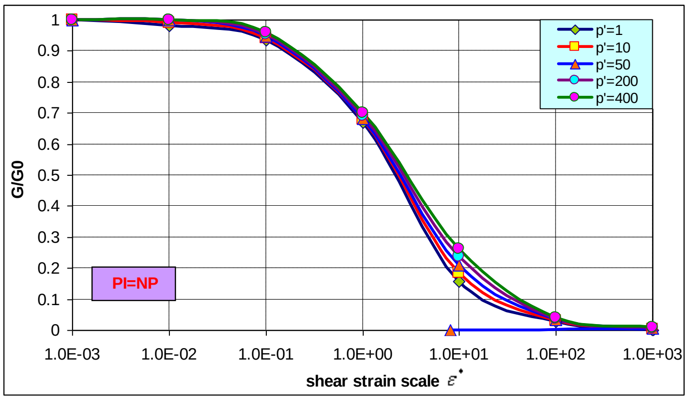

Fig. 10. Results of curves G/G_0-ε^* follow values p'.

At this moment, it is evident that the stiffness degradation curve can be affected by many factor and can not be described only by the plasticity index of soil. But it seems that all the influences of these factors can be considered in a simple form when using the concept of volumetric threshold shear strain. In other words, a unique G/G_0 curve can be defined for normalised shear strain γ/γ_t^v, as originally proposed by Santos [2]. Based on the key idea, in figure 9 are represented form the results of figure 7 in normalised shear strain scale, using the average values of γ_t^v (according to the value of PI as indicated in figure 6).

As expected, there is almost a perfect coincidence of all the previous curves for PI=NP, 15, 30 and 50%. Besides, the shear strain level $\gamma = \gamma_t^v \gamma_t^v$ correspond approximately to a stiffness degradation factor of G/G_0 (Figure 9), i.e.:

$$\gamma_t^v \cong \gamma(G/G_0 = 0.7) \qquad (9)$$

In practice, the volumetric threshold shear strain is not easy to determine and its value increase with the plasticity index and the strain rate in cohesive soils and with the main effective stress in cohesiveness soils. For normalisation purposes a reference shear strain γ_t^r is suggested and defined as:

$$\gamma_t^r = \gamma(G/G_0 = 0.7) \qquad (10)$$

The same key idea can be used to explain the influence of the main effective stress in G/G_0 curves for sands. The previous results [1] (Figure 8) are plotted in figure 10 in normalised shear strain scale $\gamma^* = \gamma/\gamma_t^r$. The curves are again almost coincident. These encouraging results show the possibility to define almost a unique relationship between G/G_0 and γ^*. “Reference [2], [3] proposed two simple equations to define the lower and upper bound values to define the lower and upper bound values of G/G_0 as a function of γ^* (for $10^{-6} \le \varepsilon \le 10^{-2}$).

$$\text{Lower bound} = \begin{cases} 1 & ;\gamma^* \le 10^{-2} \\ \dfrac{1-\tanh\left[0.48.\ln\left(\gamma^*/1.9\right)\right]}{2} & ;\gamma^* > 10^{-2} \end{cases} \qquad (11a)$$

$$\text{Upper bound} = \begin{cases} 1 & ;\gamma^* \le 10^{-1} \\ \dfrac{1-\tanh\left[0.46.\ln\left((\gamma^*-0.1)/3.4\right)\right]}{2} & ;\gamma^* > 10^{-1} \end{cases} \qquad (11b)$$

The previous values [1, 5] are plotted in Figure 11 and compared with the proposed curve.

All of the results seem to be in good agreement with the proposed stiffness degradation curves in normalised shear strains scale. Some experimental results obtained by the authors using the resonant column and the cyclic torsional shear test (with N=10) for sand and for an alluvial clay (PI=41%) are also in good agreement with the proposed curves, [2].

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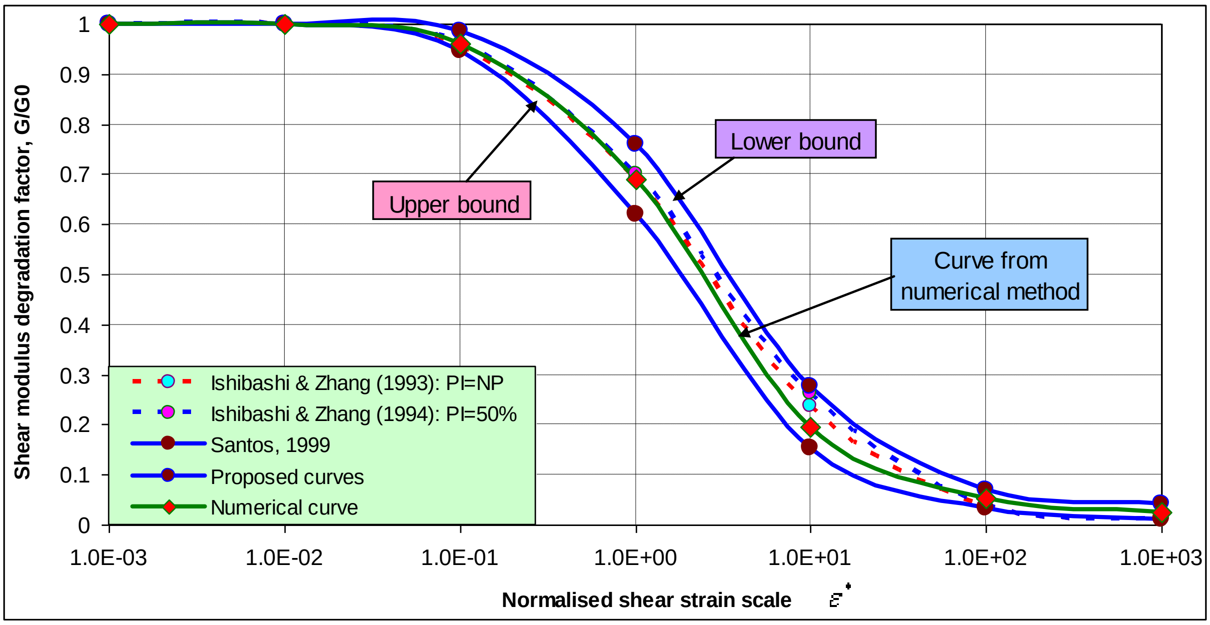

Fig. 11. Proposed stiffness degradation curves and numerial result in ε^* scale

IV. Shear Modulus Degradation Factor Numerical Analysis

One aim of this study is to use widely available finite element software and soil models to analyse the relationship between shear modulus degradation G/G_0 with normalised shear strain, and later compare and discuss the results obtained with those measures as well as other methods. Three dimension (3D) approaches to software and modelling are accessible through the exponential growth of computer technology. From the data and result of numerical method, a curve between G/G_0-ε^* plotted as in figure 11.

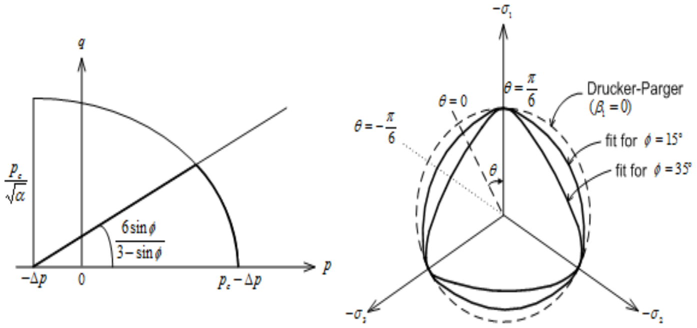

Fig. 12. Hardening soil model in Plaxis 3D.

The numerical results are nanlysed based on interaction data between shear modulus, settlement and shear strain. Therefore, hardening soil model in Plaxis 3D (version 2.1) used to study the relationship because in this model, behaviour of soils have related directly to shear modulus.

Several parameters in this model are shown as follow:

$$G_s^{'} = \frac{s_s}{\varepsilon_s} = \frac{E_s}{2(1+\mu)}\left(\mu = \frac{\varepsilon_L}{\varepsilon_v}\right) \quad (12)$$

$$\varepsilon_v = \frac{(1+\mu)(1-2\mu)\sigma_1}{E_s(1-\mu)} = \frac{1-2\mu}{2(1-\mu)}\frac{\sigma_1}{G^{'}} \quad (13)$$

$$E_{eod} = \frac{2G(1-\mu)}{1-2\mu} \quad (14)$$

Where, μ is poisson ratio; E_s is elastic modulus of soils; σ_1 is unconfined compression stress.

All of the results from numerical method seem to be in good agreement with the proposed stiffness degradation curves in normalised shear strains scale and some other experimental resultsas well as get close agreement with the proposed curves of Santos, 1999 [3].

V. Method to Identify Shear Modulus from Practice

The velocity of shear waves created as the result of an impact to a given layer of soil can be effectively determined by the cross-hole seismic survey. The principle of this technique is illustrated in figure 13, which shows two holes drilled into the ground a distance L apart. A vertical impulse is created at the bottom of one borehole by means of impulse rod. The shear waves thus generated are recorded by a vertically sensitive transducer.

The velocity of shear waves can be calculated as:

$$v_s = \frac{L}{t} \quad (15)$$

Where t = travel time of the waves

The shear modulus G of the soil at the depth at which the test is taken can be determined from the relation:

$$v_s = \sqrt{\frac{G}{(\gamma/g)}} \;\; \Rightarrow \;\; G = \frac{v_s^2\gamma}{g} \quad (16)$$

Where v_s = velocity of shear waves, γ = unit weight of soil

g = acceleration due to gravity.

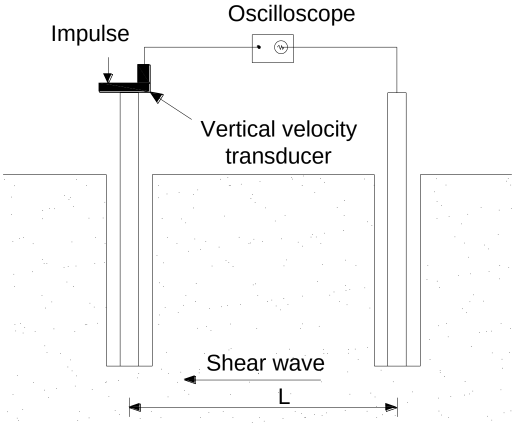

Fig. 13. Cross-hole method of seismic survey.

VI. CONCLUSION

At small deformation new unified relationships between the initial shear modulus (G_0), the void ratio and the main effective, are proposed for soils (sand, silt, clay and peat). Several well-known semi empirical correlations including some experimental results obtained by the author (resonant column tests) are analysed and compared with the new unified relationships.

At higher strain level, the investigation proceeds with the study of the shear modulus degradation. Based on recent works of some authors [Jardine, 1992] and [vucetic, 1994], the concept of volumetric threshold strain is used to explain the influence of the plasticity index and the main effective stress on the shear modulus of soils.

A new reference shear strain (γ_t^r) is defined for normalisation purposes. New unified stiffness degradation (G/G_0) curves are proposed in normalised shear strains scale (γ^*). Previous stiffness degradation (G/G_0) curves proposed by [Vucetic and Dobry, 1991] and [Ishibashi and Zang, 1993] are in good agreement with the new unified curves in normalized shear strains scale. These results also get closely with the curve from numerical analysis. The influence of loading frequency, number of cycles and stress history on the relationship G/G_0- γ^* need to investigated in the future.

In conclusion, this paper shows the possibility to define some simple unified relationship and curves that allow the assessment of shear modulus of soil under cyclic loading at small loading at small to medium level ($10^{-6} \le \varepsilon \le 10^{-2}$), for practical design purposes.

Cross-hole seismic Survey method is an experimental method that is useful and effective to identify the shear modulus of soils.

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