International Research Journal of Advanced Engineering and Science
ISSN: 2455-9024

Critical Points of Simplified Model for Ferroresonance Phenomenon in Single Phase Power Transformers

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Abstract— *This paper is aimed at proposing a simplified model of ferroresonance phenomenon in single–phase power transformers. The transformer is modelled and the non–linear equivalent resistance and inductance of the magnetizing branch are determined from open–circuit test. The ferroresonance resulting from energizing the transformer through a series capacitor is assessed and the calculated primary voltage and current values are compared with those measured experimentally. A robust meta-heurtics optimization technique, Simulated Annealing, is applied in this paper for determining for ferroresonance jump up and jump down critical points. The calculated values agreed reasonably with those measured experimentally with a deviation not exceeding 7%.*

Keywords— *Single–phase power transformers, modeling, ferroresonance, overvoltage's, overcurrent's, simulated annealing technique, critical points*.

I. INTRODUCTION

In linear circuits, resonance occurs when the capacitive reactance equals the inductive reactance at the frequency at which the circuit is driven. Iron-core inductors have a non-linear characteristic and have a range of inductance values. Therefore, there may not be a case where the inductive reactance is equal to the capacitive reactance, but yet very high and damaging overvoltages occur [1-6]. In power systems, ferroresonance occurs when a nonlinear iron-core inductor is fed from a series capacitor [7-15]. The nonlinear inductor in power system can be due to: a) The magnetic core of a wound type voltage transformer, b) Bank type transformer, c) The complex structure of a 3 limb three-phase power transformer (core type transformer), d) The complex structure of a 5 limb three-phase power transformer (shell-type transformer). The circuit capacitance in power system can be due to a number of elements, such as: a) The circuit-to-circuit capacitance, b) Parallel lines capacitance, c) Conductor to earth capacitance, d) Circuit breaker grading capacitance, e) Busbar capacitance, f) Bushing capacitance. The phenomena of ferroresonance is a name given to a situation where the nonlinear magnetic properties of iron in transformer iron core interact with capacitance existing in the electrical network to produce a nonlinear tuned circuit with an unexpected resonant frequency.

Therefore, ferroresonance phenomenon poses a hazard to an electric power system because it generates overvoltages and overcurrents. It cannot be analyzed or predicted by the computational methods based on linear approximation normally used by electrical engineers. This lack of knowledge makes it a probable culprit responsible for the unexplained destruction and malfunctioning of equipment. Different methods have been proposed in the literature for the analysis of ferroresonance phenomenon. These methods include the incremental describing function method [4], frequency damping boundaries method [5], G-1(jw)-plane method [6], the principle of harmonics balance method [7], bifurcation theory method [8]. Most of these methods are complex and need long derivations [4-7].

In this paper, the ferroresonance phenomenon in single phase power transformers is investigated. The transformer is modeled and the non–linear equivalent resistance and inductance of the magnetizing branch are determined from open–circuit test. The ferroresonance resulting from energizing the transformer through a series capacitor is assessed and the calculated primary voltage values are compared with those measured experimentally.

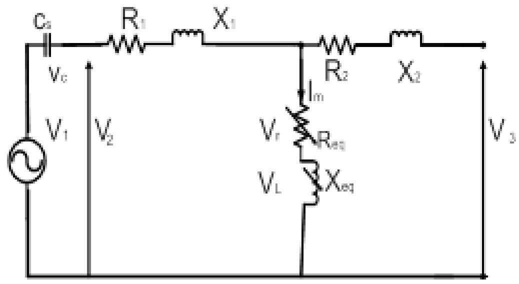
II. THE INVESTIGATED SYSTEM

The equivalent circuit of investigated system is shown in figure 1a. It is composed of an unloaded single- phase, core-type, air–cooled transformer in series with a capacitor Cs.

III. THE PROPOSED MODEL INTRODUCTION

The magnetizing branch of the transformer equivalent circuit, figure 1(a), is represented by two nonlinear elements R_{eq} and X_{eq} connected in series.

Both R_{eq} and X_{eq} can be determined experimentally from the open circuit test data: input power P_1, supply voltage V_1 and input (primary) current I_m. The series elements R_1 and X_1 are disregarded when compared with R_{eq} and X_{eq} figure 1(b).



(a)

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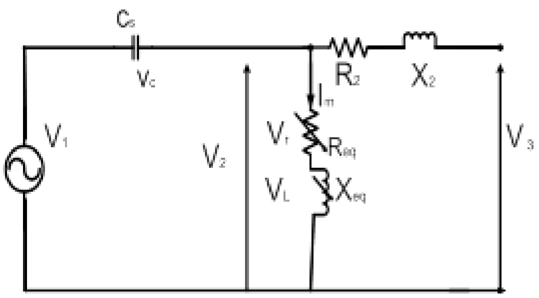
ISSN: 2455-9024



(b)
Fig. 1(a-b). Equivalent circuit of the investigated system

The equivalent resistance, the equivalent impedance and the equivalent inductance are expressed as:

$R_{eq} = P_1 / I_m^2$, $Z_{eq} = V_1 / I_m$, $X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$

Different values of R_{eq} and X_{eq} were determined at different values of the input current I_m.

A suitable fitting technique was applied to express R_{eq} and X_{eq} as polynomials of no – load input current I_m:

$R_{eq} = f_1 (I_m)$ (1)

$R_{eq} = (a_0 + a_1*I_m + a_2*I_m^2 + a_3* I_m^3 + a_4 * I_m^4)$

$X_{eq} = f_2 (I_m)$ (2)

$X_{eq} = (b_0 + b_1*I_m+b_2*I_m^2 + b_3* I_m^3 + b_4 * I_m^4)$

$V_r = I_m*R_{eq}$, $V_L= I_m*X_{eq}$ (3)

$V_c = I_m / (2\pi fC_s)$ (4)

The supply voltage V_1 is expressed as:

$V_1 = [V_r^2 + (V_L – V_c)^2]^{0.5}$ (5)

The transformer primary voltage V_2 is expressed as:

$V_2 = [V_r^2 + V_L^2]^{0.5}$ (6)

The variation of the voltage V_2 and the input current I_m with the increase of the supply voltage V_1 describes the ferroresonance phenomenon in the transformer.

IV. Experimental Set – up

Figure 2(a) shows the set – up used for open circuit testing of a 110/220V, 2kVA transformer where the supply voltage is increased in steps from zero to 120V, slightly above the rated value. Figure 2(b), shows the set – up with capacitor C_s of 40µF connected in series for assessment of ferroresonance phenomenon in the transformer. The supply voltage has been increased in steps from zero to 120V and then reduced in steps from 120 V to zero.

In each step in the forward direction as well as in the backward direction, the readings of supply voltage, V_1, the primary voltage, V_2, the primary no-load current, I_m are recorded using true RMS meters.

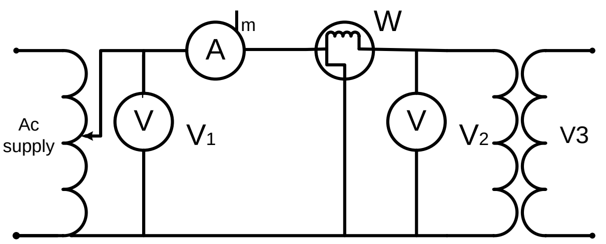


Fig. 2(a).Open circuit test

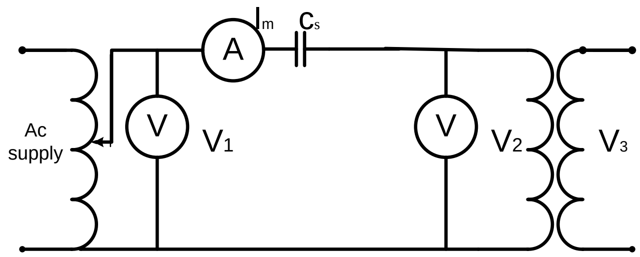


Fig. 2(b). Ferroresonance circuit under no–load condition

V. Critical Points of Jump-Up and Jump –Down of I–V Characteristics

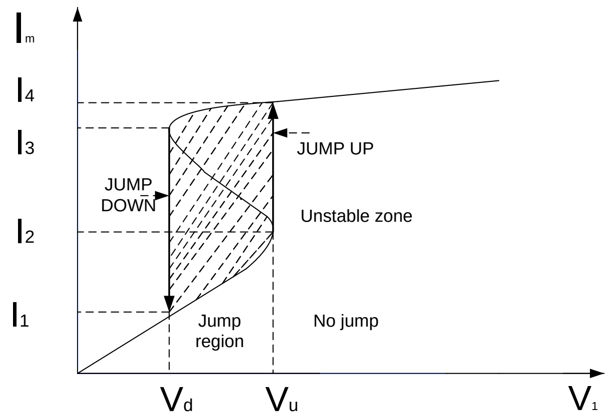


Fig. 3. Critical points of jump-up and jump-down

A. Jump Up Points

From the equivalent circuit of figure 2(b) and the equation (5)

One can write: $V_1^2 = V_r^2 + (V_L – V_C)^2$

Differentiation of equation (5) in order to seek the ferroresonance Condition as follows:

$2\partial V_1/\partial I_m = 2V_r*\partial Vr/\partial I_m + 2 (V_L – V_C)*(\partial V_L/\partial I_m - \partial V_C/\partial I_m) = 0$

$= V_r*\partial V_r/\partial I_m + (V_L – V_C)*(\partial V_L/\partial I_m - \partial V_C/\partial I_m) = 0$

$= V_r/ (V_L-V_C)*\partial V_r/\partial I_m+ \partial V_L/\partial I_m - \partial V_C/\partial I_m = 0$

$\partial V_C/\partial I_m= [Vr / (V_L-V_C)]*\partial V_r/\partial I_m+ \partial V_L/\partial I_m$ (7)

As C is constant, equation (7) takes the form:

$X_C = [R_{eq} / (X_{eq} –X_C)]*\partial Vr/\partial I_m+ \partial V_L/\partial I_m$ (8)

From equation (1);

$R_{eq} = (a_0 + a_1*I_m + a_2*I_m^2 + a_3* I_m^3 + a_4 * I_m^4)$

$V_r = I_m* R_{eq}$

$= I_m (a_0 + a_1*I_m + a_2*I_m^2 + a_3* I_m^3 + a_4 * I_m^4)$

$= (a_0*I_m + a_1* I_m^2 + a_2* I_m^{3+}a_3* I_m^4 + a_4*I_m^5)$

$\partial V_r/\partial I_m= a_0 + 2 a_1*I_m + 3 a_2*I_m^2 + 4 a_3*I_m^3 + 5 a_4*I_m^4$ (9)

Also, equation (2) gives: $X_{eq} = (b_0 + b_1*I_m+b_2*I_m^2 + b_3* I_m^3 + b_4 * I_m^4)$

$V_L = I_m* X_{eq} = I_m (b_0 + b_1*I_m + b_2*I_m^2 + b_3* I_m^3 + b_4 * I_m^4)$

$= (b_0*I_m + b_1*I_m^2 + b_2*I_m^3 + b_3* I_m^4 + b_4 * I_m^5)$ $\partial V_L/\partial I_m =b_0 +2b_1*I_m + 3b_2*I_m^2 + 4b_3*I_m^3 +5b_4*I_m^4$ (10)

Substituting equations (1), (2), (9) and (10) in equation (8) results in a nonlinear equation in the current I_m This equation is solved using a meta-heurtics technique, Simulated

Annealing technique to determine the critical jump – up value of I_2. I_4 corresponds to I_2 at the same supply voltage.

B. *Jump – Down Points*

The jump–down was occurred at the ferroresonance condition when:

$$X_C = X_{eq} \quad (11)$$

Substituting equation (2) in equation (11) results in a nonlinear equation in the current I_m.

$$X_C = (b_0 + b_1 * I_m + b_2 * I_m^2 + b_3 * I_m^3 + b_4 * I_m^4) \quad (12)$$

This equation is solved using a meta-heurtics technique, Simulated Annealing technique to determine the critical jump–down value of I_1. I_3 corresponds to I_1 at the same supply voltage.

VI. PROPOSED OPTIMIZATION ALGORITHM

Simulated annealing is a combinatorial optimization technique based on random evaluation of the objective function. The simulated annealing has the capability of finding global optimum with a high probability even for ill-conditioned functions with numerous local optima, albeit with large number of function evaluations. In general, the simulated annealing method resembles the actual cooling process of molten metals through annealing [13]. A Detailed description of the technique is given in [13], however, the technique could be understood from the flowchart below that used for solving the problem under concern. A brief description for simulated Annealing is given in the following:

- Step 1: Set Choose the parameters of the SA method. The initial temperature, the temperature reduction factor is chosen as c = 0.5, number of iterations n, machine data and current (I_m)
- Step 2: Evaluate the objective function value at (I_m) as f_1 and set the iteration number as i = 1.
- Step3: Generate a solution from the neighborhood of the current solution. Let this solution be $f_2 = f\ (I_{m2})$ and compute $\Delta f = f_2 - f_1$.
- Step4: Since the value of Δf is positive, we use the Metropolis criterion $(P\ [I_m] = e{-}\Delta f/t)$ to decide whether to accept or reject the current point. For this we choose a random number in the range (0, 1), if random number is smaller than Metropolis criterion we accept (I_m) Since Δf<0, we accept the current point as (I_{m3}) and increase the iteration number to i =3. Since i > n, we go to step5.
- Step 4: Update the iteration number as i = 2.Since the iteration number i is less than or equal to n, we proceed to step3.
- Step 5: Since a cycle of iterations with the current value of temperature is completed,
- We reduce the temperature to a new value by (t = c*t) and reset the current iteration number as i = 1 and go to step3.
- Step 6: If stop criteria is met, then STOP. Else go to Step 3.

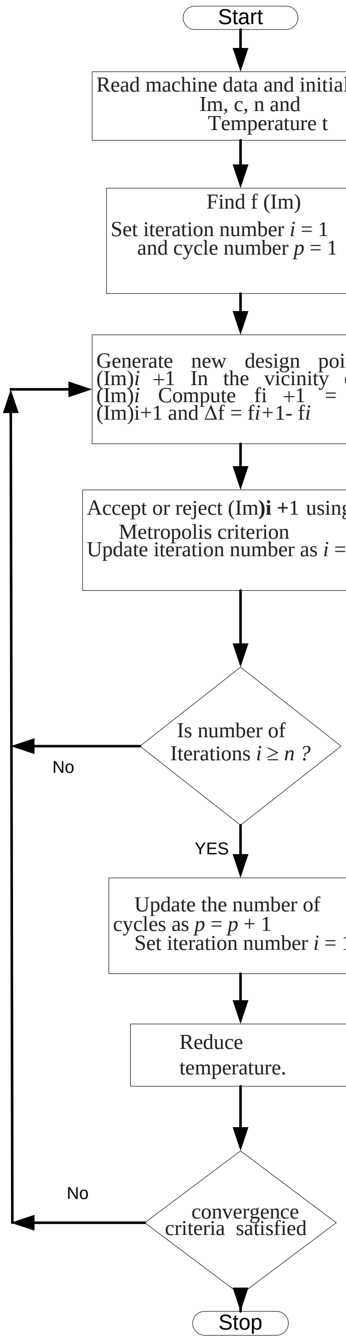


Fig. 4. Flow chart of simulated annealing algorithm

The number of iteration N in the flowchart is taken around 100, which is considered a good comprise between the accuracy and computation time.

VII. RESULTS AND DISCUSSIONS

A. *Open – Circuit Test Results*

The open–circuit test data is given in table I using experimental connection of figure 2(a). The calculated values of R_{eq}, Z_{eq} and X_{eq} for different values of the supply voltage V_1 are given in table I. The I–V characteristic of the transformer at open–circuit condition is shown in figure 5.

Radwan M. Al-Bouthigy, "Critical points of simplified model for ferroresonance phenomenon in single phase power transformers," *International Research Journal of Advanced Engineering and Science*, Volume 1, Issue 2, pp. 44-49, 2016.

TABLE I. Open–circuit test results

V_1	33.3	61	90.7	110	120.3	129.8
I_m	0.12	0.37	0.74	1.35	1.82	2.38
P_1	2.5	10	20	30	37.5	45
R_{eq}	173.61	73.72	36.52	16.46	11.32	7.94
Z_{eq}	277.5	164.87	122.57	81.48	66.10	54.54
X_{eq}	216.48	147.80	117.00	79.80	65.12	53.96

The parameters at rated voltage:

$$R_{eq} = P_1 / I_m^2 = 30 / 1.823 = 16.461\ \Omega$$

$$Z_{eq} = V_1 / I_m = 110 / 1.35 = 81.481\ \Omega$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{81.481^2 - 16.461^2}$$

$$X_{eq} = 79.801\ \Omega$$

The polynomials nonlinear $f_1\ (I_m)$ and $f_2\ (I_m)$ describing the elements R_{eq} and X_{eq} in terms of the no-load current, I_m, are:

$$R_{eq} = 11*I_m^4 - 81.5*I_m^3 + 220*I_m^2 - 270*I_m + 150 \qquad (13)$$

$$X_{eq} = -6.7*I_m^4 + 32*I_m^3 - 27*I_m^2 - 79*I_m + 180 \qquad (14)$$

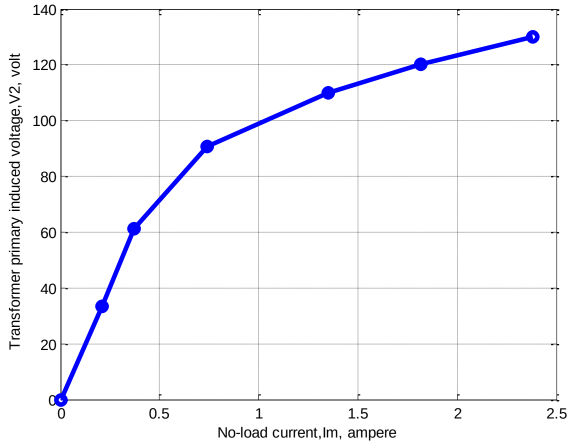


Fig. 5. Open circuit characteristic of the transformer

B. *Accuracy of the Fitting Equations of R_{eq} and X_{eq}*

Figures 6 and 7 show a satisfactory agreement of the calculated values of R_{eq} and X_{eq} using "(13)" and "(14)" with those measured experimentally at different values of the input no–load current I_m.

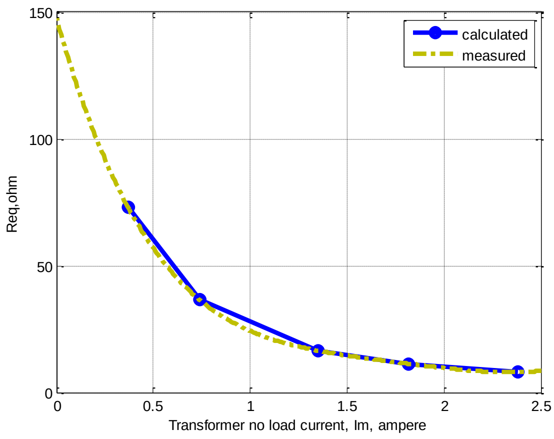


Fig. 6. R_{eq} versus primary no – load current I_m

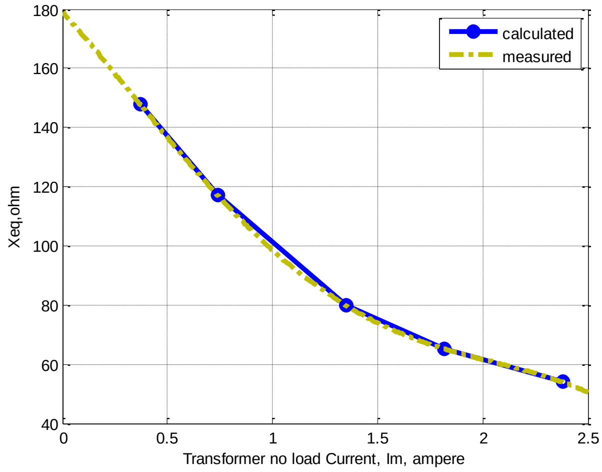


Fig. 7. X_{eq} versus the primary no – load current I_m

C. *Ferroresonance Assessment*

The value of capacitor C_s was chosen equal to 40 μF to ensure that the line characteristic of capacitor intersects the open circuit characteristic curve of the transformer in the saturation region as shown in figure 8.

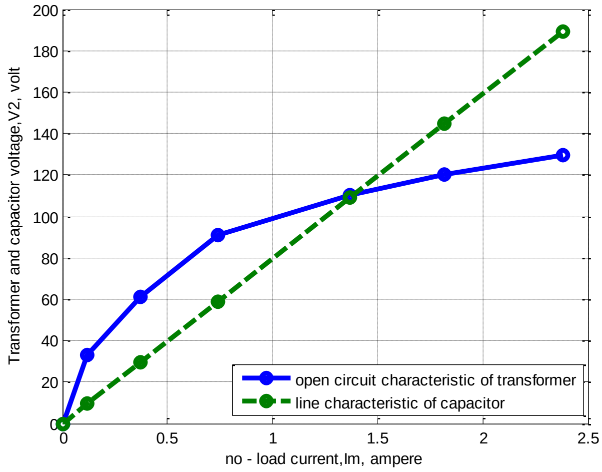


Fig. 8. Transformer open - circuit with capacitor line characteristic.

The measured values of the primary induced voltage, V_2 and the primary current, I_m are depicted respectively in figure 9 and 10 for different values of the supply voltage, V_1. The corresponding calculated values using "(5)" and "(6)" are also shown in figure 9 and 10 for different values of the input current, I_m.

Figure 9 shows the variation of the transformer primary voltage V_2 against the supply voltage V_1. Form this figure, one can conclude the following:

- The measured results on increasing the supply voltage V_1 (forward direction) dictate a sudden jump up in primary induced voltage V_2 from 82V to 133.10V due to ferroresonance at supply voltage V_1 of 44.20V.
- The measured results on decreasing the supply voltage V_1 (backward direction) dictate a sudden jump down in

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ISSN: 2455-9024

primary induced voltage V_2 from 119.7V to 42.5V due to ferroresonance at supply voltage V_1 of 28.87V.

- The calculated results dictate a jump up in primary induced voltage V_2 from 80.19V to 128.73V due to ferroresonance at supply voltage V_1 of 42.59 V.
- The calculated results dictate a jump down in primary induced voltage V_2 from 115.1V to 44.86V due to ferroresonance at supply voltage V_1 of 30.42 V.

Figure 8 shows the variation of the transformer primary current, I_m, against the supply voltage V_1. Form this figure, one can conclude the following:

- The measured results on increasing the supply voltage V_1 (forward direction) dictate a sudden jump up in primary induced current I_m from 0.55A to 2.14A due to ferroresonance at supply voltage V_1 of 44.20V.
- The measured results on decreasing the supply voltage V_1 (backward direction) dictate a sudden jump down in primary induced current I_m from 1.51A to 0.23A due to ferroresonance at supply voltage V_1 of 28.87V.
- The calculated results dictate a jump up in primary induced current I_m from 0.57A to 2A due to ferroresonance at supply voltage V_1 of 42.59V.
- The calculated results dictate a jump down in primary induced current I_m from 1.44A to 0.24A due to ferroresonance at supply voltage V_1 of 30.42 V.

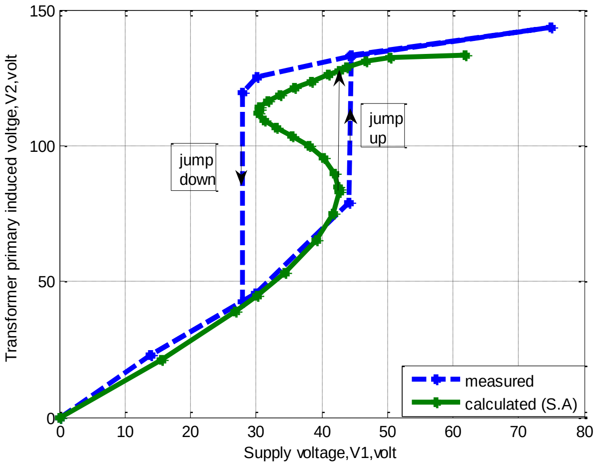

Fig. 9. The calculated and measured primary voltage, V_2, volt versus the supply voltage, V_1

The calculated and measured results of jump up and jump down voltages and currents are summarized and given in tables II and III. It is quite clear that the calculated values agreed reasonably with those measured experimentally with a deviation not exceeding 7%. This low percentage of deviation is a merit of the proposed model is characterized by simplicity with no lengthy deviation as requested by previous models [4–6].

It is worthy to mention that the two nonlinear elements R_{eq} and X_{eq} were connected in parallel in a model which failed to predict the jump–down behavior of the ferroresonance. Moreover, the predicted jump – up behavior was far from the measured one [12].

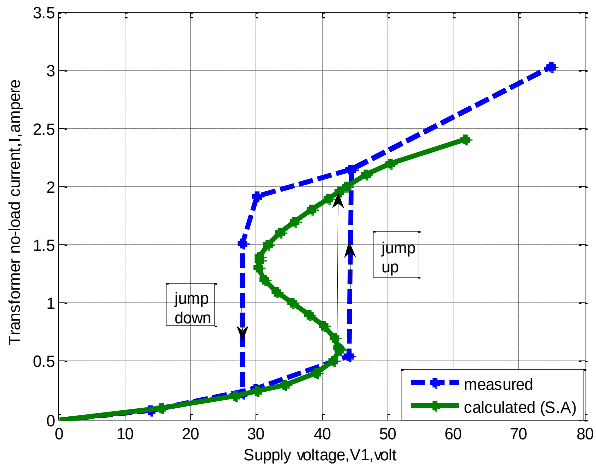

Fig. 10. The calculated and measured primary no load current, I_m, ampere versus the supply voltage, V_1.

TABLE II. Calculated and measured jump-up results

	V_1(V)	Jump-up of voltage, V_2, (V)		Jump- up of Current, I_m, (A)	
		From	**To**	**From**	**To**
Calculated (S.A)	42.59	80.19	128.73	0.57	2
Measured	44.20	82	133.1	0.55	2.14
Error	3.64%	2.2 %	3.3%	-3.6%	6.5%

TABLE III. Calculated and measured jump-down results

	V_1,(V)	Jump- down of voltage V_2, (V)		Jump- down of Current, I_m, (A)	
		From	**To**	**From**	**To**
Calculated (S.A)	30.42	115.1	44.86	1.44	0.24
Measured	28.87	119.7	42.5	1.51	0.23
Error	-5.4%	3.8%	-5.5%	4.6%	-4.3 %

VIII. CONCLUSIONS

From the measured and calculated results, one can conclude the following:

- A simplified model is proposed for ferroresonance phenomenon in single – phase transformer energized at no – load through a series capacitor. The magnetizing branch is represented by two nonlinear resistance and inductance connected in series. They are represented by fourth order polynomial functions of the magnitude of the no – load current
- A good correlation between the results of jump up and jump down points from the measured and simulated annealing Technique
- Simulated Annealing predicts with relatively small computation requirements, the critical points required for frerroresonance of power transformer
- The calculated primary voltage and current due to ferroresonance agreed with those measured experimentally with a deviation not exceeding 7%.

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